Name: _____ Period: _____

SM3 Unit 2 Factoring and Rationals Notes

2.1N – Factoring with the GCF and Grouping

Factoring: Making a polynomial into a multiplication problem. Reverse of multiplying.

Greatest Common Factor (GCF): Greatest factor can take out of all of the terms.

Prime Polynomial: Polynomial can't be factored.

Factoring Out a Common Factor:

- Find the GCF.
- Use the distributive property in reverse to "factor out" the GCF:
  - Write the GCF outside a set of parentheses.
  - Inside the parentheses, write what you are left with when you *divide* the original terms by the GCF.
  - **Note:** If the GCF is the same as one of the terms of the polynomial, there will be a 1 left inside the parentheses.
- When the leading coefficient is negative, factor out a common factor with a negative coefficient. $-x^2+3$ $-1(x^2-3)$

Examples: Factor the following expressions.

a) x^2+3x → $x(x+3)$

b) $-4n^2-20$ (leading coef. is negative) → $-4(n^2+5)$

c) $15d^2+20d^4$ → $5d^2(3+4d^2)$

d) $2a^2b^3c^4+8a^4b^8c^7-6a^3bc^5$ → $2a^2bc^4(b^2+4a^2b^7c^3-3ac)$

e) $p(q-6)+2(q-6)$ → $(q-6)(p+2)$

Factoring by Grouping (4 Terms):

1. Factor out any common factors from all four terms first.
2. Look at the first two terms and the last two terms of the polynomial separately.
3. Factor out the GCF from the first two terms, write a plus sign (or a minus sign if the GCF on the last two terms is negative), then factor out the GCF from the last two terms.
4. You should have the same thing left in both sets of parentheses after you take out the GCFs. Factor out this common binomial factor from the two groups.

Examples: Factor the following expressions.

a) $4v^3-14v^2+12v-42$ → GCF → $2(2v^3-7v^2+6v-21)$ → $v^2(2v-7)+3(2v-7)$ → $2(2v-7)(v^2+3)$

b) $15w^3z^2-20w^2z-60wz+80$ → GCF → $5(3w^3z^2-4w^2z-12wz+16)$ (negative) → $w^2z(3wz-4)-4(3wz-4)$ → $5(3wz-4)(w^2z-4)$

2.2 Factoring Trinomials

Review Examples: Multiply the following.

a) $(x+3)(x+5)$

$x^2+5x+3x+15$

$x^2+8x+15$

b) $(n-7)(n-4)$

$n^2-4n-7n+28$

$n^2-11n+28$

c) $(t-2)(t+9)$

$t^2+9t-2t-18$

$t^2+7t-18$

d) Look at your answers. How do the numbers in your answer relate to the numbers in the factors?

The 2 numbers multiply to get the last term and add to get the middle term.

Factoring a Trinomial of the Form x^2+bx+c (no leading coefficient):

1. **Always check for a GCF first!** If there is a GCF, factor it out.
2. Find two numbers that multiply to c and add to b.
3. Rewrite the middle term bx as $\mathbf{1st\ \#}\cdot x+\mathbf{2nd\ \#}\cdot x$.
4. Factor the resulting polynomial by grouping.
5. If there are no numbers that multiply to c and add to b, the polynomial is prime.

Shortcut (only works if there's no number in front of the first term).

1. Find two numbers that multiply to c and add to b.
2. The factored form of x^2+bx+c is $(x+\mathbf{1st\ \#})(x+\mathbf{2nd\ \#})$.

SHORT CUT

Examples: Factor the following polynomials.

a) $x^2+11x+30$

×	+
30	11
5	6

$(x+5)(x+6)$

b) $m^2-8m+12$

×	+
12	-8
-6	-2

$(m-6)(m-2)$

c) $-5g^2+25g-30$

$-5[g^2-5g+6]$

×	+
6	-5
-2	-3

$-5(g-2)(g-3)$

d) $t^2+6t-40$

×	+
-40	6
10	-4

$(t+10)(t-4)$

Review Examples: Multiply the following.

a) $(2x+3)(5x+4)$

$10x^2+8x+15x+12$

$10x^2+23x+12$

b) $(3v-1)(v+2)$

$3v^2+6v-v-2$

$3v^2+5v-2$

c) $(4c-3)(7c-2)$

$28c^2-8c-21c+6$

$28c^2-29c+6$

Factoring a Trinomial of the Form ax^2+bx+c by Grouping: (Leading Coefficient)

1. Always check for a GCF first! If there is a GCF, factor it out.
2. Multiply $a \cdot c$.
3. Find two numbers that multiply to your answer $(a \cdot c)$ and add to b.
4. Rewrite the middle term bx as $1\text{st}\#\cdot x + 2\text{nd}\#\cdot x$
5. Factor the resulting polynomial by grouping.
6. If there are no numbers that multiply to $a \cdot c$ and add to b, the polynomial is prime.

Examples: Factor the following polynomials.

a) $9h^2+9h+2$

×	+
18	9
6	3

$9h^2+6h+3h+2$

$3h(3h+2)+1(3h+2)$

$(3h+1)(3h+2)$

b) $2z^2-11z+12$

×	+
24	-11
-8	-3

$2z^2-8z-3z+12$

$2z(z-4)-3(z-4)$

$(2z-3)(z-4)$

GCF c) $12y^2+30y-72$

$6[2y^2+5y-12]$

×	+
-24	5
-3	8

$6[2y^2+8y-3y-12]$

$6[2y(y+4)-3(y+4)]$

$6(2y-3)(y+4)$

GCF d) $4x^2-2xy-12y^2$

$2[2x^2-xy-6y^2]$

×	+
-12	-1
-4	3

$2[2x^2-4xy+3xy-6y^2]$

$2[2x(x-2y)+3y(x-2y)]$

$2(2x+3y)(x-2y)$

Solve by factoring. (Find the x-intercepts)

a) $q^2-q-56=0$

×	+
-56	-1
-8	7

short cut

$(q-8)(q+7)=0$

$q-8=0$ $\quad q=8$

$8+7=0$ $\quad q=-7$

$(8,0)$ $\quad (-7,0)$

b) $4h^3+16h^2+12h=0$ ← short cut

GCF $4h[h^2+4h+3]=0$

$4h(h+3)(h+1)=0$

$h=0$ $\quad h=-3$ $\quad h=-1$

$(0,0)$ $\quad (-3,0)$ $\quad (-1,0)$

×	+
+3	4
3	1

not short cut

c) $4n^2-20n+25=0$

×	+
100	-20
-10	-10

$4n^2-10n-10n+25=0$

$2n(2n-5)-5(2n-5)=0$

$(2n-5)(2n-5)=0$

$2n-5=0$

$2n=5$

$n=5/2$

$\left(\frac{5}{2},0\right)$

d) $3x^2+19x=-15$

$3x^2+19x+15=0$

×	+
45	19
1	45
3	15
5	9

cant solve by factoring

2.3 Notes – Factoring Special Cases

A) Fill out the table below using the following steps:

Row 1: Write numbers 1-15

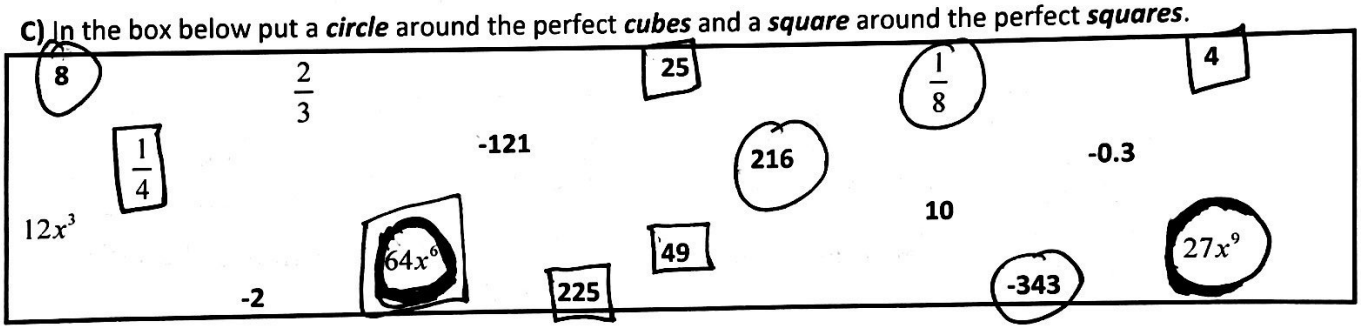
Row 2: Square the numbers from row 1

Row 3: Cube the numbers from row 1

Row 1	**Natural Numbers**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Row 2	**Perfect Squares**	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Row 3	**Perfect Cubes**	1	8	27	64	125	216	343								

B) 1. $2^3 = 8$ 2. $(-2)^3 = -8$ 3. $x^3 = x^3$ 4. $(-x)^3 = -x^3$ 5. $2x^2 = 2x^2$ 6. $(-2x)^2 = 4x^2$

C) In the box below put a ***circle*** around the perfect ***cubes*** and a ***square*** around the perfect ***squares***.



D) Multiply the following. Which one is **not** a perfect square?

1. $(x-5)(x+5)$	2. $(x+2)(x+2)$	3. $(2x-3)(2x+3)$
Dif. of squares $x^2+5x-5x-25$ x^2-25	$x^2+2x+2x+4$ x^2+4x+4	$4x^2+6x-6x-9$ Dif. of squares $4x^2-9$

E) Answer the following questions as a class.

1) If $a^2-b^2=2^2-3^2$ then $a=$ 2 and $b=$ 3 ?	**2)** If $a^2-b^2=4-9=2^2-3^2$ then $a=$ 2 and $b=$ 3 ?	**3)** If $a^2-b^2=x^2-25=x^2-5^2$ then $a=$ x and $b=$ 5 ?
4) If $a^2-b^2=(2x)^2-(3y)^2$ then $a=$ 2x and $b=$ 3y ?	**5)** If $a^2-b^2=4x^2-9y^2=(2x)^2-(3y)^2$ then $a=$ 2x and $b=$ 3y ?	**6)** If $a^2-b^2=-64c^2+d^2$ then $a=$ 8c and $b=$ d ? Factor out Negative $-(64c^2-d^2)$

F) Sum of Squares: $(x+y)^2$ or $(a+b)^2$

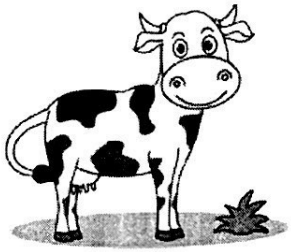
Difference of Squares: $(x+y)(x-y)$ or $(a+b)(a-b)$

G) Answer the following questions as a class.

1) If $a^3+b^3=2^3+3^3$ then $a=$ 2 and $b=$ 3 ?	**2)** If $a^3+b^3=8+27$ then $a=$ 2 and $b=$ 3 ?	**3)** If $a^3-b^3=x^3-4^3$ then $a=$ x and $b=$ 4 ?
4) If $a^3-b^3=x^3-64$ then $a=$ x and $b=$ 4 ?	**5)** If $a^3+b^3=(2x)^3+(3y)^3$ then $a=$ 2x and $b=$ 3y ?	**6)** If $a^3+b^3=8x^3+27y^3 = (2x)^3+(3y)^3$ then $a=$ 2x and $b=$ 3y ?
7)) If $a^3+b^3=125+8x^3 = 5^3+(2x)^3$ then $a=$ 5 and $b=$ 2x ?	**8)** If $a^3-b^3=y^6-216 = (y^2)^3-(6)^3$ then $a=$ y^2 and $b=$ 6 ?	**9)** If $a^3-b^3=-64c^3-d^3$ then $a=$ -4c and $b=$ d ?

H) Sum of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$



Same
Opposite
Always
Positive

I) Examples:

1) $1-9x^2$ $(1)^2-(3x)^2$ $(1+3x)(1-3x)$ Dif. of squares	**2)** $16x^2+25$ $(4x)^2+(5)^2$ sum of squares can't factor	**3)** $16x^2-25$ $(4x)^2-(5)^2$ $(4x-5)(4x+5)$ Dif. of squares	**4)** GCF $-100x^2+36$ $-4(25x^2-9)$ Dif. of squares $-4(5x+3)(5x-3)$
5) $1-x^3$ $(1)^3-(x)^3$ $(1-x)(1+x+x^2)$ dif. of cubes	**6)** m^3+8 $(m)^3+(2)^3$ $(m+2)(m^2-2m+4)$ Sum of cubes	**7)** $343-125x^3$ $(7)^3-(5x)^3$ Dif. of cubes $(7-5x)(49+35x+25x^2)$	**8)** $27a^3+8$ $(3a)^3+(2)^3$ sum of cubes $(3a+2)(9a^2-6a+4)$
9) $-125u^3+64$ $64-125u^3$ $a=4$ $b=5u$ $(4-5u)(16+20u+25u^2)$ or switch order or factor out negative	**10)** GCF $27x^4+8x$ $x(27x^3+8)$ $x((3x)^3+(2)^3)$ $x(3x+2)(9x^2-6x+4)$	**11)** $343t^3-u^3$ $(7t)^3-(u)^3$ Dif. of cubes $(7t-u)(49t^2-7tu+u^2)$	**12)** $-27a^6-y^6$ $-(27a^6+y^6)$ $-((3a^2)^3+(y^2)^3)$ $-(3a^2+y^2)(9a^4-3a^2y^2+y^4)$

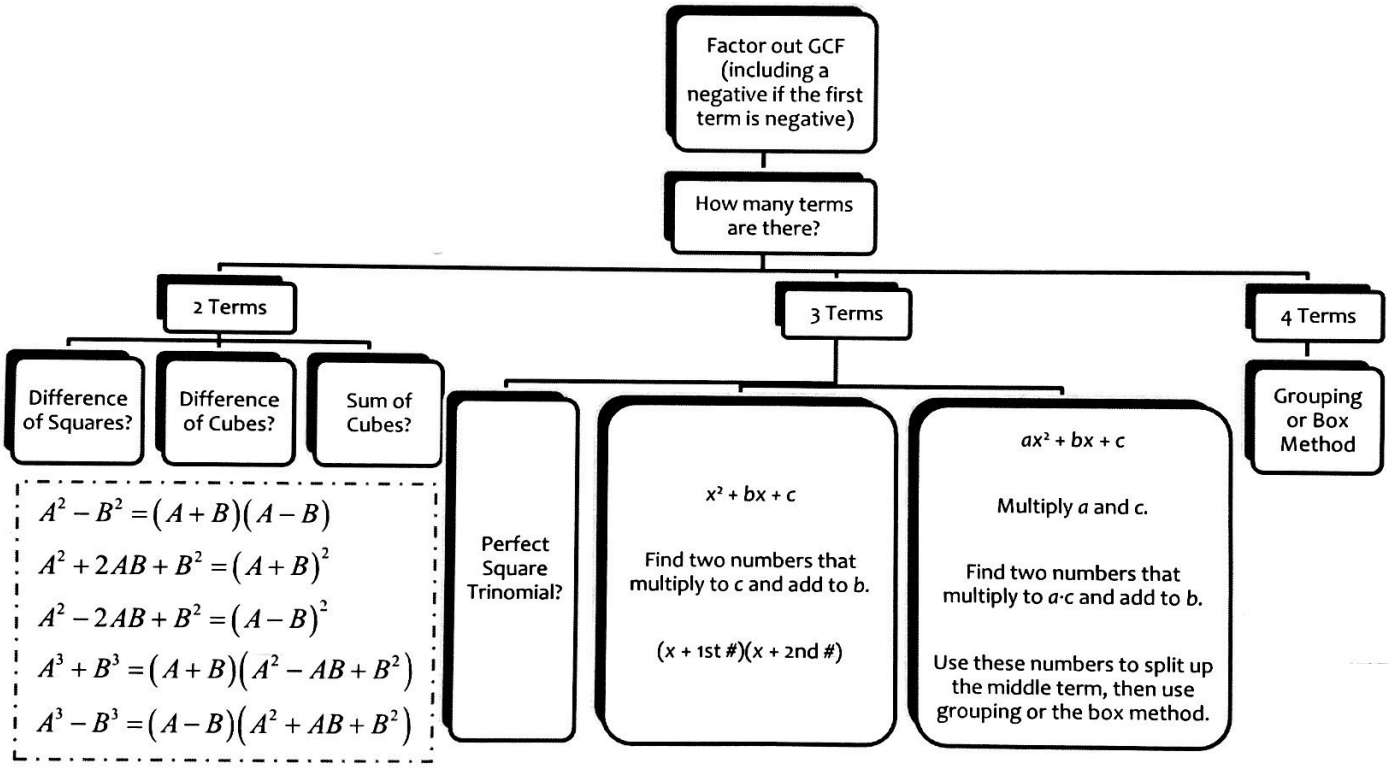
Factor out negative

$-(125u^3-64)$

$a=5u$ $b=4$

$-(5u-4)(25u^2+20u+16)$

2.4N Factoring Polynomials



SM3 Unit 4 Rational Equations Notes

2.5N – Simplifying Rational Expressions

Warm-up: Problems 16-19 on homework

Factoring Review:

1. $35r^2 - 40r$ GCF

$5r(7r-8)$

2. $m^2 + 4m - 45$ Short-cut

$(m+9)(m-5)$

3. $2v^2 + 7v + 6$ Grouping

$2v^2+3v+4v+6$

$v(2v+3)+2(2v+3)$

$(v+2)(2v+3)$

4. $3a^3 + 20a^2 - 100a$ GCF/Grouping

$a(3a^2+20a-100)$

$3a^2+30a-10a-100$

$3a(a+10)-10(a+10)$

$a(3a-10)(a+10)$

5. $9n^2 - 1$ Dif of sq.

$a=3n \quad b=1$

$(3n+1)(3n-1)$

6. $27m^3+1$ Sum of cubes

$a=3m \quad b=1$

$(3m+1)(9m^2-3m+1)$

What can you simplify? * can cancel if its multiply/not if its add

1. $\frac{6+8}{6} = \frac{14}{6} = \boxed{\frac{7}{3}}$

2. $\frac{6(8)}{6} = \boxed{8}$

3. $\frac{x+8}{x} = \boxed{\frac{x+8}{x}}$

4. $\frac{x(8)}{x} \quad \boxed{8}$

5. $\frac{x^2+8x+7}{8x+7} = \boxed{\frac{(x+1)(x+7)}{(8x+7)}}$ ← Factor, + Shortcut

6. $\frac{x^2+8x+7}{x+7} = \frac{(x+1)(x+7)}{(x+7)} \quad \boxed{x+1}$

Examples:

1. $\frac{5x^2-5x}{1-x}$ GCF, Factor out negative

$-x+1$

$\frac{5x(x-1)}{-1(x-1)} = \frac{5x}{-1}$

$= \boxed{-5x}$

2. $\frac{r-3}{r^2-6r+9}$ Short cut

$= \frac{1(r-3)}{(r-3)(r-3)}$

$\boxed{\frac{1}{r-3}}$

3. $\frac{v^2+11v+28}{v^2+15v+56}$ Short cut

$\frac{(v+7)(v+4)}{(v+8)(v+7)}$

$\boxed{\frac{(v+4)}{(v+8)}}$

4. $\frac{3b^2-20b-32}{7b-56}$ Grouping, GCF

$\frac{(3b+4)(b-8)}{7(b-8)} \quad \boxed{\frac{3b+4}{7}}$

5. $\frac{4x^2+25x-21}{4x\,4x\,16x^2-9\;\;3\cdot3}$ Grouping

$\frac{(x+7)(4x-3)}{(4x+3)(4x-3)} \quad \boxed{\frac{x+7}{4x+3}}$

6. $\frac{n^3-64}{3n^2+12n+48}$ Dif. of cubes, GCF

$\frac{3(n^2+4n+16)}{(n-4)(n^2+4n+16)}$

$= \boxed{\frac{n-4}{3}}$

GCF

$3b^2-20b-32$

$3b^2-24b+4b-32$

$3b(b-8)+4(b-8)$

$(3b+4)(b-8)$

x	+
-96	-20
1	96
2	48
3	32
4	-24

$4x^2+25x-21$

$4x^2-3x+28x-21$

$x(4x-3)+7(4x-3)$

$(x+7)(4x-3)$

x	+
-84	25
1	84
2	82
-3	28

n^3-64

$a=n \quad b=4$

SOAP

$(n-4)(n^2+4n+16)$

x	+
16	4

2.6N – Adding/Subtracting Rational Expressions

* TO FIND common denominator you must factor denominators FIRST

A. Finding the least common denominator:

1. $\frac{3\cdot\frac{2}{3\cdot 7}+\frac{4}{21}}{}$ LCD = 220

$\frac{6}{21}+\frac{4}{21}=\boxed{\frac{10}{21}}$

2. $\frac{2\cdot\frac{3}{2\cdot 5}-\frac{1\cdot 5}{2\cdot 5}}{}$

$\frac{6}{10}-\frac{5}{10}=\boxed{\frac{1}{10}}$

3. $\frac{2\cdot\frac{3}{2\cdot 10}+\frac{5\cdot 5}{4\cdot 5}}{}$

$\frac{6}{20}+\frac{25}{20}=\boxed{\frac{31}{20}}$

4. $\frac{3x\cdot\frac{3}{3x\cdot 7x}-\frac{4}{21x^2}}{}$ LCD = $21x^2$

$\frac{9x}{21x^2}-\frac{4}{21x^2}$

$\boxed{\frac{9x-4}{21x^2}}$

5. $\frac{(v+1)\frac{4}{(v+1)(v+4)}+\frac{5(v+4)}{(v+1)(v+4)}}{}$ LCD: $(v+4)(v+1)$

$\frac{4v+4}{(v+1)(v+4)}+\frac{5v+20}{(v+1)(v+4)}$

$\boxed{\frac{9v+24}{(v+1)(v+4)}}$ OR $\boxed{\frac{3(3v+8)}{(v+1)(v+4)}}$

6. $\frac{2x}{5x+5}\overset{\text{GCF } 5(x+1)}{-}\frac{5x\cdot 5}{(x+1)5}$ LCD = $5(x+1)$

$\frac{2x}{5(x+1)}-\frac{25x}{5(x+1)}$

$\boxed{\frac{-23x}{5(x+1)}}$

7. $\frac{4v}{6v-6}+\frac{6v}{v+6}$

Factor denominators

short cut

-16 | $+6$

-8 | 2

8. $\frac{2r(r+8)}{r^2-6r-16}\overset{\text{GCF}}{-}\frac{5(r-8)}{2r^2+4r}$

$2r(r-8)(r+2)$ $\quad 2r(r+2)(r-8)$

LCD: $2r(r-8)(r+2)$

$\frac{2r^2+16r}{2r(r-8)(r+2)}+\frac{-5r+40}{2r(r-8)(r+2)}$

$\boxed{\frac{2r^2+11r+40}{2r(r-8)(r+2)}}$

x | $+$

80 | 11

16 | 5

9. $\frac{3\cdot 5}{3x+12}+\frac{7}{9x+36}$ LCD: $9(x+4)$

$3(x+4)$ $\quad 9(x+4)$

$\frac{15}{9(x+4)}+\frac{7}{9(x+4)}$

$\boxed{\frac{22}{9(x+4)}}$

10. $\frac{2(x-6)}{x^2+11x+30}+\frac{-4(x+5)}{x^2-36}$

LCD: $(x+5)(x+6)(x-6)$

$+$ | 11

5 | 6

$(x-6)(x+5)(x+6)$

$4x$ $\quad 66$

$(x+6)(x-6)(x+5)$

$\frac{2x-12}{(x+5)(x+6)(x-6)}+\frac{-4x-20}{(x+6)(x-6)(x+5)}$

$\boxed{\frac{-2(x+16)}{(x+5)(x+6)(x-6)}}$ or $\boxed{\frac{2(x+16)}{(x+5)(x+6)(x-6)}}$

11. $\frac{x-2}{2x^2+x-10}+\frac{(x+4)(x-2)}{2x+5}$ LCD = $(2x+5)(x-2)$

$(2x+5)(x-2)$ $\quad (x-2)$

$\frac{(x-2)}{(2x+5)(x-2)}+\frac{x^2+2x-8}{(2x+5)(x-2)}$

short cut $x^2+3x-10$

-10 | $+3$

5 | -2

$(2x+5)(x-2)$

Grouping $2x^2+x-10$ $\quad -20$ | x | $+1$ | -4

$2x^2+5x-4x-10$

$x(2x+5)-2(2x+5)$

$(2x+5)(x-2)$

$\frac{(x+5)(x-2)}{(2x+5)(x-2)}$

$\boxed{\frac{x+5}{2x+5}}$

$(x+4)(x-2)$

$x^2-2x+4x-8$

x^2+2x-8

B. Go back through each problem and perform the indicated operation. Don't forget to simplify when possible.

2.7 N – Multiply and Divide Rational Expressions

Lesson 4.1

A. Practice Simplifying

1. $\frac{14}{35} \div \frac{7}{7} = \frac{2}{5}$

2. $\frac{12x^4}{14x^3} = \frac{6}{7x}$

3. $\frac{8x^3y^2z}{15xy^2z^5} = \frac{x^2}{3z^4}$

4. $\frac{3x^2+27x+42}{18x^2+9x-54}$ must be factored to cancel

$3(x^2+9x+14)$ ← short cut

$9(2x^2+x-6)$ ← GCF 9 ← Grouping

$\frac{3(x+7)(x+2)}{9(2x-3)(x+2)} = \frac{x+7}{3(2x-3)}$

$2x^2+x-6$

$2x^2+4x-3x-6$

$2x(x+2)-3(x+2)$

$(2x-3)(x+2)$

B. Practice multiplication review

1. $\frac{3}{4} \cdot \frac{20}{9} = \frac{60}{36} = \frac{5}{3}$

2. $\frac{3}{14} \cdot \frac{7}{27} = \frac{1}{9}$

Factor First Then simplify

3. $\frac{9y^3}{25y^2} \cdot \frac{8y^{5-2}}{18x^3} = \frac{y^3}{10x^2}$

4. $\frac{9v-81}{10v^2} \cdot \frac{10}{v^2-81}$ new $9(v-9)$

$\frac{9(v-9)}{10v^2} \cdot \frac{10}{(v+9)(v-9)} = \frac{9}{v^2(v+9)}$

5. $\frac{n^2-12n+35}{n-5} \cdot \frac{n+4}{n-7}$ short cut $35 \mid -12$, $-7, -5$

$\frac{(n-7)(n-5)}{(n-5)} \cdot \frac{(n+4)}{(n-7)} = n+4$

6. $\frac{5x+45}{7x-7} \cdot \frac{21x^2+14x-35}{15x+25}$

$5(x+9)$ $7(3x^2+2x-5)$ GCF

$7(x-1)$ $5(3x+5)$

$\frac{5(x+9)}{7(x-1)} \cdot \frac{7(x-1)(3x+5)}{5(3x+5)} = x+9$ Grouping $-15 \mid 2$, $5, -3$

$3x^2+2x-5$

$3x^2-3x+5x-5$

$3x(x-1)$ $5(x-1)$

$(x-1)(3x+5)$

C. Practice Division ← same as multiply by the reciprocal

1. $\frac{5}{18} \div \frac{15}{9}$

$\frac{5}{18} \cdot \frac{9}{15} = \frac{1}{6}$

2. $\frac{35x^2}{x+10} \div \frac{2x-10}{x^2+5x-50}$ ← mult. by reciprocal

$\frac{35x^2}{x+10} \cdot \frac{x^2+5x-50}{2x-10}$ ← short cut $-50 \mid 5$, $10, -5$

$\frac{35x^2}{(x+10)} \cdot \frac{(x+10)(x-5)}{2(x-5)} = \frac{35x^2}{2}$

3. $\frac{7x^2+35x+28}{x+1} \div \frac{x^2-16}{x^2+6x-7}$ Flip it

$7(x^2+5x+4)$ short cut $4 \mid 5$, $4, 1$

x^2+6x-7 $-7 \mid 6$, $7, -1$

x^2-16

$\frac{7(x+4)(x+1)}{(x+1)} \cdot \frac{(x+7)(x-1)}{(x+4)(x-4)} = \frac{7(x+7)(x-1)}{(x-4)}$