11.6 N - Solving Logarithmic Equations

A. Review

1) $\log_3 x = 4$

$3^4 = x$

$x = 81$

2) $\frac{27}{27}\left(\frac{1}{3}\right)^{x/5} = \frac{1}{27}$

$\left(\frac{1}{3}\right)^{x/5} = \frac{1}{3^3}$

$\left(3^{-1}\right)^{x/5} = 3^{-3}$

$-\frac{x}{5} = -3$

$x = 15$

3) $\frac{16 \cdot 4^{x/3}}{16} = \frac{1024}{16}$

$4^{x/3} = 64$

$4^{x/3} = 4^3$

$\frac{x}{3} = 3$

$x = 9$

B. Use the Properties of Logarithms and Exponents to solve equations.

- To avoid extraneous solutions, determine the domain of the variable first.
- Use the *properties of logarithms and exponents* to manipulate the equations.
- Try rewriting as an exponential or logarithmic function: $y = \log_a x \Leftrightarrow x = a^y$
- Remember the properties: $\log_a M = \log_a N \Leftrightarrow M = N$ and $a^u = a^v \Leftrightarrow u = v$ (Make the bases the same).
- Check your solution by substituting into the original equation.

Rewrite as log & use change of base Formula

a) $4^x = 37$

$\log_4 37 = x$ change base

$\frac{\log 37}{\log 4} = x$

$2.6047 \approx x$

b) $2.05^x = 4.36$

$\log_{2.05} 4.36 = x$

$\frac{\log 4.36}{\log 2.05} = x$

$2.0513 \approx x$

c) $30e^{0.014x} = 600$

$e^{.014x} = 20$

$\ln 20 = .014x$

$\frac{\ln 20}{.014} = x$

$213.9809 \approx x$

d) $8 - 5e^{-x} = -12$

$-5e^{-x} = -20$

$e^{-x} = 4$

$\ln 4 = -x$

$-\ln 4 = x$

$-1.3863 \approx x$

e) $2^{4-x} - 7 = 14$

$2^{4-x} = 21$

$\log_2 21 = 4 - x$

$\frac{\log 21}{\log 2} - 4 = -x$

$.3923 = -x$

$-.3923 = x$

f) $\log_4 x = \log_4 (3x - 8)$

$x = 3x - 8$

$-2x = -8$

$x = 4$

g) $\ln x^2 = 8$

$\sqrt{e^8} = \sqrt{x^2}$

$\pm e^{8/2} = x$

$\pm e^4 = x$

or

$x = \pm 54.582$

h) $-4\log(x+5) - 3 = -4$

$-4\log(x+5) - 3 = -4$

$-4\log(x+5) = -1$

$\log(x+5) = \frac{1}{4}$

$10^{1/4} = x+5$

$x = -5 + 10^{1/4}$ or $x = -3.2217$

i) $-2\log_4 x = \log_4 9$

$\log_4 x^{-2} = \log_4 9$

$x^{-2} = 9$

$\frac{1}{x^2} = \frac{9}{1}$

$9x^2 = 1$

$x^2 = 1/9$

$x = \pm 1/3$ ($-1/3$ extraneous)

$x = \frac{1}{3}$

j) $3\log_2(x-1) + \log_2 4 = 5$

$\log_2 (x-1)^3 \cdot 4 = 5$

$2^5 = (x-1)^{3.4}$

$32 = (x-1)^{3.4}$

$\sqrt[3]{8} = \sqrt[3]{(x-1)^3}$

$2 = x-1$

$3 = x$

k) $\ln(5x) - \ln(10) = 5$

$\ln \frac{5x}{10} = 5$

$\ln \frac{x}{2} = 5$

$e^5 = \frac{x}{2}$

$2e^5 = x$ or $x = 296.8263$

l) $\log_6(x+4) + \log_6(x+3) = 1$

$\log_6 (x+4)(x+3) = 1$

$6^1 = (x+4)(x+3)$

$6 = x^2 + 7x + 12$

$0 = x^2 + 7x + 6$

$0 = (x+6)(x+1)$

$x = -6$ (EXTRANEOUS) $x = -1$

m) $\log_2 x + \log_2(x-2) = \log_2(x+4)$

$\log_2 x(x-2) = \log_2(x+4)$

$x(x-2) = x+4$

$x^2 - 2x = x+4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ $x = -1$ (EXTRANEOUS)