11.2N – Exponential Functions

A. Warm-up: Practice ***Laws of Exponents.***

$a^s \cdot a^t =$ $a^s \cdot a^t = a^{s+t}$ $(a^s)^t = a^{s \cdot t}$ $(ab)^s = a^s b^s$ $1^s =$ 1 $a^{-s} = \frac{1}{a^s}$ $a^0 =$ 1

B. Properties of Exponential Functions

An ***exponential function*** is a function of the form $y = a^x$ where a is a positive real number $(a > 0)$ and $a \neq 1$. The domain of f is the set of all real numbers.

Examples: Determine if the functions below are exponential and explain why or why not.

x	$f(x)$
−1	12
0	4
1	4/3
2	4/9
3	4/27

÷3 or × 1/3 ÷3

yes. Mult. by $\frac{1}{3}$ or ÷3

x	$f(x)$
−1	2
0	5
1	8
2	11
3	14

+3 +3 +3 +3

no – Linear

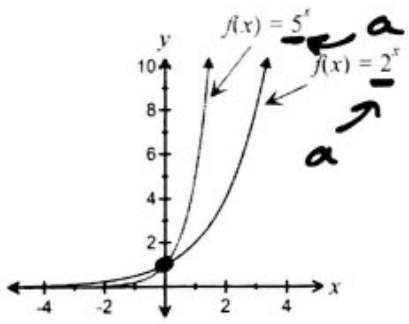
x	$f(x)$
−1	2/3 · 3/2 = 1
0	1 · 3/2 = 3/2
1	3/2 · 3/2 = 9/4
2	9/4
3	27/8

yes. Mult. by 3/2

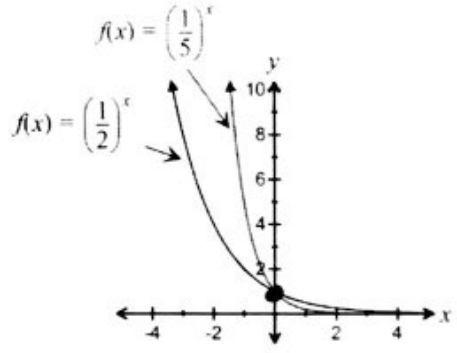
Properties of the Exponential Function $f(x) = a^x,\ a > 0,\ a \neq 1$

- Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
- There are no x int. ; the y-intercept is 1.
- The x-axis $(y = 0)$ is a asymptote.
  - For $a > 1$, the graph approaches the x-axis as x gets closer to $-\infty$
  - For $0 < a < 1$, the graph approaches the x-axis as x gets closer to ∞
- $f(x) = a^x$ is one-to-one.
  - For $a > 1$, $f(x) = a^x$ is an increasing and positive function.
  - For $0 < a < 1$, $f(x) = a^x$ is a decreasing and positive function.
- The graph of f contains the points $(0, 1)$ y int, $(1, a)$, and $(-1, 1/a)$
- * a is the base

$a > 1$

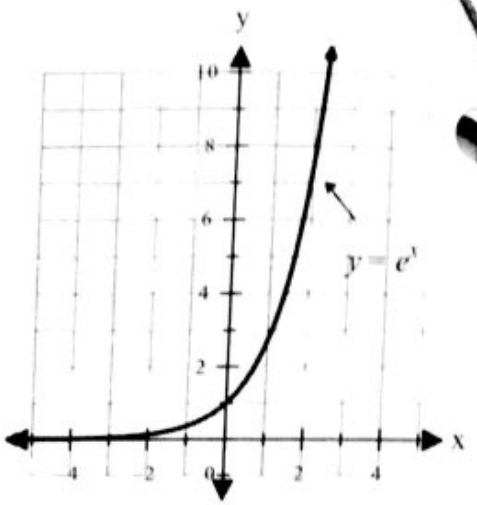


$0 < a < 1$



C. The number e

- The **number e** (approximately 2.71828...) is defined as the number that the expression $\left(1+\frac{1}{n}\right)^n$ approaches as $n \to \infty$. In calculus, this is expressed using limit notation as $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. $e^x = (2.71828...)^x$



- Find e^2 = 7.389056...

D. Review Transformations. (No Calculators!)

- The general equation for an exponential function is $y = b \cdot a^{c(x-h)} + k$
  List the transformation that corresponds with each variable.

b = vertical stretch or shrink c = horizontal stretch or shrink (opp) h = horizontal shift (opp) k = vertical shift

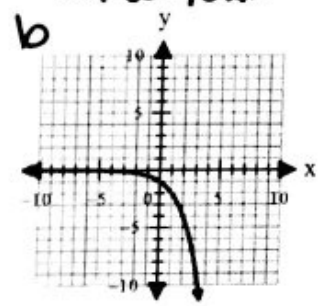
Negative function Reflection over x axis Negative exponent Reflection over y axis

- Without a Calculator, match each equation to the appropriate graph.

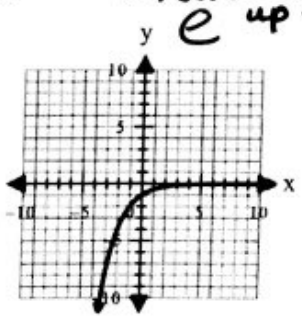
a) $y = 2^x$ *b)* $y = -2^x$ reflect over x axis *c)* $y = 2^{-x}$ reflect y axis *d)* $y = 2^x - 1$ down 1

e) $y = -2^{-x}$ reflect x axis reflect y axis *f)* $y = 2^{x-1}$ right 1 *g)* $y = 1 - 2^x$ reflect x axis up 1 *h)* $y = 2^{1-x}$ reflect over y axis left 1

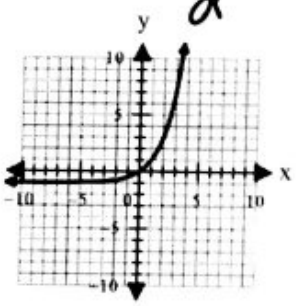
1. b



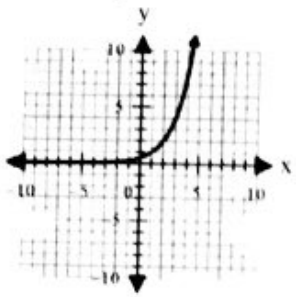
2. e



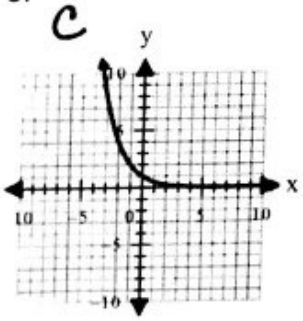
3. d



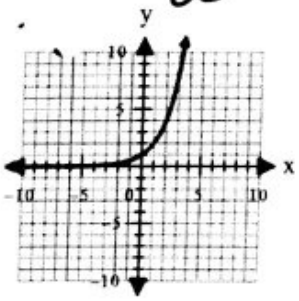
4. f



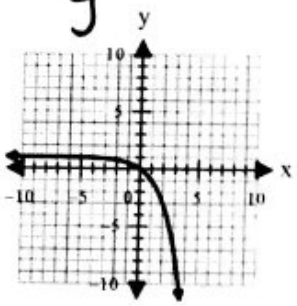
5. c



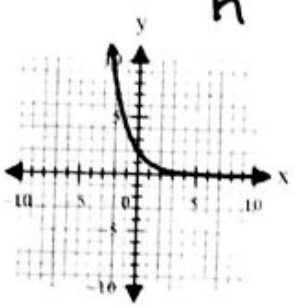
6. a



7. g



8. h



E. Graphing using transformations and 3 key points.

* Key Points

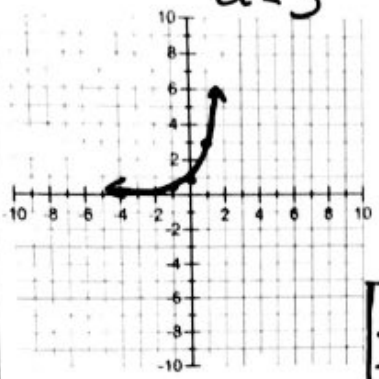
x	f(x)
-1	ya
0	1
1	a

a is base

Examples: Use 3 key points and transformations to graph. (No Calculators!) Find domain, range, and horizontal asymptote.

a) Graph $f(x) = 3^x$.

a=3



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

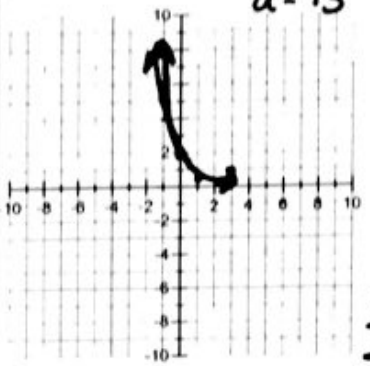
Horizontal asymptote: x axis or y=0

Key points and transformations:

x	f(x)
-1	1/3
0	1
1	3

b) Graph $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$.

a=1/3



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: y=0

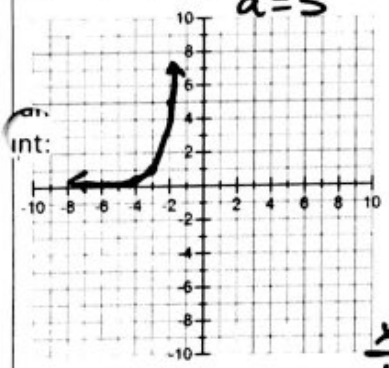
Key points and transformations: Stretch by 2

x	f(x)
-1	3·2
0	1·2
1	1/3·2/1

x	f(x)
-1	6
0	2
1	2/3

c) Graph $f(x) = 5^{x+3}$.

a=5



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: y=0

Key points and transformations: left 3

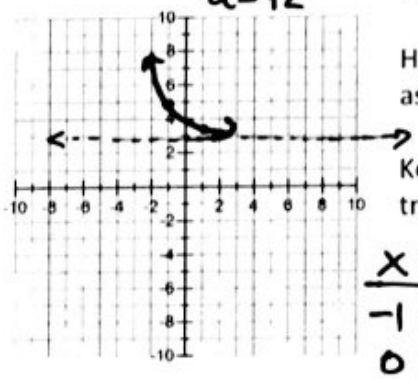
x	f(x)
-1	1/5
0	1
1	5

-3+ -3+ -3+

x	f(x)
-4	1/5
-3	1
-2	5

d) Graph $f(x) = \left(\frac{1}{2}\right)^x + 3$.

a=1/2



Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

Horizontal asymptote: y=3

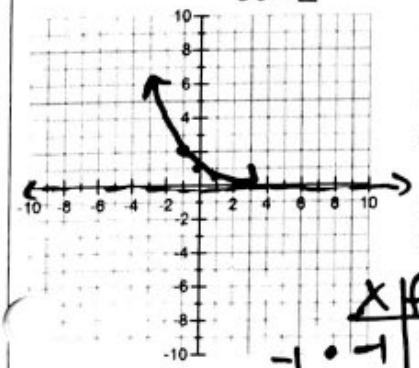
Key points and transformations: up 3

x	f(x)
-1	2+3
0	1+3
1	1/2+3

x	f(x)
-1	5
0	4
1	3.5

e) Graph $f(x) = 2^{-x}$.

a=2



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal asymptote: y=0

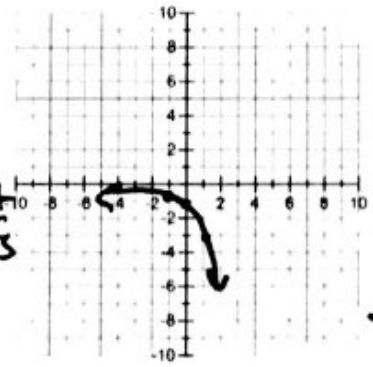
Key points and transformations: reflect over y axis

x	f(x)
-1	1/2
0	1
1	2

-1· -1· -1·

x	f(x)
1	1/2
0	1
-1	2

f) Graph $f(x) = -3^x$



Domain: $(-\infty, \infty)$

Range: bottom to top $(-\infty, 0)$

Horizontal asymptote: y=0

Key points and transformations: Reflected x axis

x	f(x)
-1	1/3·-1
0	1·-1
1	3·-1

x	f(x)
-1	-1/3
0	-1
1	-3