

## 11.2N – Exponential Functions

A. Warm-up: Practice **Laws of Exponents**.

$$a^s \cdot a^t = a^{s+t} \quad (a^s)^t = a^{s \cdot t} \quad (ab)^s = a^s b^s \quad 1^s = 1 \quad a^{-s} = \frac{1}{a^s} \quad a^0 = 1$$

B. Properties of Exponential Functions

An **exponential function** is a function of the form  $y = a^x$  where  $a$  is a positive real number ( $a > 0$ ) and  $a \neq 1$ . The domain of  $f$  is the set of all real numbers.

**Examples:** Determine if the functions below are **exponential** and explain why or why not.

$x$	$f(x)$
-1	12
0	4
1	$\frac{4}{3}$
2	$\frac{4}{9}$
3	$\frac{4}{27}$

yes. Mult. by  $\frac{1}{3}$  or  $\times \frac{1}{3}$

Properties of the Exponential Function  $f(x) = a^x$ ,  $a > 0$ ,  $a \neq 1$

$x$	$f(x)$
-1	2
0	5
1	8
2	11
3	14

no - Linear

$x$	$f(x)$
-1	$\frac{2}{3} \cdot \frac{3}{2} = 1$
0	$1 \cdot \frac{3}{2} = \frac{3}{2}$
1	$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$
2	$\frac{9}{4} \cdot \frac{3}{2} = \frac{27}{8}$
3	$\frac{27}{8} \cdot \frac{3}{2} = \frac{81}{16}$

yes. Mult. by  $\frac{3}{2}$

- Domain:  $(-\infty, \infty)$  Range:  $(0, \infty)$

• There are no  $x$  int.; the  $y$ -intercept is 1.

• The  $x$ -axis ( $y = 0$ ) is a asymptote.

o For  $a > 1$ , the graph approaches the  $x$ -axis as

$x$  gets closer to  $-\infty$

o For  $0 < a < 1$ , the graph approaches the  $x$ -axis as

$x$  gets closer to  $\infty$

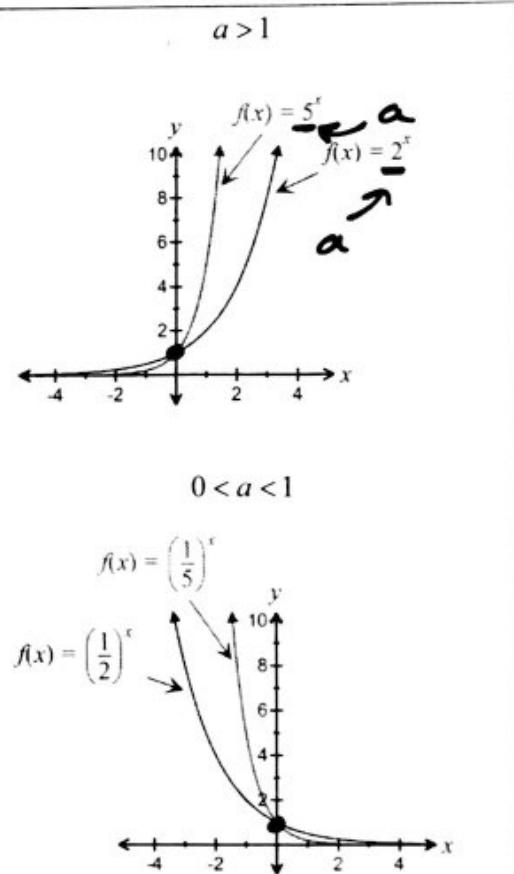
•  $f(x) = a^x$  is one-to-one.

o For  $a > 1$ ,  $f(x) = a^x$  is an increasing function.  
and positive

o For  $0 < a < 1$ ,  $f(x) = a^x$  is a decreasing function.  
and positive

\* The graph of  $f$  contains the points  $(0, 1)$ , y int  
 $(1, a)$  and  $(-1, \frac{1}{a})$

\*  $a$  is the base



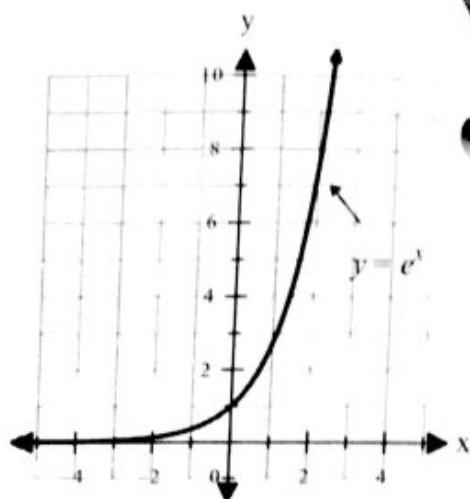
C. The number  $e$

- The number  $e$  (approximately 2.71828...) is defined as the number that the expression  $\left(1 + \frac{1}{n}\right)^n$  approaches as  $n \rightarrow \infty$ . In calculus, this is expressed using limit notation as

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n.$$

$$e^x = (2.71828\ldots)^x$$

- Find  $e^2 = 7.389056\ldots$



D. Review Transformations. (No Calculators!)

- The general equation for an exponential function is  $y = b \cdot a^{c(x-h)} + k$

List the transformation that corresponds with each variable.

$$b = \begin{array}{l} \text{vertical stretch or shrink} \\ \text{horizontal stretch or shrink} \end{array} \quad c = \begin{array}{l} \text{horizontal stretch or shrink} \\ \text{(opposite)} \end{array} \quad h = \begin{array}{l} \text{horizontal shift} \\ \text{negative exponent} \end{array} \quad k = \begin{array}{l} \text{vertical shift} \\ \text{reflection over y-axis} \end{array}$$

Negative function Reflection over x axis

Negative exponent Reflection over y axis

- Without a Calculator, match each equation to the appropriate graph.

a)  $y = 2^x$

b)  $y = -2^x$  reflect over x axis

c)  $y = 2^{-x}$  reflect over y axis

d)  $y = 2^x - 1$  down 1

e)  $y = -2^{-x}$  reflect x axis right 1

f)  $y = 2^{x-1}$  reflect x axis up 1

g)  $y = 1 - 2^x$  reflect y axis left 1

h)  $y = 2^{1-x}$  reflect over y axis left 1

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E. Graphing using transformations and 3 key points.

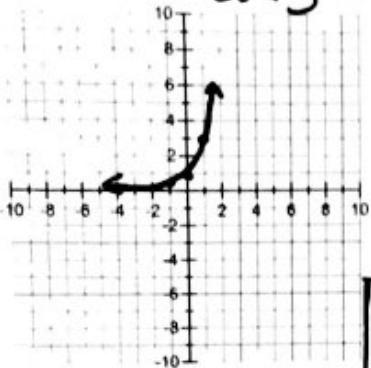
**Examples:** Use 3 key points and transformations to graph. (No Calculators!) Find domain, range, and horizontal asymptote.

\* Key Points  

$$\begin{array}{|c|c|} \hline x & f(x) \\ \hline -1 & 0 \\ 0 & a \\ 1 & a \\ \hline \end{array}$$
 / a is base

a) Graph  $f(x) = 3^x$ .

$$a=3$$



Domain:

$$(-\infty, \infty)$$

Range:

$$(0, \infty)$$

Horizontal asymptote:

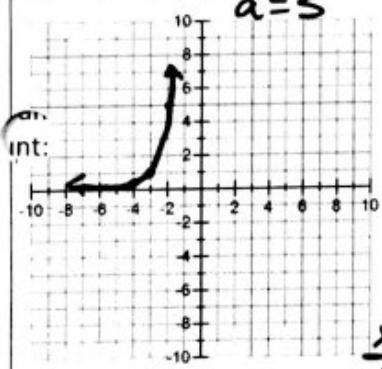
$$x \text{ axis or } y=0$$

Key points and transformations:

x	f(x)
-1	1/3
0	1
1	3

c) Graph  $f(x) = 5^{x+3}$ .

$$a=5$$



Domain:

$$(-\infty, \infty)$$

Range:

$$(0, \infty)$$

Horizontal asymptote:

$$y=0$$

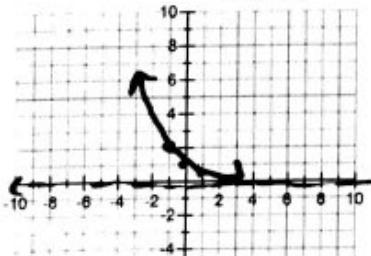
Key points and transformations:

left 3	
-3 +	-1 1/5
-3 +	0 1
-3 +	1 5

x	f(x)
-4	1/5
-3	1
-2	5

e) Graph  $f(x) = 2^{-x}$ .

$$a=2$$



Domain:

$$(-\infty, \infty)$$

Range:

$$(0, \infty)$$

Horizontal asymptote:

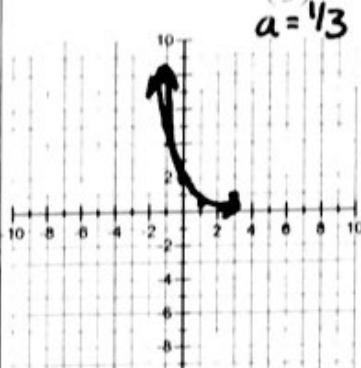
$$y=0$$

Key points and transformations:

x	f(x)
-1	1/2
0	1
1	2

b) Graph  $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$ .

$$a=1/3$$



Domain:

$$(-\infty, \infty)$$

Range:

$$(0, \infty)$$

Horizontal asymptote:

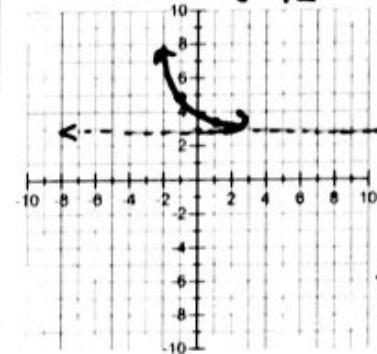
$$y=0$$

Key points and transformations:

x	f(x)
-1	3/2
0	1
1	2/3

d) Graph  $f(x) = \left(\frac{1}{2}\right)^x + 3$ .

$$a=1/2$$



Domain:

$$(-\infty, \infty)$$

Range:

$$(3, \infty)$$

Horizontal asymptote:

$$y=3$$

Key points and transformations:

x	f(x)
-1	2+3
0	1+3
1	1/2+3

x	f(x)
-1	5
0	4
1	3.5

f) Graph  $f(x) = -3^x$ .

Domain:

$$(-\infty, \infty)$$

Range:

$$\text{bottom to top } (-\infty, 0)$$

Horizontal asymptote:

$$y=0$$

Key points and transformations:

x	f(x)
-1	1/3 - 1
0	1 - 1
1	3 - 1

x	f(x)
-1	-1/3
0	-1
1	-3