11.3N – Logarithmic Functions

A. Finding the inverse of a logarithmic function.

- **$\log_2 x$ means "the exponent to which we raise 2 to get x."**
  Pronounced "the logarithm, base 2, of x" or "log, base 2, of x"

★ LOGARITHMS ARE EXPONENTS! ★

- ***Logarithm:*** $\log_b a$ means the ***exponent*** to which we raise b to get a.
  b is called the ***base*** of the logarithm (the number being raised to the exponent).
  a is called the ***argument*** of the logarithm (the number you get when you raise the base to the exponent).

The ***logarithmic function of base b***, where $b > 0$ and $b \neq 1$ is denoted by $y = \log_b x$ and is defined by

exponent $y = \log_b x$ if and only if $x = b^y$.

base

Example: Change each exponential expression to an equivalent expression involving a logarithm.

a) $5^x = 625$ answer

base $\log_5 625 = x$ exp. base

b) $x^3 = 64$

$\log_x 64 = 3$

c) $3^2 = x$

$\log_3 x = 2$

Example: Change each logarithmic expression to an equivalent expression involving an exponent.

a) $\log_3 x = 5$

$3^5 = x$

b) $\log_e 5 = x$

$e^x = 5$

c) $\log_m 2 = n$

$m^n = 2$

B. Evaluating Logarithms

- Instead of "$\log_2 8 = x$," think, what power of 2 equals 8? Or 2 to what power equals 8?
  - $2^x = 8$
  - The answer would be 3 because $2^3 = 8$.

Example: Find the exact value of each logarithm without using a calculator. rewrite & solve

a) $\log_3 9 = x$ ans. exp. base

$3^x = 9$
$3^x = 3^2$
$x = 2$

b) $\log_2 32 = x$

$2^x = 32$
$2^x = 2^5$
$x = 5$

c) $\log_6 1 = x$

$6^x = 1$
$x = 0$
anything to zero power is 1

d) $\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$
get bases same
$5^x = \frac{1}{5^3}$
$5^x = 5^{-3}$
$x = -3$

e) $\log_7 \sqrt{7} = x$

$7^x = \sqrt[2]{7}$
$7^x = 7^{1/2}$
$x = 1/2$

C. Inverses of exponential functions.

- The logarithmic function $y = \log_a x$ is the inverse of the exponential function $a^y = x$
- Domain $y = a^x$: $(-\infty, \infty)$ Range $y = a^x$: $(0, \infty)$
- Domain $y = \log_a x$: $(0, \infty)$. Range $y = \log_a x$: $(-\infty, \infty)$

Domain of the logarithmic function =

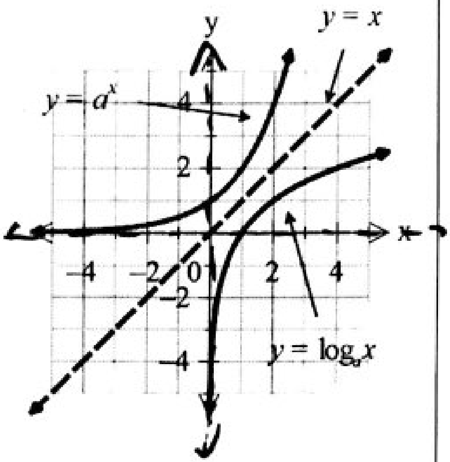
Range of the exponential function $= (0, \infty)$

Range of the logarithmic function =

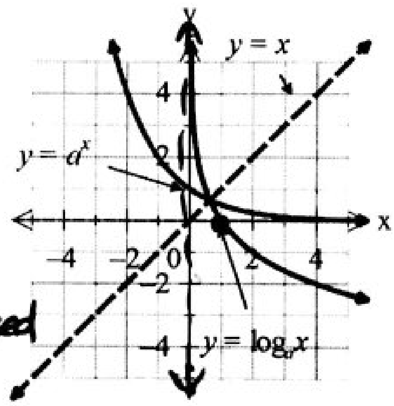
Domain of the exponential function $= (-\infty, \infty)$

★ **Caution!** You can't take the log of zero or of a negative because it is impossible to get zero or a negative by raising a positive base to an exponent. **The argument of a logarithmic function must be greater than zero.**

$a > 1$



$0 < a < 1$



Properties of the Logarithmic Function $f(x) = \log_a x$

- The x-intercept is 1. There is no y-intercept.
- The vertical asymptote of the graph is $x = 0$.
- The logarithmic function is decreasing if $0 < a < 1$ and increasing if $a > 1$. The function is one-to-one.
- Since $y = \log_a x$ is the inverse of $y = a^x$ and the graph $y = a^x$ contains the points $\boxed{\left(-1, \frac{1}{a}\right), (0,1), \text{ and } (1, a)}$ then the graph of $y = \log_a x$ contains the points

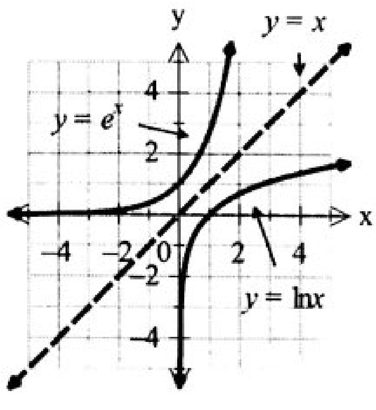
Key points reversed

$(1/a, -1), (1, 0)$, and $(a, 1)$.

Common Logarithmic Function: If the base of a logarithmic function is the number 10, then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is, $y = \log_{10} x$ if and only if $x = 10^y$.

Natural Logarithms: If the base of a logarithmic function is the number e, then we have the natural logarithm function (abbreviated ln). That is, $y = \ln x$ if and only if $x = e^y$.

e



- $y = \ln x$ is the inverse of $y = e \wedge x$

=e

$e^y = x$

inverses

To Find Domain set argument > 0

D. Finding the domain of logarithmic functions.

1. $f(x) = \log_2(x+3)$ ← argument
   $x+3 > 0$
   $-3 \quad -3$
   $x > -3$
   Domain $\boxed{(-3, \infty)}$

2. $h(x) = -\log_3 x$
   $x > 0$
   $\boxed{(0, \infty)}$
   left to right

3. $g(x) = \ln(-x-5)$
   $-x-5 > 0$
   $+5 \quad +5$
   $\frac{-x}{-1} > \frac{5}{-1}$
   $x < -5$
   $\boxed{(-\infty, -5)}$

E. Graphing logarithmic functions.

Steps for Graphing Logarithmic Functions:

1. Find the domain
2. Find the asymptotes
3. Graph the asymptotes
4. Find the 3 key points $\boxed{(1,0),\ (a,1), \text{ and } \left(\frac{1}{a}, -1\right)}$ and apply the appropriate transformations. *
5. Plot your points and connect them to form a smooth curve.
6. Find the range

Key points

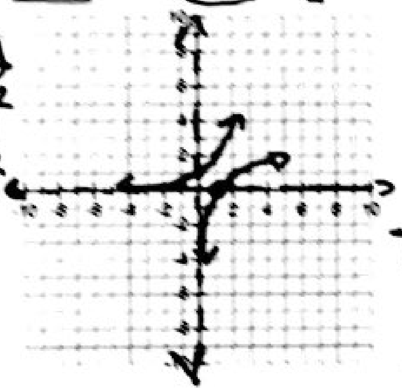
x	f(x)
1/a	-1
1	0
a	1

log ← base 10
ln ← base e

Examples: Graph the following functions.

a) $y = 2^x$ and $y = \log_2 x$ inverse $2^y = x$

x	y
-1	1/2
0	1
1	2



Domain: $(0, \infty)$

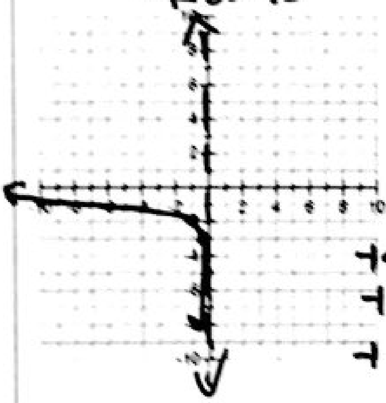
Asymptotes: $x = 0$

Key points and transformations:

x	y
1/2	-1
1	0
2	1

Range: $(-\infty, \infty)$

b) $y = \log(-x) - 2$ 10 ← a=10



Domain: $(-\infty, 0)$

Asymptotes: $x = 0$

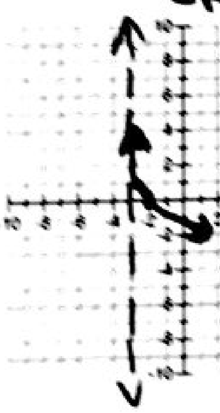
Key points and transformations: − reflect over y axis, − down 2

x	y
-1·1/10	-1 -2
-1·1	0 -2
-1·10	1 -2

x	y
-1/10	-3
-1	-2
-10	-1

Range: $(-\infty, \infty)$

c) $f(x) = -\ln(x+3)$ base e ← a=e



Domain: $(-3, \infty)$

Asymptotes: $x = -3$

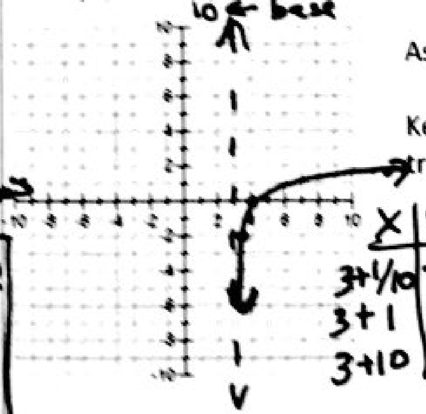
Key points and transformations: reflect x axis, left 3

x	y
-3 + 1/e	-1 · -1
-3 + 1	0 · -1
-3 + e	1 · -1

x	y
-2.63	1
-2	0
-.28	-1

Range: $(-\infty, \infty)$

d) $f(x) = 2\log(x-3)$ 10 ← base



Domain: $(3, \infty)$

Asymptotes: $x = 3$

Key points and transformations: mult. y's 2, right 3

x	y
3+1/10	-1·2
3+1	0·2
3+10	1·2

x	y
3 1/10	-2
4	0
13	2

Range: $(-\infty, \infty)$