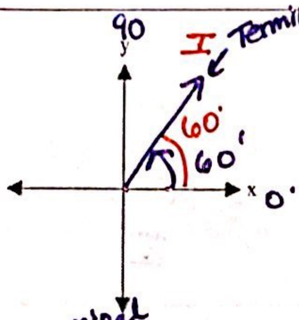
## 6.1 N – Coterminal and Reference Angles in Degrees

Red : Reference angle
Blue : Given angle

**A.** Drawing Angles

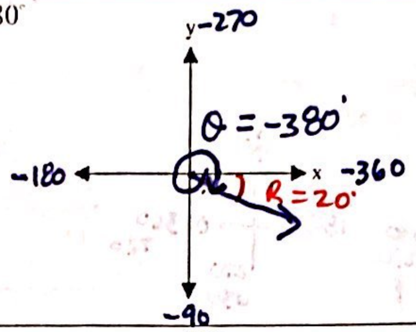
- label the *terminal* side of the angle
- Find the *reference angle*. (A **reference angle** is formed by the *terminal side* of the angle and the *x-axis*. This means the angle will always be less than 90°.). Always be positive

1. 60°

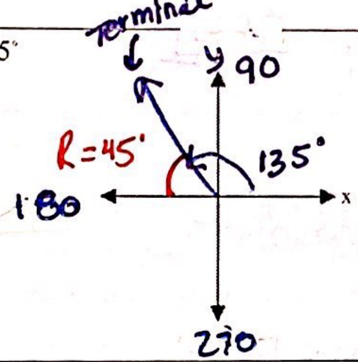



If angle is in First Quadrant the ref. ∠ and the given angle are the same

2. −380°

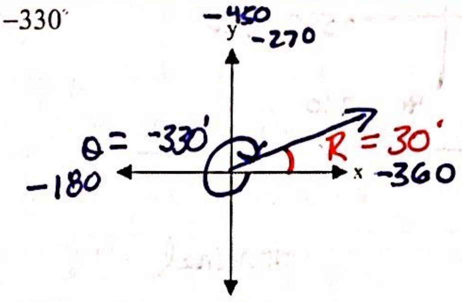



3. 135°

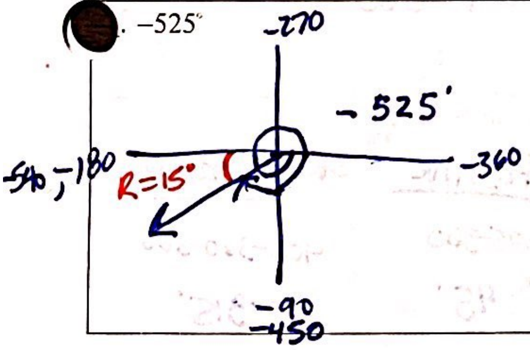



$$\begin{array}{r} 180 \\ -135 \\ \hline 45 \end{array}$$

4. −330°



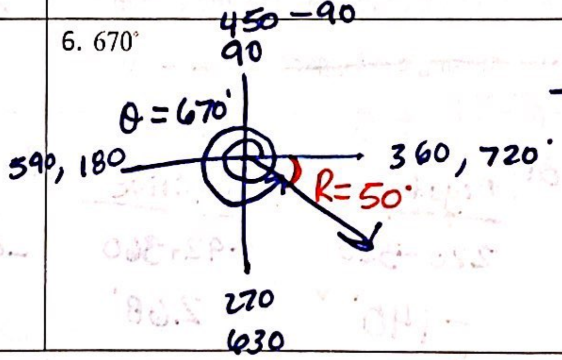

5. −525°




Ref. ∠

$$\begin{array}{r} 540 \\ -525 \\ \hline 15° \end{array}$$

6. 670°

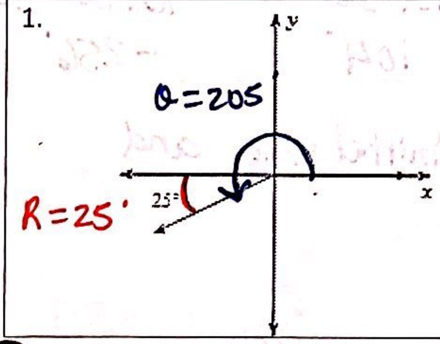



$$\begin{array}{r} 720 \\ -670 \\ \hline 50° \end{array}$$

**B.** Measures of Angles

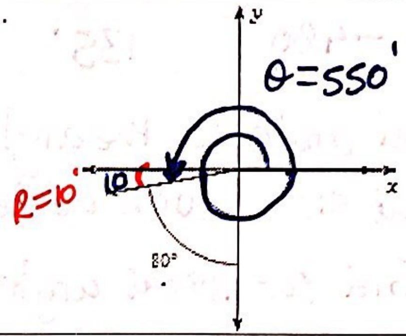
Find the measure of each angle and then find the reference angle.

1.

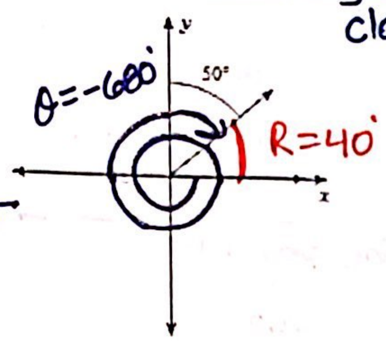
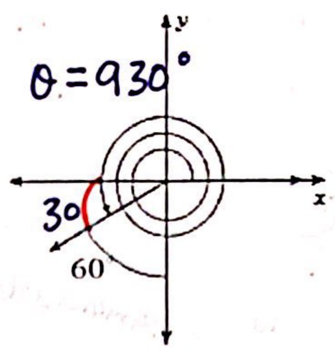



$$\begin{array}{r} 180 \\ +25 \\ \hline 205° \end{array}$$

2.



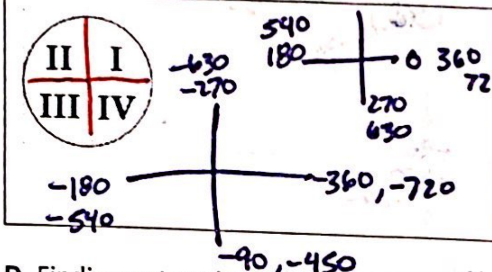

$$\begin{array}{r} 360 \\ +180 \\ 10 \\ \hline 550° \end{array}$$

| | |
|---|---|
| 3. negative clockwise<br>360<br>180<br>90<br>50<br>—<br>-680°<br><br>360<br>360<br>180<br>30<br>—<br>930° | 4.<br> |





**C.** Determine the quadrant.





| | |
|---|---|
| 1. 480° II | 2. 405° I |
| 3. −420° IV | 4. −256 II |

**D.** Finding coterminal angles between 0° and 360°.

- Watch Youtube video: (Stop at 2:40) https://www.youtube.com/watch?v=dz5YpNFuhRQ

Coterminal angles share the ___terminal___ side of the angle.

What happens when you go around a circle more than once? add or subtract 360°

~~How do you know if the angle is bigger than 360°?~~

| | | |
|---|---|---|
| 1. 580° Positive OR negative<br>580-360 220-360<br>220° -140° | 2. −92° Positive OR negative<br>-92+360 -92-360<br>268° -452° | 3. 405° Positive OR Negative<br>405-360 405-360-360<br>45° -315° |
| 4. −120° Positive OR negative<br>-120+360 -120-360<br>240° -480° | 5. −225° Positive OR Negative<br>-225+360 -225-360<br>135° -585° | 6. 464° Positive OR Negative<br>464-360 464-360-360<br>104° -256° |

Coterminal angle is the angle with same initial side and terminal side with added rotations.

How to find coterminal angles.

- add 360 as many times as needed to get positive
- subtract 360 as many times as needed to get negative

6.2N

# V – Coterminal and Reference Angles in Radians

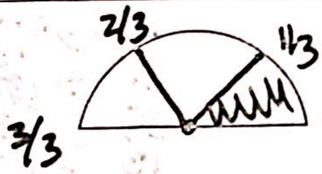
$\pi$ 3.14


1/2 way around circle is $\pi$

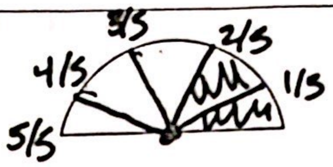
Full circle $2\pi$

**A.** Shade the appropriate portion of the semi-circle.

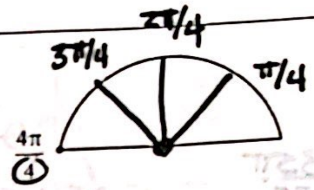
1. $\frac{1}{3}$



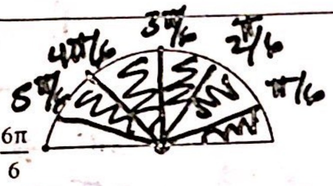

2. $\frac{2}{5}$



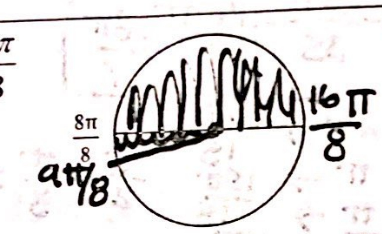

3. $\frac{\pi}{4}$



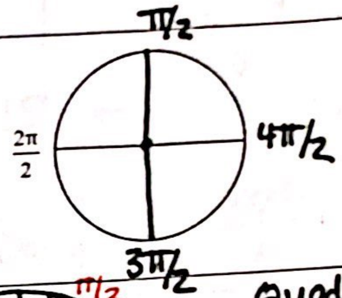

4. $\frac{5\pi}{6}$

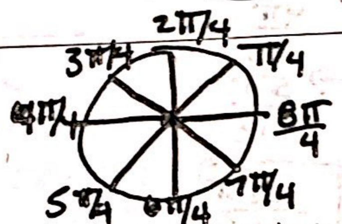


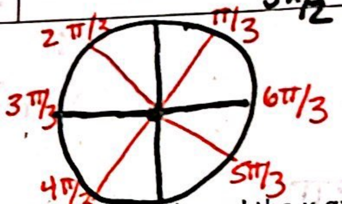

5. $\frac{9\pi}{8}$

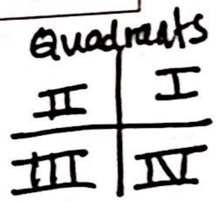



6. $\frac{3\pi}{2}$



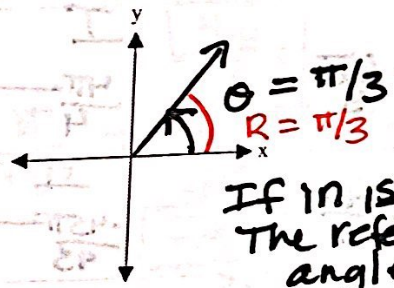




**B.** Drawing Angles

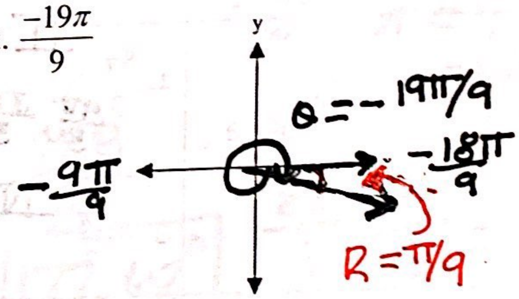
- label the *terminal* side of the angle
- Find the *reference angle*. (A **reference angle** is formed by the *terminal side* of the angle and the *x-axis*. This means the angle will always be less than 90°.) Always positive

1. $\frac{\pi}{3}$

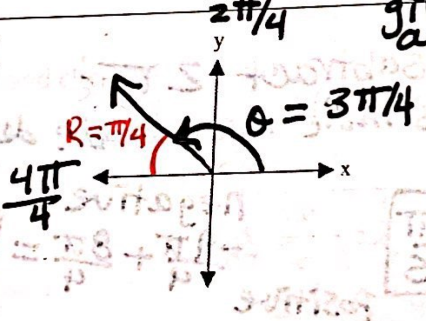



If in 1st Quad. The reference angle and given angle are the same

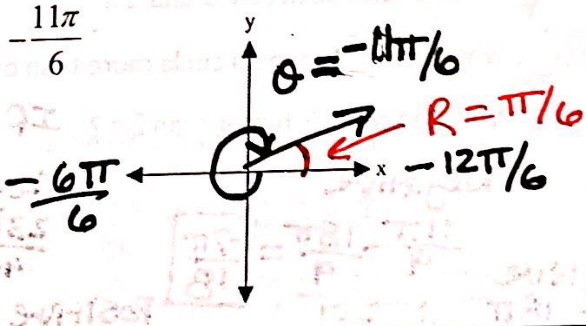
2. $\frac{-19\pi}{9}$



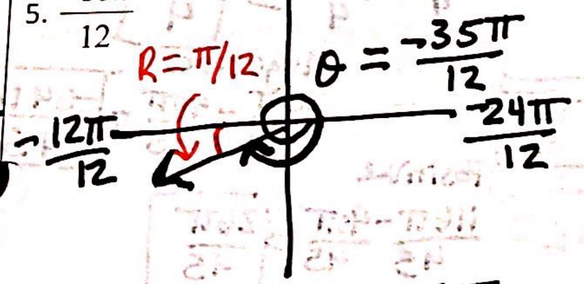

3. $\frac{3\pi}{4}$




4. $-\frac{11\pi}{6}$

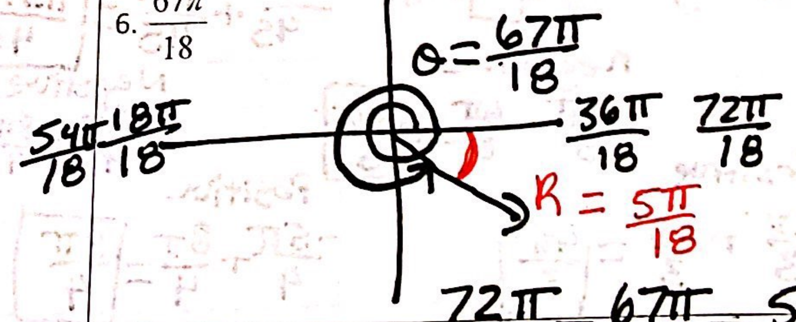



5. $\frac{-35\pi}{12}$




$$R = \frac{36\pi}{12} - \frac{35\pi}{12} = \frac{\pi}{12}$$

6. $\frac{67\pi}{18}$

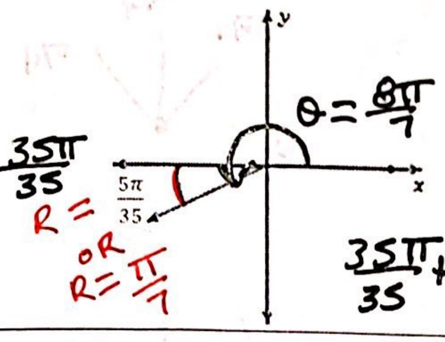



$$\frac{72\pi}{18} - \frac{67\pi}{18} = \frac{5\pi}{18}$$

**C.** Measures of Angles $90° = \pi/2$

Find the measure of each angle and then find the reference angle. 180° → 0° 90° $\frac{2\pi}{2}$ $\pi/2$

1.

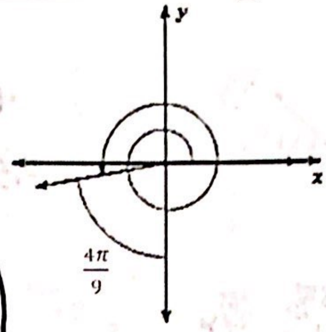


$\frac{35\pi}{35}$ $\theta = \frac{8\pi}{7}$

R = $\frac{5\pi}{35}$ or R = $\frac{\pi}{7}$

$\frac{35\pi}{35} + \frac{5\pi}{35} = \boxed{\frac{40\pi}{35} = \frac{8\pi}{7}}$

2.



or

$2\pi \to \frac{36\pi}{18}$

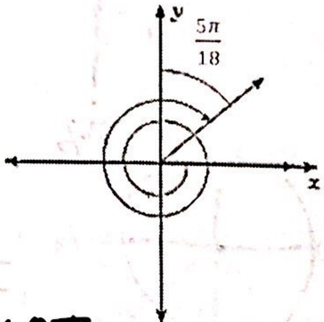
$\pi \to \frac{18\pi}{8}$

$\frac{\pi}{18} \to \frac{\pi}{18}$

$+ \frac{\pi}{18}$

$\boxed{\frac{55\pi}{18}}$

3.



$2\pi \to \frac{36\pi}{18}$

$\pi \to \frac{18\pi}{18}$

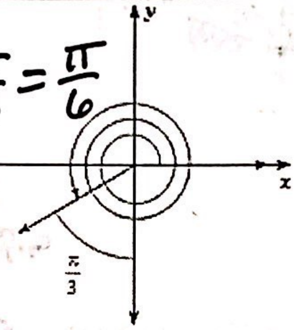
$\frac{\pi}{2} \to \frac{9\pi}{18}$

$+ \frac{5\pi}{18} \to \frac{5\pi}{18}$

$\boxed{\frac{68\pi}{18}}$

$-\frac{68\pi}{18}$ or $-\frac{34\pi}{9}$

4.



$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$2\pi \to \frac{12\pi}{6}$

$2\pi \to \frac{12\pi}{6}$

$\pi \to \frac{6\pi}{6}$

$\frac{\pi}{6} \to \frac{\pi}{6}$

$\boxed{\frac{31\pi}{6}}$

**D.** Determine the quadrant.

| | | |
|---|---|---|
| II \| I / III \| IV | 1. $\frac{8\pi}{3}$ II $\frac{9\pi}{3}, \frac{3\pi}{3}$ 7.5 $\frac{6\pi}{3}$ | 2. $\frac{9\pi}{4}$ I $\frac{4\pi}{4}$ $\frac{8\pi}{4}$ |
| | 3. $-\frac{7\pi}{3}$ IV $-\frac{3\pi}{3}$ $-\frac{6\pi}{3}$ | 4. $\frac{-64\pi}{45}$ II 67.5 $-\frac{45\pi}{45}$ $-\frac{90\pi}{45}$ |

**E.** Finding *coterminal angles* between $0$ and $2\pi$.

What happens when you go around a circle more than once? add or subtract $2\pi$

How do you know if the angle is bigger than $2\pi$ ? If numerator is more than double denominator

| | | |
|---|---|---|
| 1. $\frac{29\pi}{9}$ Negative $\frac{11\pi}{9} - \frac{18\pi}{9} = \boxed{\frac{-7\pi}{18}}$ Positive $\frac{29\pi}{9} - \frac{18\pi}{9} = \boxed{\frac{11\pi}{9}}$ | 2. $\frac{23\pi}{45}$ Negative $\frac{23\pi}{45} - \frac{90\pi}{45} = \boxed{\frac{-67\pi}{45}}$ Positive $\frac{23\pi}{45} + \frac{90\pi}{45} = \boxed{\frac{113\pi}{45}}$ | 3. $\frac{-9\pi}{4}$ Negative $-\frac{9\pi}{4} + \frac{8\pi}{4} = \boxed{-\frac{\pi}{4}}$ Positive $-\frac{\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{7\pi}{4}}$ |
| 4. $-\frac{2\pi}{3}$ Negative $-\frac{2\pi}{3} - \frac{6\pi}{3} = \boxed{\frac{-8\pi}{3}}$ Positive $-\frac{2\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{4\pi}{3}}$ | 5. $\frac{-5\pi}{4}$ Negative $-\frac{5\pi}{4} - \frac{8\pi}{4} = \boxed{\frac{-13\pi}{4}}$ Positive $-\frac{5\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{3\pi}{4}}$ | 6. $\frac{116\pi}{45}$ Negative $\frac{26\pi}{45} - \frac{90\pi}{45} = \boxed{\frac{-64\pi}{45}}$ Positive $\frac{116\pi}{45} - \frac{90\pi}{43} = \boxed{\frac{26\pi}{45}}$ |

# 6.3 N – Converting Angles, Arc Length, Sector Area

**A.** Review: Multiplying fractions

1. $\frac{12}{25} \cdot \frac{15}{8} = \frac{180}{200} = \frac{9}{10}$

2. $\frac{\cancel{10}}{\cancel{7}_1} \cdot \frac{\cancel{28}}{\cancel{15}} = \frac{8}{3}$

3. $\frac{\cancel{20}}{3\cancel{7}} \cdot \frac{\cancel{8}}{\cancel{40}_4} = \frac{2}{12} = \frac{1}{6}$

**B.** Converting time: 1 minute = 60 seconds

more seconds multiply

1. 3 minutes = <u>180</u> seconds

$3(\cancel{min}) \cdot \frac{60\ (sec.)}{1\ (\cancel{min})} = 180\ sec.$

2. 900 seconds = <u>15</u> minutes

$900(\cancel{sec.}) \cdot \frac{1\ (min)}{60\ (sec)} = \frac{900}{60}$

$= 15 min$

$\frac{\pi}{180}$ or $\frac{180}{\pi}$

**C.** Converting Radians to degrees and degrees to radians: If $\pi = 180^\circ$ then $2\pi = 360^\circ$.

https://commons.wikimedia.org/wiki/File:Circle_radians.gif#/media/File:Circle_radians.gif

$\pi$ or $180^\circ$ — $2\pi$ $360^\circ$

Radians

1. $\frac{45^\circ}{1} \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \boxed{\frac{\pi}{4}}$

Degrees

2. $\frac{2\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}} = \frac{360}{3} = \boxed{120^\circ}$

3. $\boxed{-\frac{11\pi}{6}}$

$-\frac{11\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}} = \frac{-1980}{6}$ $\frac{\pi}{180} = 1$

$= \boxed{-330^\circ}$

4. $\frac{-935^\circ}{1} \cdot \frac{\pi}{180} = \frac{-935\pi}{180} = \boxed{\frac{-187\pi}{36}}$

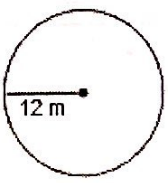
5. $\frac{3.1}{1} \cdot \frac{180}{\pi} = \frac{558}{\pi} = \boxed{177.62^\circ}$

6. $80^\circ$

$\frac{80^\circ}{1} \cdot \frac{\pi}{180} = \frac{80\pi}{180} = \boxed{\frac{4\pi}{9}}$

**D.** Area and circumference of a circle: $\boxed{A = \pi r^2 \quad C = 2\pi r}$

1. Find the area of the following circle.




$A = \pi \cdot 12^2$

$= 144\pi\ m^2$

OR $= 452.39\ m^2$

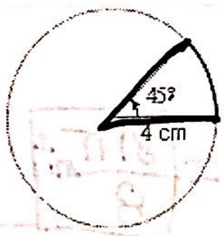
2. Find the circumference of the following circle.


$C = 2\pi \cdot 5$

$C = 10\pi\ km$

OR $= 31.42\ KM$

**E.** What fraction of the circle is $45^\circ$?

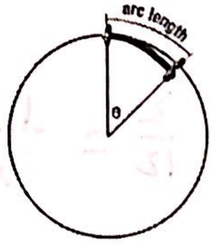



1. $\frac{45}{360} = \frac{1}{8}$

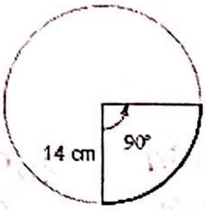
Scanned with CamScanner

**F.** Find *Arc Length* of a circle. is part of circumference

$C=2\pi r$




Arc length is ... a fraction of the circumference of a circle. S stands for arc length

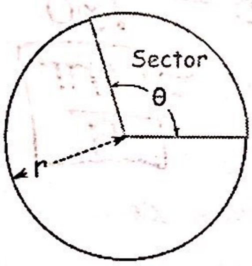
| | |
|---|---|
|   1. $\frac{90}{360}\cdot\frac{2\pi\cdot 14}{1}$ <br> $\frac{2520\pi}{360}=7\pi cm$ <br> or 22cm | 2. $r=17cm,\ \theta=\frac{7\pi}{6}rad$; find $s$ <br> $\frac{\frac{7\pi}{6}}{\cancel{2\pi}}\cdot\frac{\cancel{2\pi}\cdot 17}{1}=\frac{7\pi}{6}\cdot\frac{17}{1}=\frac{119\pi}{6}cm$ <br> OR 62.3cm |
| 3. $s=4m,\ r=2m$; find $\theta$ Degree <br> $4=\frac{\theta}{360}\cdot\frac{2\pi\cdot 2}{1}$ <br> $360\cdot 4=\frac{4\pi\theta}{\cancel{360}}\cdot\cancel{360}$ <br> $\frac{1440}{(4\pi)}=\frac{\cancel{4\pi}\theta}{\cancel{4\pi}}$ <br> $\boxed{\theta=114.6^\circ}$ | 4. $s=9in,\ \theta=6rad$; find $r$ <br> $9=\frac{6}{\cancel{2\pi}}\cdot\cancel{2\pi}r$ <br> $\frac{9}{6}=\frac{6r}{6}$ <br> $\boxed{r=1.5in}$ |

Equations for *arc length*: Fraction of circle • circumference

Degrees: $S=\frac{\theta}{360}\cdot 2\pi r$

Radians: $S=\frac{\theta}{2\pi}\cdot 2\pi r$

**F.** Find the *area of a sector* of a circle.

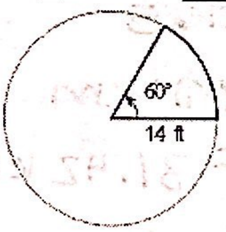



Degrees

$\frac{\theta}{360}\cdot\pi r^2$

Radians

$\frac{\theta}{2\pi}\cdot\pi r^2$

| | |
|---|---|
|   1. $\frac{60}{360}\cdot\frac{\pi\cdot 14^2}{1}$ <br> $\frac{11760\pi}{360}=\boxed{\frac{98\pi}{3}ft^2}$ | 2. $r=9mi,\ \alpha=\frac{\pi}{4}$ <br> $\frac{\frac{\pi}{4}}{2\pi}\cdot\pi\cdot 9^2$ |

OR $\boxed{102.6ft^2}$

$\frac{\pi}{4}\div 2\pi$

same as

$\frac{\cancel{\pi}}{4}\cdot\frac{1}{2\cancel{\pi}}\cdot\frac{\pi 9^2}{1}=\boxed{\frac{81\pi}{8}mi^2}$

OR $\boxed{31.8m^2}$

| | |
|---|---|
| 3. $r = 10km$, $\alpha = 300°$ $\frac{300}{360} \cdot \frac{\pi\, 10^2}{1} = \frac{30000\pi}{360}$ $\frac{250\pi}{3} km^2$ or $261.8 km^2$ | 4. $r = 10ft$, $\alpha = 60°$ $\frac{60}{360} \cdot \frac{\pi \cdot \overset{100}{10^2}}{1} = \frac{6000\pi}{360} = \frac{50\pi}{3} ft^2 = 52.4 ft^2$ |

Equations for *Area of a Sector*: Fraction of circle $\cdot$ Area of circle

$\pi \cdot r^2$

Degrees:

$$\frac{\theta}{360} \cdot \frac{\pi \cdot r^2}{1}$$

Radians:

$$\frac{\theta}{2\pi} \cdot \pi r^2$$

# 6.4 N – Solving Trigonometric Equations

**A.** Review Solving

**Solve each equation.**

$-\frac{15}{2} + \frac{3}{1}$    $-\frac{15}{2} \quad \frac{6}{2}$

1. $6x = -15x - 20$

$+15x \quad +15x$

$\frac{21x}{21} = \frac{-20}{21}$

$x = \frac{-20}{21}$

2. $5x - 16 = 13x$

$-5x \quad -5x$

$\frac{-16}{8} = \frac{8x}{8}$

$-2 = x$

3. $-\frac{15}{2} = -3 + x$

$+3 \quad +3$

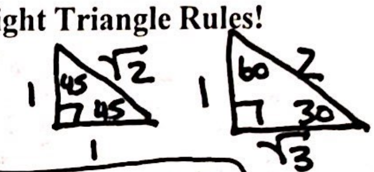
$-\frac{9}{2} = x$

II S | A I   positive

III T | C IV

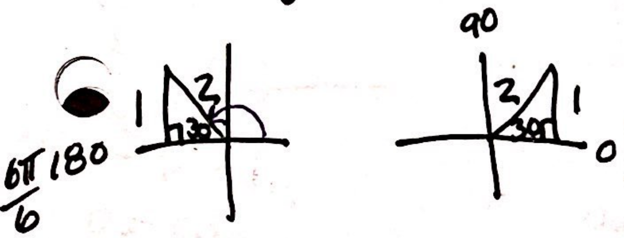
**B.** Review Solving Special Trig Equations

Things to Remember: SOH CAH TOA, All Students Take Calculus, and **Special Right Triangle Rules!**



**Find all angles in the interval $[0°, 360°)$ and $[0, 2\pi)$ that satisfy each equation.**

4. $\sin x = \frac{1}{2}$ SOH $\frac{\text{opp}}{\text{hyp}}$   $30° = \frac{\pi}{6}$



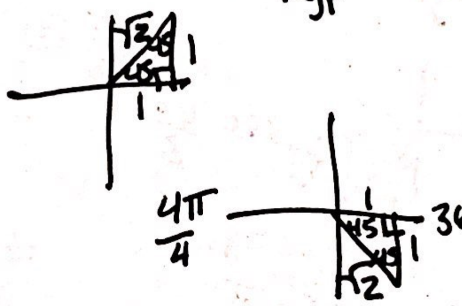
$180 - 30 = 150$

$\frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

Degrees: $30°, 150°$ Radians: $\frac{\pi}{6}, \frac{5\pi}{6}$

5. $\cos x = \frac{1}{\sqrt{2}}$ CAH $\frac{\text{adj.}}{\text{hyp.}}$

$\boxed{\frac{\pi}{4} = 45° \quad \frac{\pi}{3} = 60° \quad \frac{\pi}{6} = 30°}$

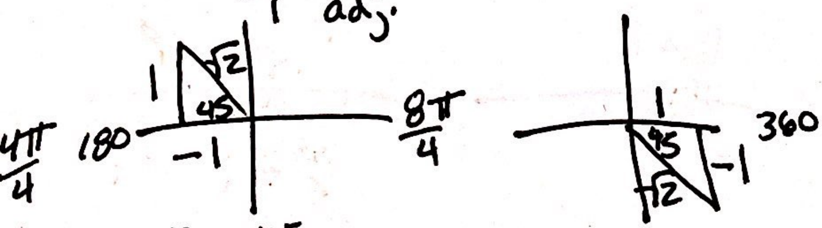


$360 - 45°$

$\frac{8\pi}{4} - \frac{\pi}{4}$

Degrees: $45°, 315°$ Radians: $\frac{\pi}{4}, \frac{7\pi}{4}$

6. $\tan(x) = -1$ TOA $\frac{\text{opp}}{\text{adj.}}$   $\frac{\pi}{4} = 45°$



$180 - 45$

$\frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$

$360 - 45$

$\frac{8\pi}{4} - \frac{\pi}{4}$

Degrees: $135°$, $315°$ Radians: $\frac{3\pi}{4}$, $\frac{7\pi}{4}$

## C. Solving harder Trig

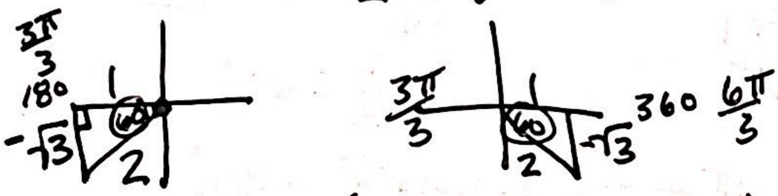
Steps:

1. Solve for the trigonometric function → Get sin/cos/tan alone
2. Draw 2 triangles $\frac{S \mid A}{T \mid C}$
3. Decide whether Δ is 45 45 90 or 30 60 90 and label Δ.
4. Find the angles in degrees & radians

**Find all angles in the interval $[0°, 360°)$ that satisfy each equation.**

7. $\frac{2\sin\theta}{2} = \frac{-\sqrt{3}}{2}$ SOH

$\sin\theta = \frac{-\sqrt{3}}{2}$ $\frac{\text{opp}}{\text{hyp}}$ $\frac{\pi}{3} = 60°$



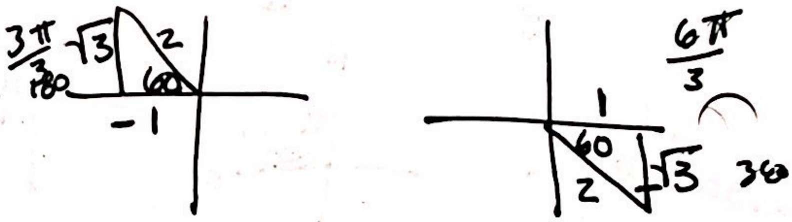

$180 + 60 = 240°$ $\frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$

$360 - 60 = 300°$ $\frac{6\pi}{3} - \frac{\pi}{3}$

Degrees: 240°, 300° Radians: $\frac{4\pi}{3}, \frac{5\pi}{3}$

8. $\tan\theta + \sqrt{3} = 0$ TOA $\frac{\pi}{3} = 60°$

$\tan\theta = \frac{-\sqrt{3}}{1}$ $\frac{\text{opp}}{\text{adj.}}$




$180 - 60$ $\frac{3\pi}{3} - \frac{\pi}{3} =$

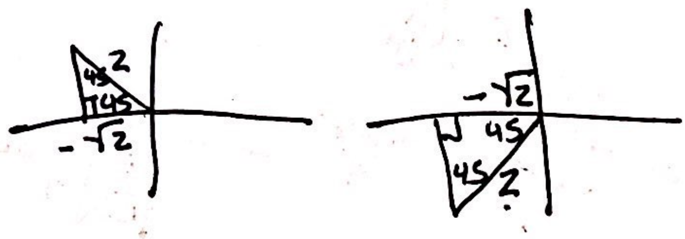
$360 - 60$ $\frac{6\pi}{3} - \frac{\pi}{3}$

Degrees: 120°, 300° Radians: $\frac{2\pi}{3}, \frac{5\pi}{3}$

9. $2\cos\theta + \sqrt{2} = 0$
$-\sqrt{2}$ $-\sqrt{2}$ CAH $\frac{S \mid A}{T \mid C}$

$\frac{2\cos\theta}{2} = \frac{-\sqrt{2}}{2}$

$\cos\theta = \frac{-\sqrt{2}}{2}$ $\frac{\text{adj.}}{\text{hyp}}$




Try it

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

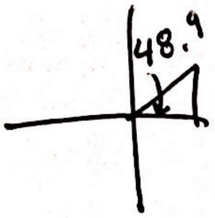
Degrees: 135, 225° Radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

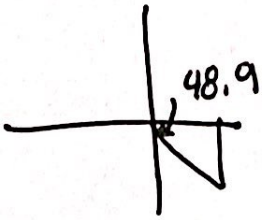
$\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$ S | A / T | C

Find all angles in the interval $[0°, 360°)$ that satisfy each equation. Round approximations to the nearest tenth of a degree (use your calculator). Mode in degrees

10. $\cos\theta = 0.657$

$\cos^{-1}(.657) = 48.9°$




360−48.9

Degrees: 48.9, 311.1°

11. $\sqrt{5}\sin\theta = -2$

$\frac{\sqrt{5}\sin\theta}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$

180+63.4


$\sin\theta = \frac{-2}{\sqrt{5}}$


360 − 63.4

$\sin^{-1}(-2/\sqrt{5}) = -63.4°$

Degrees: 243.4°, 296.6

S | A / T | C

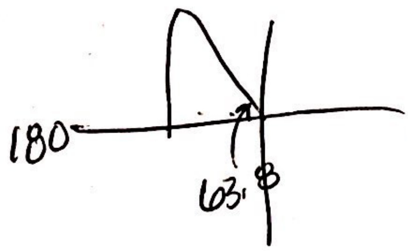
12. $\tan\theta + 2 = -.036$
$-2 \quad -2$

$\tan\theta = -.036 \cancel{+2}$

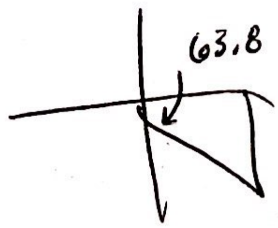
$\tan\theta = -2.036$

negative

$\tan^{-1}(-2.036) = -63.8$




180 − 63.8




360 − 63.8

Degrees: 116.2°
296.2°

Scanned with CamScanner

# 6.5N – Special Right Triangles

**A.** Review:

Simplify the following radicals.

1. $\frac{\sqrt{4}}{2\sqrt{64}} = \frac{2}{2\cdot 8} = \frac{2}{16} = \frac{1}{8}$

2. $\frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$

3. $\frac{2\sqrt{15}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{2\sqrt{75}}{5} = \frac{2\cdot 5\sqrt{3}}{5} = 2\sqrt{3}$

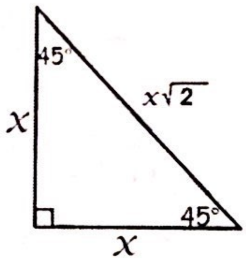
$\sqrt{25}$ $\cancel{5\cdot 5}$ $\sqrt{5\cdot 5\cdot 3}$ 10

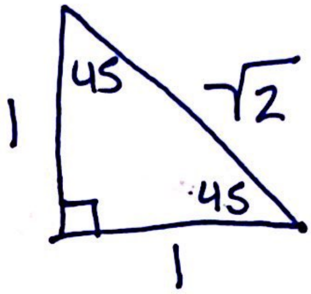
**B.** Special Right Triangles

In a **45°-45°-90° Triangle** the hypotenuse is $\sqrt{2}$ times as long as each leg. So, the ratio is

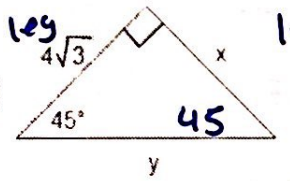
**leg : leg : hypotenuse**

1 : 1 : $\sqrt{2}$




**Example #1:**

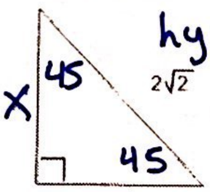



$x = 4\sqrt{3}$

$y = 4\sqrt{3}\cdot\sqrt{2}$

$y = 4\sqrt{6}$

**Example #2:**

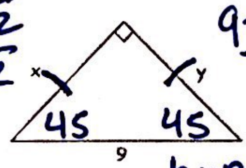



$x = 2$

$x = \frac{2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$x = 2$

**Example #3:**



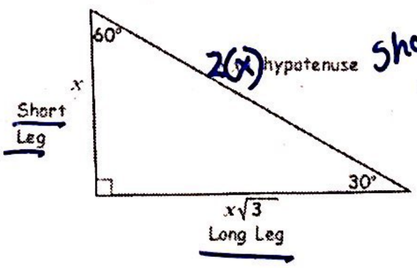

$\frac{9}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{9\sqrt{2}}{2}$

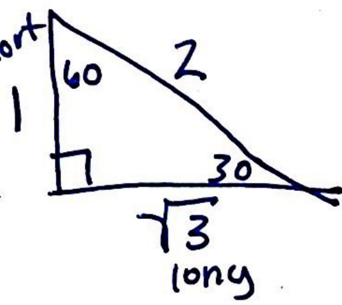
<u>30°- 60°- 90° Triangles</u>

In a **30°- 60°-90° Triangle** the hypotenuse is 2 times as long as the shortest leg and the longest leg is $\sqrt{3}$ tim longer than the shorter leg . So, the ratio is

**short leg : long leg : hypotenuse**

1 : $\sqrt{3}$ : 2




**Example #4:**

long leg $2\sqrt{9}$

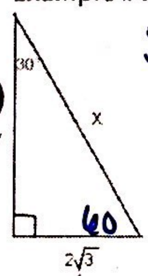
$y = 2\sqrt{3}\cdot\sqrt{3} = 2\cdot 3$

$y = 6$

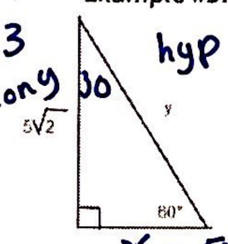
hyp

$x = 2\cdot 2\sqrt{3}$

$x = 4\sqrt{3}$




**Example #5:**



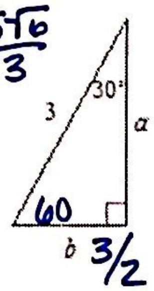

$x = \frac{5\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{6}}{3}$

$y = \frac{5\sqrt{6}}{3}\cdot\frac{2}{1}$

$y = \frac{10\sqrt{6}}{3}$

short $x = \frac{5\sqrt{6}}{3}$

**Example #6:**



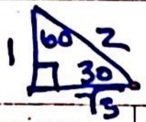

$b = \frac{3}{2} = \frac{3}{2}$

$a = \frac{3}{2}\cdot\frac{\sqrt{3}}{1} = \frac{3\sqrt{3}}{2}$

**Special Right Triangle Rules:**


**45-45-90**

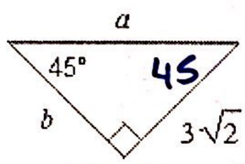
| *if this measure is given:* | *and you want this measure:* | *then do this:* |
|---|---|---|
| the leg | hypotenuse | multiply the leg by $\sqrt{2}$ |
| hypotenuse | the leg | divide hypotenuse by $\sqrt{2}$ |




**30-60-90**

| *if this measure is given:* | *and you want this measure:* | *then do this:* |
|---|---|---|
| short leg | hypotenuse | multiply short leg by 2 |
| short leg | long leg | multiply short leg by $\sqrt{3}$ |
| long leg | short leg | divide long leg by $\sqrt{3}$ |
| hypotenuse | short leg | divide hypotenuse by 2 |

1.



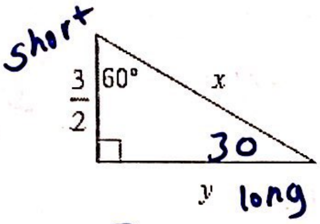

$\boxed{b = 3\sqrt{2}}$

$a = 3\sqrt{2} \cdot \sqrt{2}$

$a = 3 \cdot 2$

$\boxed{a = 6}$

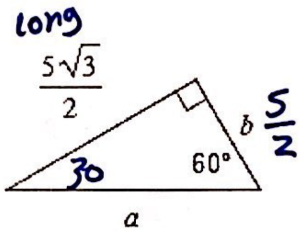
2.




$y = \frac{3}{2} \cdot \frac{\sqrt{3}}{1} = \frac{3\sqrt{3}}{2}$

$x = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = 3$

3.

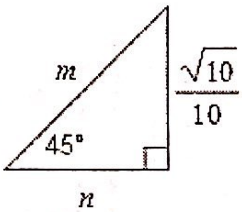



$b = \frac{5\sqrt{3}}{2} \div \sqrt{3}$

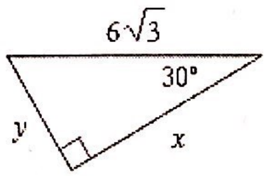
$\frac{5\cancel{\sqrt{3}}}{2} \cdot \frac{1}{\cancel{\sqrt{3}}} = \boxed{\frac{5}{2}}$

$a = \frac{5}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = 5$

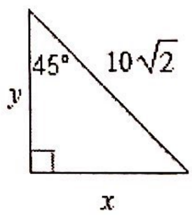
4.




5.

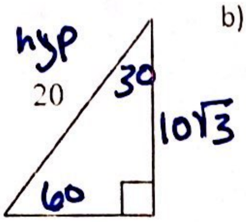



6.



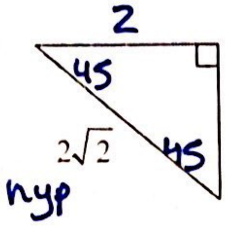
c. Given the sides of the right triangle, decide which type of special right triangle it is, ($30° - 60° - 90°$ or $45° - 45° - 90°$). Then write the degree measures of the missing 2 angles in the correct spot. **Triangles are not drawn to scale. Hint: Use the rules for special right triangles to help you.**

a)



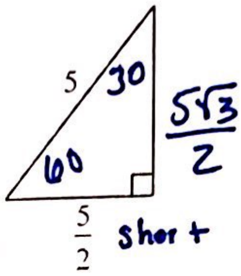

$20 \div 2 = 10$

b)



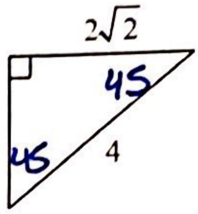

$\frac{2\sqrt{2}}{\sqrt{2}} = 2$

c)




$\frac{5}{2}$

d)



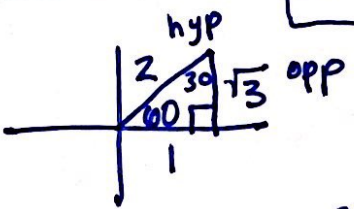

~~4 30 60 90 = 2~~

45 45 90

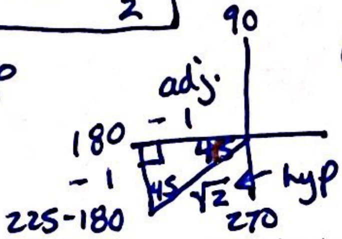
$\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{2}}$

Draw a reference triangle for the given angle. Pick a number for the hypotenuse. Decide which type of special triangle it is and use the rules to find the missing sides. Give the exact value of each trig function without using a calculator.

e) $\sin 60° =$ SOH $\boxed{\sin 60 = \frac{\sqrt{3}}{2}}$




f) $\cos 225° =$ CAH

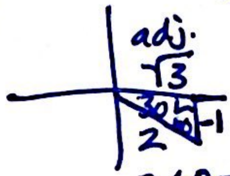



$\cos 225 = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\cos 225 = \frac{-\sqrt{2}}{2}}$

g) $\tan 330° =$ TOA

$\tan 330° = \frac{-1}{\sqrt{3}}$




$\frac{-1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{-\sqrt{3}}{3}$

360-330

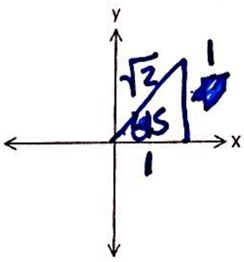
Find the acute angles, $\theta$, that satisfy the given equation by drawing the triangles. Give $\theta$ in both **degrees and radians.**
**You should do these problems without a calculator.**

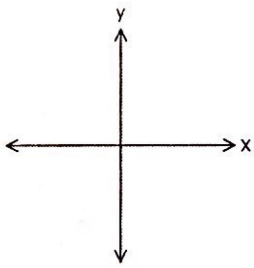
$\boxed{\frac{\pi}{4} = 45° \quad \frac{\pi}{3} = 60° \quad \frac{\pi}{6} = 30°}$

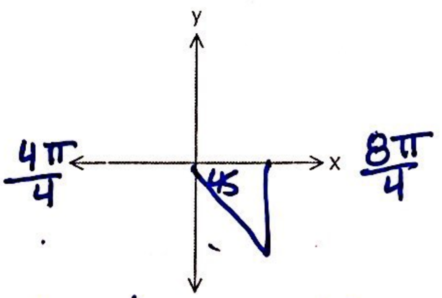
h) $\cos\theta = \frac{1}{\sqrt{2}}$ CAH $\frac{\text{adj.}}{\text{hyp}}$

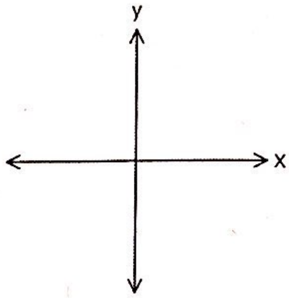
S | A.
T | C

i) $\sec\theta = 2$




$\theta$ = 45°, 315°  360-45°

and $\theta$ = $\frac{\pi}{4}$, $\frac{7\pi}{4}$  $\frac{8\pi}{4} - \frac{\pi}{4}$

$\theta$ = ____________

and $\theta$ = ____________