Linear Functions

Name ______________________ Per_____

4.5 Notes

I Am Transformed!

Goal: Identify the effect on the equation when we change aspects of a real-world function. Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *kf(x)*, *f(kx)*, and *f(x+k)* for specific values of *k*.

In this exploration we will see how transformations of a function tell different, but related stories.

1. Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let *p = f(E)* where *p* is the pounds of fish needed and *E* is the expected number of customers. What would the expressions *f(E +15)* and *f(E) + 15* mean?

- These two expressions are similar in that they both involve adding 15. However, for *f(E + 15)*, the 15 is added on the inside, so 15 is added to the number of customers expected. Therefore, *f(E + 15)* gives the number of pounds of fish needed for 15 extra customers.
- The expression *f(E) + 15* represents an outside change. We are adding 15 to *f(E)*, which represents pounds of fish, not expected number of customers. Therefore, *f(E) + 15* means that we have 15 more pounds of fish than we need for E expected customers.

Match each story below to one of the expressions at the right:

Matching Function Number	Situation
2	The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case.
1	On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for 2 nights.
3	The owner of the restaurant planned to host his 2 fish-loving parents for dinner at the restaurant.

Function	Transformation
1) *p = 2f(E)*	Vertical Stretch by 2
2) *p = f(E) + 2*	Shift up 2
3) *p = f(E + 2)*	Shift Left 2

2. Suppose every day I take a taxi around town. The trip is ***x* miles**, so the **cost** for the trip is **C = f(x)**. ("C is a function of x." or, "The cost of the trip is a function of the miles driven.")

Match each story to a function which represents the amount paid to the taxi driver.

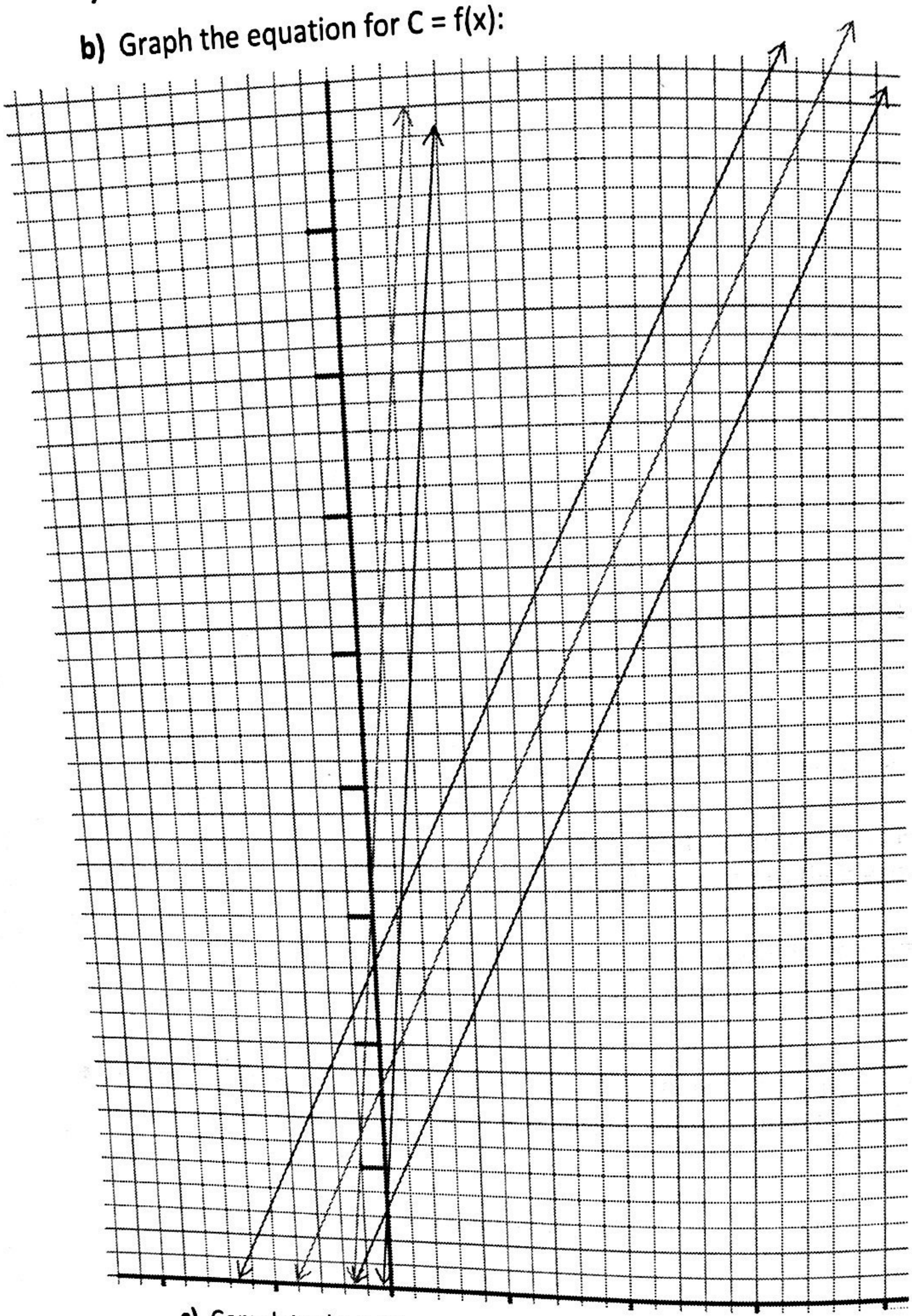
Matching Function Number	Situation
2	I received a raise yesterday, so today I gave my driver a five dollar tip.
4	I had a new driver today and he got lost. He drove five extra miles and charged me for it.
1	I haven't paid my driver all week. Today is Friday and I'll pay what I owe for the week.
3	The meter in the taxi went crazy and showed five times the number of miles I actually traveled.

Function	Transformation
1) *C = 5f(x)*	Vertical Stretch by 5
2) *C = f(x) + 5*	Shift up 5
3) *C = f(5x)*	Horizontal Shrink by 1/5
4) *C = f(x + 5)*	Shift Left 5

. The taxi driver charges a $3 flat fee in addition to $2.00 per mile.

a) Write the equation for C = f(x): ______2x+3______________________

b) Graph the equation for C = f(x):



Original 2x+3

Domain: $[0,\infty)$

Range: $[3,\infty)$

X-intercept: (-1.5, 0) – not in domain

Y-intercept(0, 3)

Situation 1 2x+8

Domain: $[0,\infty)$

Range: $[8,\infty)$

X-intercept: (-4, 0) – not in domain

Y-intercept: (0, 8)

Situation 2 2x+13

Domain: $[0,\infty)$

Range: $[13,\infty)$

X-intercept: (-6.5, 0) – not in domain

Y-intercept: (0, 13)

Situation 3 10x+15

Domain: $[0,\infty)$

Range: $[15,\infty)$

X-intercept: (-1.5, 0) – not in domain

Y-intercept: (0, 15)

Situation 4 10x+3

Domain: $[0,\infty)$

Range: $[3,\infty)$

X-intercept: (-0.3, 0) – not in domain

Y-intercept: (0, 3)

c) Complete the table:

Situation	Function Notation	Equation	Transformation
1) I received a raise yesterday, so today I gave my driver a five dollar tip.	*f(x)+5*	*2x+8*	Shift up 5
2) I had a new driver today and he got lost. He drove five extra miles and charged me for it.	*f(x+5)*	*2x+13*	Shift Left 5
3) I haven't paid my driver all week. Today is Friday and I'll pay what I owe for the week.	*5f(x)*	*10x+15*	V. Stretch by 5
4) The meter in the taxi went crazy and showed five times the number of miles I actually traveled.	*f(5x)*	*10x+3*	H. Shrink by 1/5

Name ______________________

I Am Transformed! (continued)

Goal: Identify the effect on the equation when we change aspects of a real-world function. Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *kf(x)*, *f(kx)*, and *f(x+k)* for specific values of *k*.

In this exploration we will see how transformations of a function tell different, but related stories.

3. Let $W = p(t)$ be the average weight (in pounds) of a puppy who is t months old. The weight, W of a particular puppy named Cecee is given by the function $W = p(t) + 2$, and the weight of another puppy named Brutus is given by the function $W = p(t + 4)$.

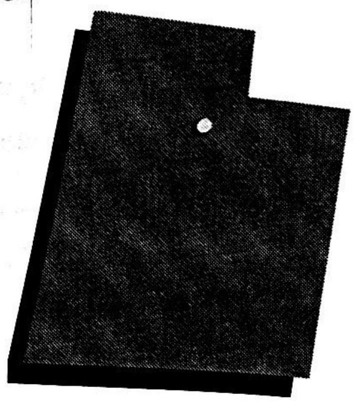
Compare the growth rate for each of these puppies to the growth rate of the average puppy.

The growth rate for Cecee and Brutus is the same as the average puppy. (Shifting a graph up 2 and left 4 will not change the rate.)

Describe how Cecee's and Brutus' weights compare to the average weight of puppies their age at any given time.

Cecee is 2 pounds heavier than the average puppy.
Brutus weighs as much as a puppy that is 4 months older.

4. Suppose $y = P(t)$ is a function that gives the population of the Salt Lake valley in thousands of people, where t is in years from today. Write an expression that would represent each of the following statements:



The population 20 years before today.

P(t-20)

Twenty thousand more people live in Salt Lake City than we have today.

P(t)+20

Twenty percent of the population we have today.

0.2P(t)

Goal: Discover what happens to a quadratic function when we change aspects of a real-world quadratic situation. Identify the effect on the graph, table and equation by replacing *H(v)* with ***H(v)+k, H(v) – k, H(v+k), H(v-k), aH(v), and H(av)*** for specific values of *k* and *a*.

		Function Notation	**Equation**
H(v)	Last year as a senior Lily was a pole vaulter for Raviloli High. The equation $H(v) = -.05(v-9.5)^2 + 4.5$ models the average *height* of her jumps, *H* (meters) in terms of *velocity, v* (ft/sec).	**H(v)**	$H(v) = -0.05(v-9.5)^2 + 4.5$
Situation 1	To prepare for Olympic tryouts, Lily increased her weight lifting/sprinting routine and ate healthier foods which increased her velocity by a factor of 1.2 ft/sec.	*H(1.2v)*	$-0.05(1.2v-9.5)^2 + 4.5$
Situation 2	During the Olympic tryouts Lily had a burst of adrenaline and the average height of her jumps increased 0.3 meters which qualified her for the Olympics.	*H(v)+0.3*	$-0.05(v-9.5)^2 + 4.8$
Situation 3	The day of the Olympics, during her first vault, Lily stumbled and decreased her velocity by 0.5 seconds.	*H(v-0.5)*	$-0.05(v-10)^2 + 4.5$

1. Identify the variables in situations above.

Independent variable (input): __v__, ________*Velocity*____________
Dependent variable (output): __*H(v)*__, _______*Height*____________

2. Based on the situations above when you transform the ***original*** (parent) function to:

a. ***Situation 1***, is the transformation happening to the input (v) or the output H(v)? ___Input________
Explain how you know: __Lily's velocity increased.________________________________

b. ***Situation 2***, is the transformation happening to the input (v) or the output H(v)? _Output________
Explain how you know: ___Lily's height increased.________________________________

c. ***Situation 3***, is the transformation happening to the input (v) or the output H(v)? __Input________
Explain how you know: __Lily's velocity decreased.________________________________

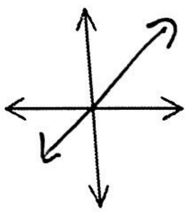
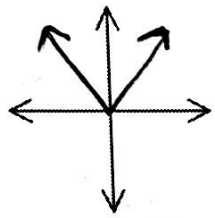
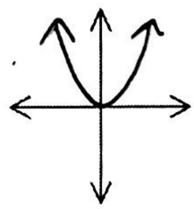
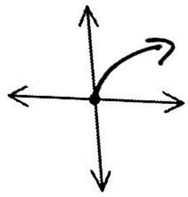
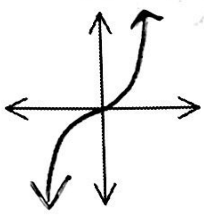
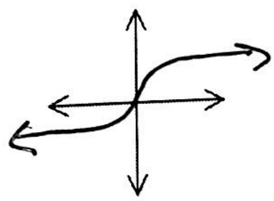
3. In which situation is Lily's average height the highest? __Situation 2___________

Why do you think this is in terms of the story? _Situation 2 is the only one that refers to Lily's height increasing. (The equations are in vertex form, and situation 2 has the highest y-value.)** You may want to show your students the graphs of each equation at this point and discuss each transformation.

4.6 Transformations Notes

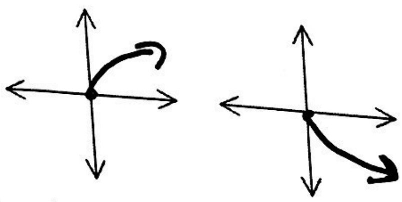
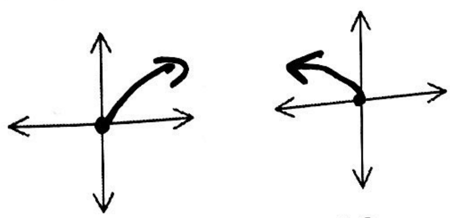
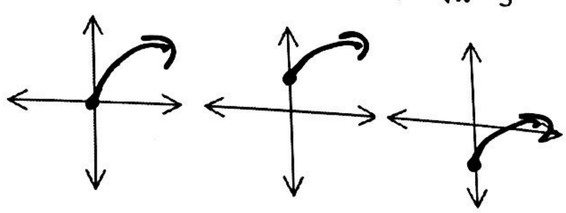
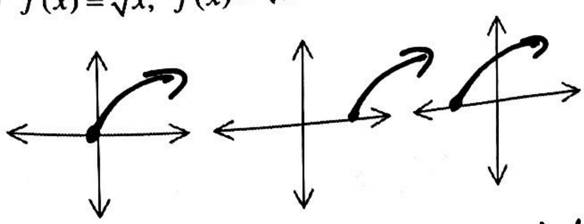
A. Review Parent Functions

1) Draw a sketch of each parent function.
2) Write the general equation for each parent function.

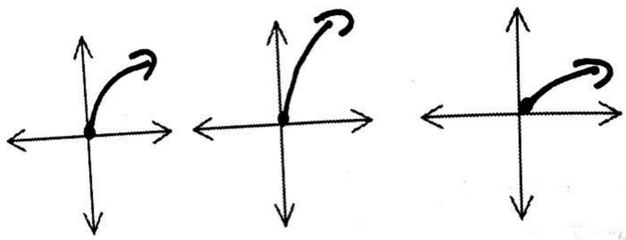
Linear $f(x)=x$  **General Equation:** $y=x$	**Absolute Value** $f(x)=\lvert x\rvert$  **General Equation:** $y=\lvert x\rvert$	**Quadratic** $f(x)=x^2$  **General Equation:** $y=x^2$
Square Root $f(x)=\sqrt{x}$  **General Equation:** $y=\sqrt{x}$	**Cubic** $f(x)=x^3$  **General Equation:** $y=x^3$	**Cube Root** $f(x)=\sqrt[3]{x}$  **General Equation:** $y=\sqrt[3]{x}$

B. Transformations

As a class, graph the following functions and answer the questions.

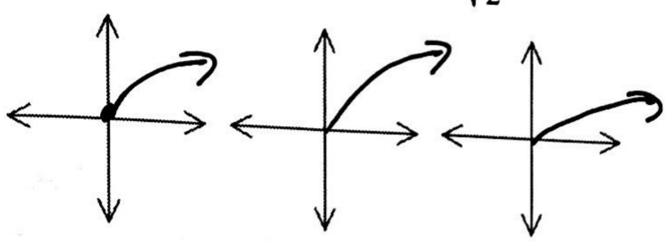
1) $f(x)=\sqrt{x},\ f(x)=-\sqrt{x}$  What does the negative in front of the entire function $f(x)$ do? reflect over x-axis	2) $f(x)=\sqrt{x},\ f(x)=\sqrt{-x}$  What does the negative in front of x do? reflect over y axis
3) $f(x)=\sqrt{x},\ f(x)=\sqrt{x}+3,\ f(x)=\sqrt{x}-3$  What does the k do? Translate up or down	4) $f(x)=\sqrt{x},\ f(x)=\sqrt{x-5},\ f(x)=\sqrt{x+5}$  What does the h do? Translate left or right

5) $f(x)=\sqrt{x},\ f(x)=2\sqrt{x},\ f(x)=\frac{1}{2}\sqrt{x}$



What does the a do?

6) $f(x)=\sqrt{x},\ f(x)=\sqrt{2x},\ f(x)=\sqrt{\frac{1}{2}x}$



What does the b do?

C. Write an equation.

Given the parent function and a list of transformations, write an equation for the function.

1) **Parent function:** $f(x)=x^2$

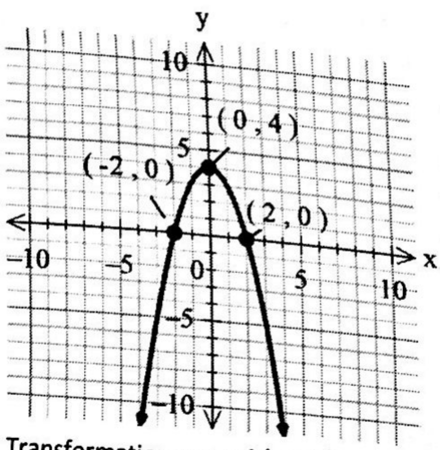
Transformations: reflect over x-axis, translate up 3 $f(x) = -x^2+3$

2) **Parent function:** $f(x)=\sqrt[3]{x}$

Transformations: Vertical shrink of $\frac{1}{3}$, translate left 3 and down 6 $f(x) = \frac{1}{3}\sqrt[3]{x+3} - 6$

Determine the transformations used to change the given parent function to the function that is graphed. Then write an equation for the function graphed.

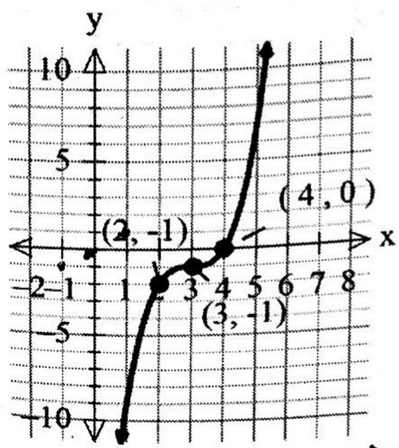
1) $f(x)=x^2$



Transformations: reflect over x-axis up 4

Equation: $f(x) = -x^2+4$

2) $f(x)=x^3$



Transformations: right 3, down 1

Equation: $f(x) = (x-3)^3 - 1$

4.7

4.7 Notes: Transformations of Points

General Form: $g(x) = f(x)$

$$g(x) = f\left(-\frac{1}{2}(x+2)\right) - 1$$

Write transformations in words:
1. Reflect over y axis
2. Horizontal Stretch
3. Left 2
4. Down 1

Original ordered pairs

1. $(1,0)$
2. $(-4,5)$
3. $(-4,-5)$
4. $(1,0)$
5. $\left(-\frac{1}{2}, 1\frac{1}{2}\right)$
6. $(-4,-2)$
7. $(-4,1)$
8. $(-2,3)$

Parent

	x	f(x)
−2 •	1	0
−2 •	−4	5
−2 •	−4	−5
−2 •	1	0
−2 •	−1/2	1 1/2
−2 •	−4	−2
−2 •	−4	1
−2 •	−2	3

Reflect and Stretch

	x	f(x)	
−2+	−2	0	−1
−2+	8	5	−1
−2+	8	−5	−1
−2+	−2	0	−1
−2+	1	1 1/2	−1
−2+	8	−2	−1
−2+	8	1	−1
−2+	4	3	−1

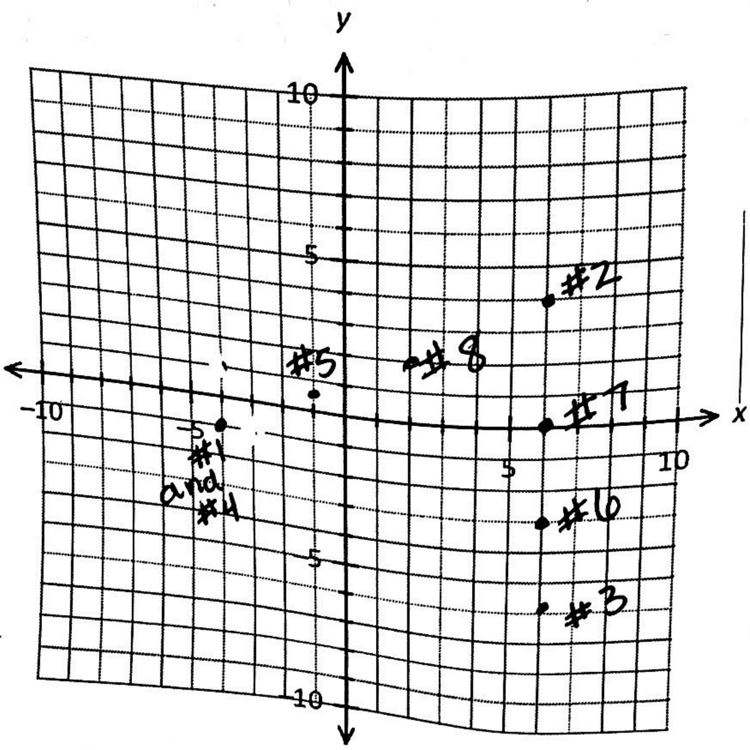
~~Stretches/ Compressions~~

x	f(x)

Left 2
Down 1

Translations (Shifts)

x	f(x)	
−4	−1	#1
6	4	#2
6	−6	#3
−4	−1	#4
−1	1/2	#5
6	−3	#6
6	0	#7
2	2	#8



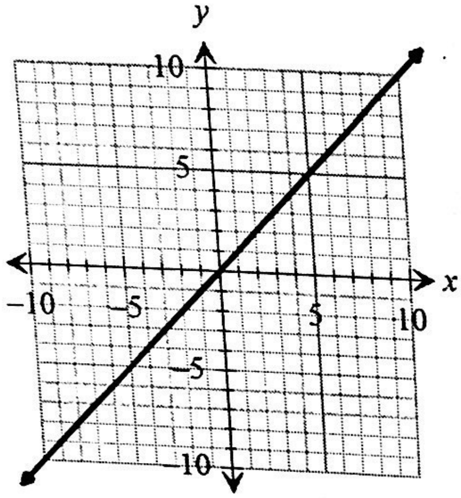
4.8

Name ______________________ Date ____________ Period ________

SM3 4.8 Notes—Parent Functions

Graph each function. Then determine the key features of each parent function.

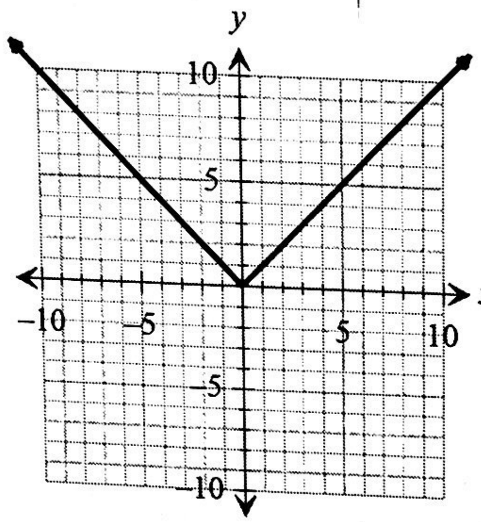
1. $f(x)=x$ **LINEAR**



$y=x$

x	y
-2	-2
-1	-1
0	0
1	1
2	2

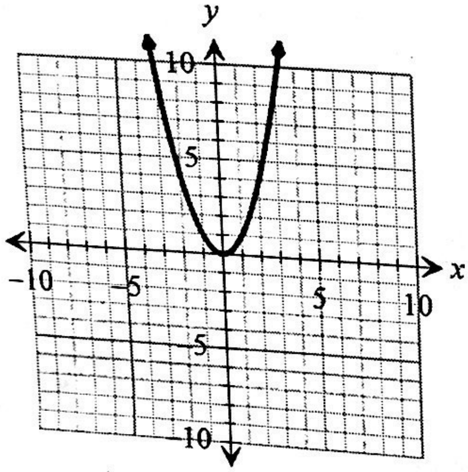
2. $f(x)=|x|$ **ABSOLUTE VALUE**



$y=|x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

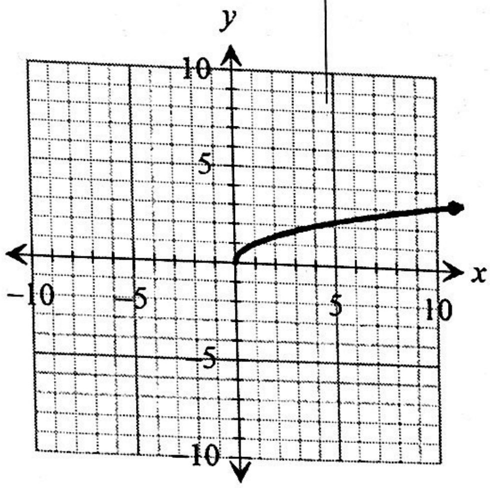
3. $f(x)=x^2$ **QUADRATIC**



$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

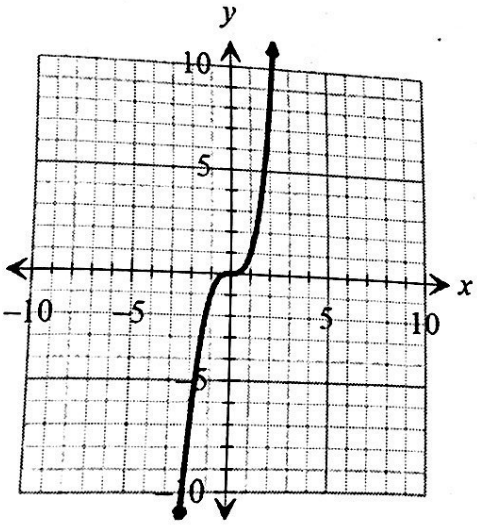
4. $f(x)=\sqrt{x}$ **SQUARE ROOT**



$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

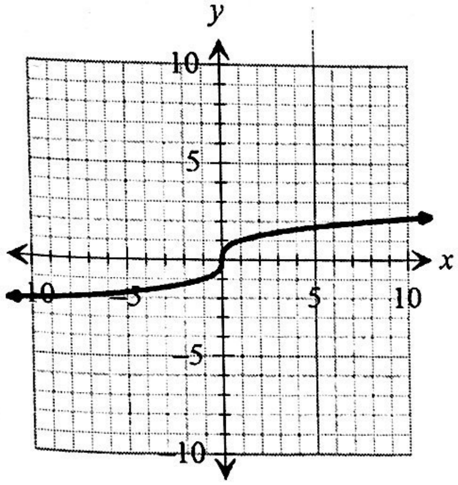
5. $f(x)=x^3$ **CUBIC**



$y=x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

6. $f(x)=\sqrt[3]{x}$ **CUBE ROOT**



$y=\sqrt[3]{x}$

x	y
-8	-2
-1	-1
0	0
1	1
8	2