ln ← base is e
log ← base is 10

11.4N – Solving Logarithmic Equations

A. Review

Change each logarithmic statement into an equivalent exponential statement.

1. $\log_8 64 = 2$ $8^2 = 64$
2. $\log_2 \frac{1}{16} = -4$ $2^{-4} = \frac{1}{16}$
3. $\log 8 = x$ $10^x = 8$
4. $\ln x = 5$ $e^5 = x$

Change each exponential statement into an equivalent logarithmic statement.

1. $4^x = 27$ ← argument (exp, base) $\log_4 27 = x$
2. $3^{-4} = \frac{1}{81}$ $\log_3 \frac{1}{81} = -4$
3. $9^x = 3.2$ $\log_9 3.2 = x$

Solve the following equation using the laws of exponents.

1. $16^{m+2} = 64$
   $(4^2)^{m+2} = 4^3$
   $2(m+2) = 3$
   $2m + 4 = 3$
   $2m = -1$
   $\boxed{m = -1/2}$
2. $9^{-3n} = 243$
   $(3^2)^{-3n} = 3^5$
   $2(-3n) = 5$
   $\frac{-6n}{-6} = \frac{5}{-6}$
   $\boxed{n = -5/6}$

B. Solving Logarithmic and Exponential Equations

- Use the properties of logarithms and exponents to manipulate the equations.
  a. Remember the *exponential* property: $a^u = a^v \Leftrightarrow u = v$.
- Try rewriting as an *exponential* function: $y = \log_a x \Leftrightarrow x = a^y$ or as a *logarithmic* equation: $x = a^y \Leftrightarrow y = \log_a x$

Examples: rewrite in exp. form | rewrite in log form

a) $\log_{18} 324 = x$

$18^x = 324$
$18^x = 18^2$
$\boxed{x = 2}$

b) $6^{x-4} = 11$

$\log_6 11 = x - 4$
$+4 \quad +4$
$\boxed{x = \log_6 11 + 4}$

c) $\ln e^{2x} = 6$

$e^6 = e^{2x}$
$6 = 2x$ $\boxed{x = 3}$

d) $\frac{3 \cdot (10)^{3-x}}{3} = \frac{7}{3}$

$(10)^{3-x} = \frac{7}{3}$
rewrite as log
$\log_{10} \frac{7}{3} = 3 - x$
$-3 \quad -3$
$+x = \log \frac{7}{3} + 3$
$\boxed{x = -\log \frac{7}{3} + 3}$

e) $\log_3(3x-1) = 2$

$3^2 = 3x - 1$
$9 = 3x - 1$
$\frac{10}{3} = \frac{3x}{3}$
$\boxed{x = \frac{10}{3}}$

f) $2^{-x} = 1.5$

$-\log_2 1.5 = +x$
$\boxed{x = -\log_2 1.5}$

g) $\log_6 216 = (3x+2)$

$6^{3x+2} = 216$
$6^{3x+2} = 6^3$
$3x + 2 = 3$
$3x = 1$
$\boxed{x = 1/3}$

h) $e^{4x+3} = 9$

base e so use ln instead of log

$\ln 9 = 4x + 3$
$-3 \quad -3$
$\frac{-3 + \ln 9}{4} = \frac{4x}{4}$
$\boxed{x = \frac{-3 + \ln 9}{4}}$ or $\boxed{x = -.2}$