11.5N – Properties of Logarithms

- ***Remember:*** *Definition of Logarithm:* $y = \log_a x \Leftrightarrow a^y = x$

$\log_a x = y$

A. Properties of Logarithms

For any positive numbers M, N, and a, where $a \neq 1$ and r is any real number:

If $a^0 = 1$ then
- $\log_a 1 =$ 0 (rewrite as log)

If $a^1 = a$ then
- $\log_a a =$ 1 (same)

If $a^M = \log_a M$ then
- $a^{\log_a M} =$ M (same)

If $a^r = a^r$ then
- $\log_a a^r =$ r (same)

- $\log_a(MN) = \log_a M + \log_a N$ (Exponent: $x^M \cdot x^N = x^{M+N}$)
- $\log_a M^r = r \log_a M$ (Exponent: $(x^m)^n$ mult. exp.)
- $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ (Exponent: $\frac{x^M}{x^N}$)
- $\log_a M = \log_a N \Leftrightarrow M = N$ (Exponent: $a^m = a^n$, $m = n$)

* ***Change of Base Formula:***

- $\log_a M = \frac{\log_b M}{\log_b a}$
- $\log_a M = \frac{\log M}{\log a}$
- $\log_a M = \frac{\ln M}{\ln a}$

Examples: Find the exact value of each expression. (Do not use a calculator).

a) $\log_4 1 = 0$

b) $5^{\log_5 3} = 3$ (same)

c) $\log_7 7^{-1} = -1$ (same)

d) $\ln e^1 = 1$ (same)

e) $\log_2 64$
$\log_2 2^6 = 6$ (same)

f) $\log_7 \frac{1}{\sqrt[3]{49}}$
$\log_7 \frac{1}{49^{1/3}}$
$\log_7 (7^2)^{-1/3}$
$\log_7 7^{-2/3}$ (same) $-2/3$

Expand

Examples: Write each expression as a *sum/difference* of logarithms. Express powers as factors.

a) $\log 5x$
$\log 5 + \log x$

b) $\ln \frac{3}{x}$
$\ln 3 - \ln x$

c) $\log_7(x^5)$
$5 \log_7 x$

d) $\ln(x^2 e^x)$
$\ln x^2 + \ln e^x$
$2 \ln x + x \ln e$

e) $\log \frac{\sqrt[4]{x}}{\sqrt[4]{y}} = \log \frac{x^{1/4}}{y^{1/4}}$
$\log x^{1/4} - \log y^{1/4}$
$\frac{1}{4} \log x - \frac{1}{4} \log y$

f) $\ln \frac{y^4}{x^5} = \ln y^4 - \ln x^5$
$4 \ln y - 5 \ln x$

Condense it

Examples: Write each expression as a single logarithm.

a) $\ln 8 + \ln x$ → $\ln(8x)$

b) $\log u - \log v$ → $\log\left(\frac{u}{v}\right)$

c) $\frac{1}{4}\log x$ → $\log(x^{1/4})$

d) $\log_7 u + 3\log_7 v$ → $\log_7(uv^3)$

e) $4\ln(uv) - 3\ln(vw)$ → $\ln\left(\frac{(uv)^4}{(vw)^3}\right)$ → $\ln\left(\frac{u^4v}{w^3}\right)$

f) $\log(x-4)(6x+5)$ FOIL it

$\log(x-4)(6x+5)$

$\log(6x^2+5x-24x-20)$

$\log(6x^2-19x-20)$

Examples: Use the *change of base* formula to evaluate each logarithm.

a) $\log_6 9 = \frac{\log 9}{\log 6} = 1.2263$

b) $\log_{\sqrt{2}} 7 = \frac{\log 7}{\log\sqrt{2}} = 5.6147$

c) $\log_\pi \sqrt{3} = \frac{\log\sqrt{3}}{\log\pi} = .4799$

Examples: Write the expression using only *natural logarithms*. ln

a) $\log_7 30 = \frac{\ln 30}{\ln 7} = 1.7479$

b) $\log_4 10 = \frac{\ln 10}{\ln 4} = 1.6610$

Examples: Write the expression using only *common logarithms*. log change of base

a) $\log_6 y = \frac{\log y}{\log 6}$

b) $\log_2(d+e) = \frac{\log(d+e)}{\log 2}$

Examples: Use properties of logarithms to find the exact value of each expression. (Do not use a calculator).

a) $\log_7 21 - \log_7 3$

$\log_7\left(\frac{21}{3}\right)$

$\log_7(7)$

$\log_7 7^1$

$\boxed{1}$

b) $5^{\log_5 6 + \log_5 7}$

$5^{\log_5(6\cdot 7)}$

$5^{\log_5 42}$

$\boxed{42}$

c) $\log_4 11 \cdot \log_{11} 256$

different bases

can't combine