4.1 Notes – Zeros/Quadratic Formula/Complex zeros

Review: Simplify the following radicals.

a. $\sqrt{28}$ $\sqrt{2\cdot 2\cdot 7}$

$\boxed{2\sqrt{7}}$

b. $2\pm\sqrt{36}$ $6\cdot 6$

2 ± 6

$2+6$ $\boxed{8}$ $2-6$ $\boxed{-4}$

c. $\sqrt{-81}$

$\sqrt{-1\cdot 9\cdot 9}$

$\boxed{9i}$

d. $\dfrac{4\pm\sqrt{-80}}{2}$ $i\sqrt{2\cdot 2\cdot 2\cdot 2\cdot 5}$

$\dfrac{4\pm 4i\sqrt{5}}{2}$

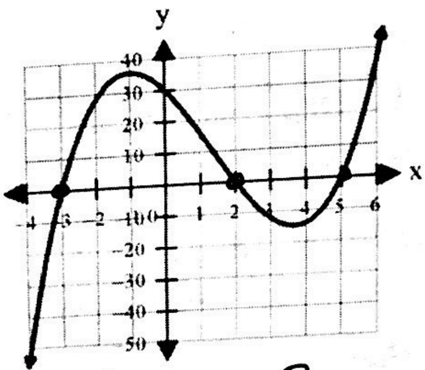
$\boxed{2\pm 2i\sqrt{5}}$

Definitions of **Zeros**: 1. Solutions (answers) when solving a polynomial for x.

2. The x-intercepts (where the graph of the polynomial crosses the x-axis).

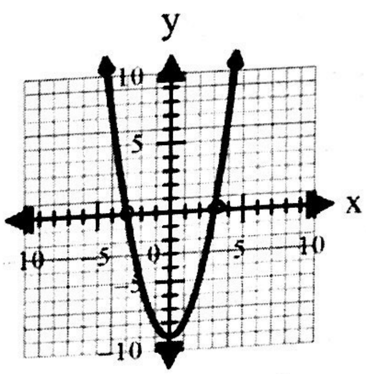
Determine the **number of zeros** for each of the polynomials and **state the degree.**

a. $f(x)=x^3-4x^2-11x+30$



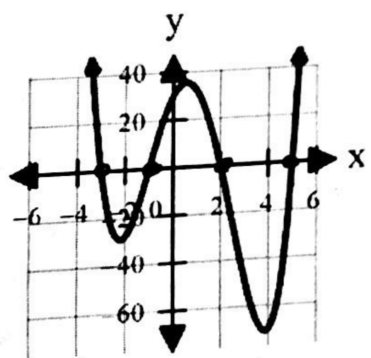
Zeros 3 Degree 3

b. $f(x)=x^2-9$



Zeros 2 Degree 2

c. $f(x)=x^4-3x^3-15x^2+19x+30$



Zeros 4 Degree 4

What do you notice about the number of zeros and the degree of the polynomial? # zeros = degree (same)

Factor example (b) $f(x)=x^2-9$, then solve each factor for x.

$(x-3)(x+3)$

x= 3, -3

Compare your answers to the graph from example (b). Describe what you noticed? The answers are the x intercepts

The zeros of $f(x)=x^2-9$ are x = 3, -3.

*HINT: How can you solve a quadratic equation other than factoring?

Summary:

To find the zeros from a graph you look at the x intercepts

To find the zeros from an equation you find the factors and solve for *x* or use the quadratic formula

To find the x-intercepts from and equation you find the factors and solve for *x* or use the quadratic formula

Two other names for zeros are: 1) x intercepts 2) solutions 3) roots

Remember: Two other names for zeros are: 1) X intercepts 2) solutions 3) Roots

When asked to write zeroes or x-intercepts you must write them in the following way:

Zeroes: $x =$ $x =$

X-intercepts: $(\ ,0)$ or $x =$

Examples:

Write an equation in *factored form* for the function with the given zeros.

a) $x = 3, -5, 4$

$f(x) = (x-3)(x+5)(x-4)$

b) $x = 5, -5, 4, 8$

$f(x) = (x-5)(x+5)(x-4)(x-8)$

Write an equation in *standard form* for the function with the given zeros.

a) $x = 3, -4$

$f(x) = (x-3)(x+4)$

$f(x) = x^2+4x-3x-12$

$f(x) = x^2+x-12$

b) $x = -1, 7$

$f(x) = (x+1)(x-7)$

$f(x) = x^2-7x+x-7$

$f(x) = x^2-6x-7$

Solve for x by *factoring*. (Find the zeros for each polynomial.)

a) $0 = x^2 - 5x - 14$ SHORT CUT

$0 = (x-7)(x+2)$

x + : -14 | -5 ; -7 | 2

$\boxed{x=7 \quad x=-2}$

OR

$(7,0)$ $(-2,0)$

b) $0 = 4x^2 - 9$ DIF OF SQ.

$0 = (2x+3)(2x-3)$

$\boxed{x=-3/2 \quad x=3/2}$

OR

$(-3/2,0)$ $(3/2,0)$

c) $0 = 3x^2 - 2x - 5$ Grouping

x + : -15 | -2 ; -5 | 3

$0 = 3x^2+3x-5x-5$

$0 = 3x(x+1)-5(x+1)$

$0 = (3x-5)(x+1)$

$\boxed{x=5/3 \quad x=-1}$

OR

$(5/3,0)$ $(-1,0)$

Use the *Quadratic Formula* to find the zeros of each polynomial.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Do you remember the song?

a) $f(x) = 3x^2 - 5x + 1$

$0 = 3x^2-5x+1$

$a=3 \quad b=-5 \quad c=1$

$$x = \frac{5 \pm \sqrt{\overset{25-12}{25-4(3)(1)}}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{13}}{6}$$

b) $f(x) = 2x^2 + 3$

$0 = 2x^2+3$

$a=2 \quad b=0 \quad c=3$

$$x = \frac{0 \pm \sqrt{\overset{-24}{0-4(2)(3)}}}{2(2)}$$

$$x = \frac{0 \pm i\sqrt{2\cdot2\cdot2\cdot3}}{4}$$

$$x = \pm\frac{2i\sqrt{6}}{4} = \pm\frac{i\sqrt{6}}{2}$$

The Remainder Theorem:

1. a) What is the remainder of $5\overline{)12}$? $\boxed{2}$

$$\begin{array}{r} 2 \\ 5\overline{)12} \\ -10 \\ \hline 2 \end{array}$$

b) Is **5** a factor of ***12***? NO

2. a) What is the remainder of $5\overline{)15}$? $\boxed{0}$

$$\begin{array}{r} 3 \\ 5\overline{)15} \\ -15 \\ \hline 0 \end{array}$$

b) Is **5** a factor of ***15***? yes

3. a) What is the remainder when you divide $f(x)$ by $g(x)$? $f(x)=2x^2-5x-1$ $\quad g(x)=x-3$

$$\begin{array}{r} 2x+1 \\ x-3\overline{)2x^2-5x-1} \\ -2x^2+6x \\ \hline x-1 \\ -x+3 \\ \hline \boxed{2} \end{array}$$

← remainder

4. a) What is the remainder when you divide $f(x)$ by $g(x)$? $f(x)=x^2-3x-4$ $\quad g(x)=x-4$

$$\begin{array}{r} x+1 \\ x-4\overline{)x^2-3x-4} \\ -x^2+4x \\ \hline x-4 \\ -x+4 \\ \hline \boxed{0} \end{array}$$

← remainder

b) Find $f(3)$

$f(3)=2(3)^2-5(3)-1$

$18-15-1=\boxed{2}$

b) Find $f(4)$

$f(4)=4^2-3(4)-4$

$16-12-4=\boxed{0}$

5. What do you notice about the answers to parts ***a*** and ***b*** in questions ***3*** and ***4***?

They are the same

6. Is $x-3$ a factor of $2x^2-5x-1$? How do you know?

No, because the remainder is 2.

7. Is $x-4$ a factor of x^2-3x-4? How do you know?

yes - remainder is 0

8. Which of the following are factors of $f(x)=3x^3-11x^2+4x+6$

a) $x-2$ ($x=2$) NO

b) $x+3$ ($x=-3$) NO

c) $x-3$ ($x=3$) yes

a) $f(2)=3(2)^3-11(2)^2+4(2)+6=-6$

$24-44+8+6$

b) $f(-3)=3(-3)^3-11(-3)^2+4(-3)+6=-186$

$-81-99-12+6$

→ c) $f(3)=3(3)^3-11(3)^2+4(3)+6=\underline{\underline{0}}$

SM3 Notes – Graphing Polynomials

2.5 Graphing Polynomials –

Graphing Polynomials-

A. Start by finding the zeros. (Factor when necessary)

B. Multiplicity: The number of times the factor appears in the polynomial.

   a. Even Multiplicity- touches

   b. Odd Multiplicity- crosses

   *Hint: Look at the exponent of the factors

C. End Behavior depends on the degree of the polynomial.

* Hint: To find the degree if it is in standard form look at the exponent of the first term

Degree

$x^4+3x^3-2x^2+x+4$

To find the degree if it is in factored form, add the exponents of the factors

$(x-4)^4(x+1)^3$

$4+3=7$

Degree 7

If the degree is:

- Even- End behavior is up/up

  Example $y=x^2$

  Limit Notation: $\lim_{x\to-\infty} f(x)=\infty$ Left Behavior

  $\lim_{x\to\infty} f(x)\ \infty$ Right Behavior

- Negative Even- End behavior is down/down because it is reflected because it is reflected over the x axis.

  Example $y=-x^2$

  Limit Notation: Left side $\lim_{x\to-\infty} f(x)=-\infty$

  Right side $\lim_{x\to\infty} f(x)=-\infty$

- Odd- End behavior is down/up

  Example $y=x^3$

  Limit Notation: Left side $\lim_{x\to-\infty} f(x)=-\infty$

  Right side $\lim_{x\to\infty} f(x)=\infty$

- Negative Odd- End behavior is up/down because it is reflected over the x axis.

  Limit Notation:

  Example $y=-x^3$

  Left side $\lim_{x\to-\infty} f(x)=\infty$

  Right side $\lim_{x\to\infty} f(x)=-\infty$

① Mult. is even touch
② mult. is odd crosses

SM3 2.5 Graphing Polynomials – Class Notes

Graph each function without a calculator. Start by factoring to find the zeros. Then fill out the chart for multiplicity and determine whether each zero touches or crosses the x-axis. Find the end behavior. Finally sketch the graph (don't worry about the height of the maximums and minimums).

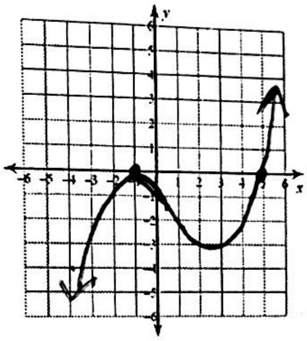
1. $f(x) = (x+1)^4(x-5)^3$ Degree = 4+3 = 7 ODD

X int look at exp.

Zero	Multiplicity	Touch/Cross
-1	4	touch
5	3	cross

below left
$\lim_{x\to-\infty} f(x) = -\infty$

above Right
$\lim_{x\to+\infty} f(x) = \infty$



negative even

flipped over x axis

Degree 2+3+1 = 6

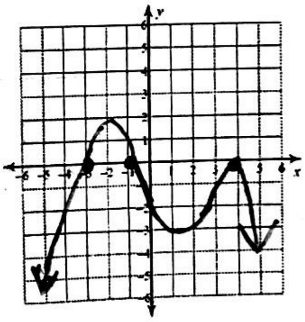
2. $f(x) = -(x-4)^2(x+1)^3(x+3)^1$

Exp

Zero	Multiplicity	Touch/Cross
4	2	touch
-1	3	cross
-3	1	cross

below Left
$\lim_{x\to-\infty} f(x) = -\infty$

below Right
$\lim_{x\to+\infty} f(x) = -\infty$



$(x+4)^1(x-3)^1$ shortcut $\begin{array}{c|c} \times & + \\ -12 & 1 \\ 4 & -3 \end{array}$

3. $f(x) = x^2 + x - 12$

End Behavior

Factor to find zeros

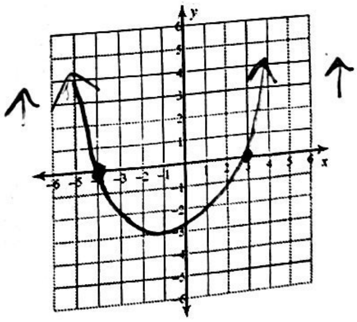
x int exp.

Zero	Multiplicity	Touch/Cross
-4	1	cross
3	1	Cross

opposite of parentheses

above Left
$\lim_{x\to-\infty} f(x) = \infty$

above Right
$\lim_{x\to+\infty} f(x) = \infty$



Degree 1+1 = 2 (Even)

Degree

4. $f(x) = x^3 - 16x$ a=x b=4

$x(x^2-16) = \boxed{x(x+4)(x-4)}$

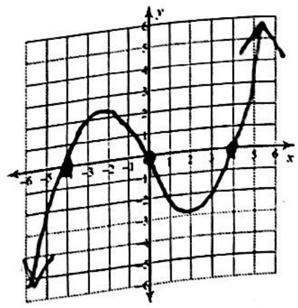
above left

Zero	Multiplicity	Touch/Cross
0	1	cross
-4	1	cross
4	1	cross

$\lim_{x\to-\infty} f(x) = -\infty$

below Right
$\lim_{x\to+\infty} f(x) = \infty$

Degree is 3



4.3 Notes – Complex Zeroes

Objective:

A **Complex Number** is of the form $a+bi$ where *a* is the real part and *bi* is the imaginary part.

Remember *the number* i: i is the number whose square is -1. That is, $i=\sqrt{-1}$ and $i^2=-1$.

A) Review

Simply the following: (Simplifying Radicals)

1. $\sqrt{-25}$

$5i$

2. $\sqrt{-40}$

$i\sqrt{2\cdot2\cdot2\cdot5}$

$2i\sqrt{10}$

40
8 5
4 2
2 2

3. $\sqrt{-180}$

$i\sqrt{2\cdot2\cdot3\cdot3\cdot5}$

$2\cdot3i\sqrt{5}$

$6i\sqrt{5}$

180
18 10
6 3 2 5
2 3

Distribute (Multiply) the following. Make sure you write in standard form:

4. $(3i)(-5i)$

$-15i^2$

$-15(-1)$

$\boxed{15}$

5. $(x-4i)(2x+5i)$

$2x^2+5xi-8xi-20i^2$

$2x^2-3xi-20(-1)$

$\boxed{2x^2-3xi+20}$

6. $(x-6-i)(x-6+i)$

$x^2-6x+\cancel{xi}$

$-6x+36-\cancel{6i}$

$-\cancel{xi}+\cancel{6i}-i^2$

$x^2-12x+36-i^2$

$+(+1)$

$\boxed{x^2-12x+37}$

Find the zeroes by using the quadratic formula:

a) $f(x)=x^2-2x+5$ $a=1$ $b=-2$ $c=5$

$x=\dfrac{2\pm\sqrt{4-4(1)(5)}}{2(1)}$ -16

$x=\dfrac{2\pm4i}{2}$ $\div2$

$\boxed{x=1\pm2i}$

b) $f(x)=x^2-4x+13$

$a=1$ $b=-4$ $c=13$

$x=\dfrac{4\pm\sqrt{16-4(1)(13)}}{2(1)}$ $16-52$

$x=\dfrac{4\pm\sqrt{-36}}{2}$

$x=\dfrac{4\pm6i}{2}$ $\div2$

$\boxed{x=2\pm3i}$

Factor each polynomial over the complex numbers. **Write your answer in factored form. What are the zeroes?**

Examples:

c) $f(x)=x^2+1$

$a=1$
$b=0$
$c=1$

$x=\dfrac{0\pm\sqrt{0-4(1)(1)}}{2(1)}$ -4

$x=\dfrac{\pm2i}{2}$ $x=\pm i$

Factored Form: $f(x)=(x+i)(x-i)$

Zeroes: $x=i$ $x=-i$

d) $f(x)=x^2+121$

$a=1$
$b=0$
$c=121$

$x=\dfrac{0\pm\sqrt{0-4(1)(121)}}{2(1)}$ -484

$x=\pm\dfrac{22i}{2}$

$x=\pm11i$

Factored Form: $f(x)=(x+11i)(x-11i)$

Zeroes: $x=11i$ $x=-11i$

Remember the short cut? 1. Set equation equal to zero
2. Solve for x by taking the square root.
3. Write in factored form with the solutions you found for x.

Identify the zeros of the function and the x-intercepts of its graph. Write the polynomial in standard form. Show work!

e) $f(x) = (x - 7i)(x + 7i)$

Zeros: $x = 7i$ $x = -7i$

x-intercepts: none

Standard form: $f(x) = x^2 + \cancel{7xi} - \cancel{7xi} - 49i^2$

$f(x) = x^2 - 49(-1)$

$f(x) = x^2 + 49$

f) $f(x) = (x - 3)(x + 2)(x + i)(x - i)$

Zeros: $x = 3$ $x = -2$ $x = -i$ $x = i$

x-intercepts: $(3,0)$ $(-2,0)$

$(x-3)(x+2)$ $(x+i)(x-i)$

Standard form: $f(x) = (x^2 - x - 6)(x^2 + 1)$

$f(x) = x^4 + x^2 - x^3 - x - 6x^2 - 6$

$f(x) = x^4 - x^3 - 5x^2 - x - 6$

Write a polynomial function of minimum degree in factored form with real coefficients whose zeros include those listed, find the degree of the polynomial (# of zeros) and identify the x-intercepts.

g) $2 - i$ and $2 + i$

Zeros: $x = 2 - i$ $x = 2 + i$

x-intercepts: none

Factored form:

$f(x) = (x - 2 + i)(x - 2 - i)$

h) 4,7, and $2i$

Zeros: $x = 4$ $x = 7$ $x = 2i$ $x = -2i$

x-intercepts: $(4,0)$ $(7,0)$

$x = 4$ $x = 7$

Factored form:

$f(x) = (x-4)(x-7)(x-2i)(x+2i)$

Find the degree of the polynomial (# of zeros), the zeros and identify the x-intercepts. Then write the polynomial in factored form.

i) 2 (multiplicity of 3), −5 (multiplicity of 2)

Zeros: $x = 2$ $x = -5$

Factored form:

$f(x) = (x-2)^3(x+5)^2$

Degree: 5

x-intercepts: $x = 2$ $x = -5$ or

$(2,0)$ $(-5,0)$

j) 1 (multiplicity of 4), 5-2i (multiplicity of 1)

Zeros: $x = 1$ $x = 5 - 2i$ $x = 5 + 2i$

Factored form:

$f(x) = (x-1)^4(x - 5 + 2i)(x - 5 - 2i)$

Degree: 5

x-intercepts: $x = 1$ or $(1,0)$

Secondary Math 3

4.4 Key Features of Functions Notes

FILLED IN
* NOTES *

Domain: All possible values of x in a function.

To find the domain of a function, we must first realize that there are <u>two</u> things that will give us difficulty: a function that has a **denominator** and a function with an **even root radicals**. Today we will look at functions with **even root radicals.**

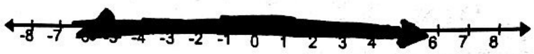
Even Root Radicals: Functions that have even roots cannot have negative numbers under the radical sign. (Negative values under the square root are imaginary, and will not exist in our domain of *real* numbers.)

**** When a function does not have an even root or a denominator, then the domain is all real numbers. ****

1. $f(x)=3(x+2)-2$

Interval Notation: $(-\infty,\infty)$

Set Notation: $\{x \mid x \in R\}$

Graph: 

Eliminating restricted values in even root radicals:

- Set whatever is under the radical greater than or equal to zero (≥ 0).
- Solve for x
- The values from the inequality are values that are allowed in the domain.

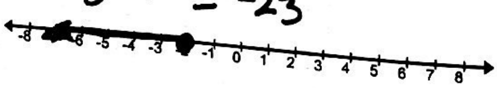
Examples:

2. $f(x)=\sqrt{2x+4}$

$2x+4 \geq 0$
$2x \geq -4$
$x \leq -2$

Interval Notation: $(-\infty, -2]$

Set Notation: $\{x \mid x \leq -2\}$

Graph: 

3. $f(x)=\sqrt{3-x}$

$3-x \geq 0$
$3 \geq x$

Interval Notation: $(-\infty, 3]$

Set Notation: $\{x \mid x \leq 3\}$

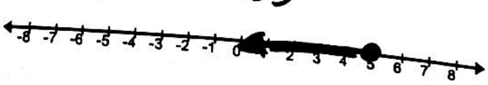
Graph: 

4. $f(x)=(x+2)\sqrt{5-x}$

$5-x \geq 0$
$5 \geq x$

Interval Notation: $(-\infty, 5]$

Set Notation: $\{x \mid x \leq 5\}$

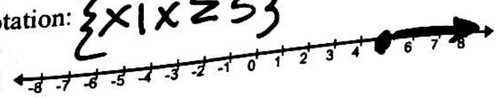
Graph: 

5. $f(x)=-(x+2)^3\sqrt{2x-10}$

$2x-10 \geq 0$
$2x \geq 10$
$x \geq 5$

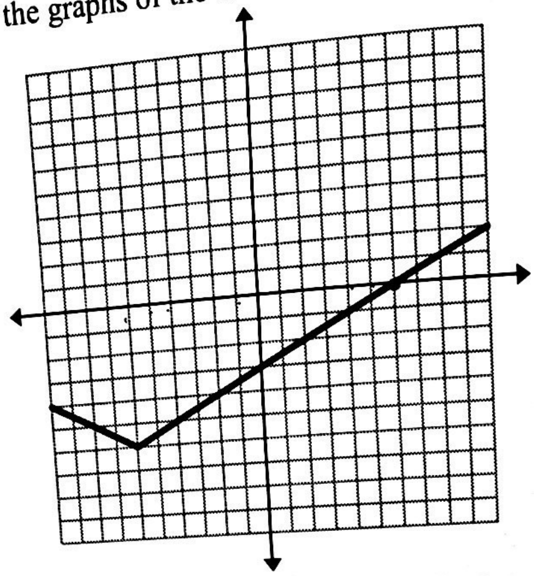
Interval Notation: $[5, \infty)$

Set Notation: $\{x \mid x \geq 5\}$

Graph: 

Given the graphs of the functions below, determine the key features.

6.



Domain: $(-\infty,\infty)$

Range: $[-6,\infty)$

x-intercept(s): $(6,0)$

y-intercept: $(0,-3)$

Increasing: $(-6,\infty)$

Decreasing: $(-\infty,-6)$

Constant: NA

Positive: $(6,\infty)$

Negative: $(-\infty,6)$

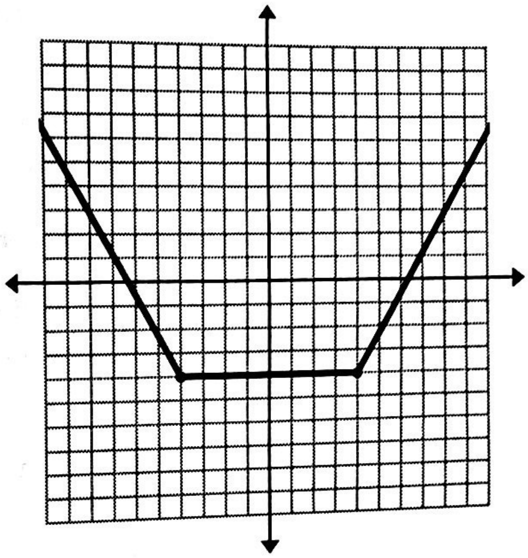
Maximums / minimums: min $(-6,-6)$

Symmetry: NA

End Behavior:

$\lim_{x\to-\infty} f(x) = \infty$ $\quad \lim_{x\to\infty} f(x) = \infty$

7.



Domain: $(-\infty,\infty)$

Range: $[-4,\infty)$

x-intercept(s): $(6.3,0)$ $(-6.3,0)$

y-intercept: $(0,-4)$

Increasing: $(4,\infty)$

Decreasing: $(-\infty,-4)$

Constant: $(-4,4)$

Positive: $(-\infty,-6.3)\cup(6.3,\infty)$

Negative: $(-6.3,6.3)$

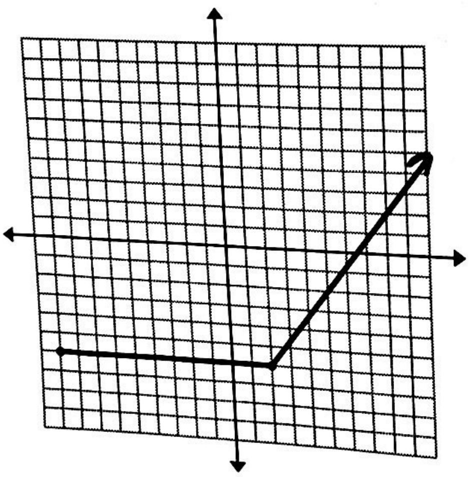
Maximums / minimums: value -4

Symmetry: even

End Behavior:

$\lim_{x\to-\infty} f(x) = \infty$ $\quad \lim_{x\to\infty} f(x) = \infty$

8.



Domain: $[-9,\infty)$

Range: $[-6,\infty)$

x-intercept(s): $(6.5,0)$

y-intercept: $(0,-6)$

Increasing: $(2,\infty)$

Decreasing: NA

Constant: $(-9,2)$

Positive: $(6.5,\infty)$

Negative: $[-9,6.5)$

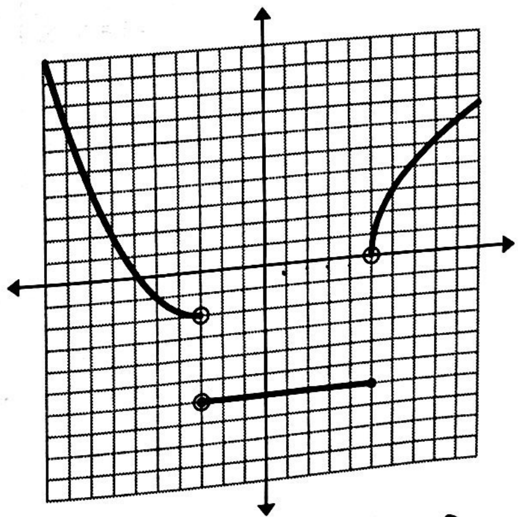
Maximums / minimums: min value -6

Symmetry: NA

End Behavior:

$\lim_{x\to-\infty} f(x) =$ DNE or NA $\quad \lim_{x\to\infty} f(x) = \infty$

9.



Domain: $(-\infty,-3)\cup(-3,\infty)$

Range: $[-6,-6]\cup(-2,\infty)$

x-intercept(s): $(-6,0)$

y-intercept: $(0,-6)$

Increasing: $(5,\infty)$

Decreasing: $(-\infty,-3)$

Constant: $(-3,5]$.

Positive: $(-\infty,-6)$

Negative: $(-6,-3)\cup(-3,5]$

Maximums / minimums: min value -6

Symmetry: NA

End Behavior:

$\lim_{x\to-\infty} f(x) = \infty$ $\quad \lim_{x\to\infty} f(x) = \infty$