11.1N – Exponents Review and Solving by Changing Base

A. Basic Properties of Exponents

Review Properties of Exponents

1.	$b^0 = 1$	Zero Property	1) $11^0 = 1$
2.	$b^{-n} = \frac{1}{b^n}$ or $\frac{1}{b^{-n}} = b^n$	Negative Exponent Property	1) $5^{-3} = \frac{1}{5^3}$ 2) $\frac{1}{2^{-3}} = \frac{2^3}{2\cdot2\cdot2} = 8$ 3) $\left(\frac{1}{6}\right)^{-2} = \frac{1^{-2}}{6^{-2}} = \frac{6^2}{1^2}$ 4) $9 = 3^{\boxed{2}} = \left(\frac{\boxed{1}}{\boxed{3}}\right)^{-2}$
3.	$(b^m)(b^n) = b^{m+n}$	Product Rule add exp.	1) $x^6x^8 = x^{14}$
4.	$\frac{b^m}{b^n} = b^{m-n}$	Quotient Rule	1) $\frac{x^4}{x^2} = x^2$ 2) $\frac{x^6}{x^7} = \frac{1}{x^1} = \frac{1}{x}$
5.	$(b^m)^n = b^{m \cdot n}$	Power to a Power Rule	1) $(4x)^2 = 4^2x^2 = 16x^2$ 2) $4x^2 = 4x^2$
6.	$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	Positive Rational Exponents	1) $16^{\frac{3}{2}} = (\sqrt{16})^3$ $4^3 = 64$ 2) $\frac{1}{8^{-\frac{4}{3}}} = 8^{4/3} = (\sqrt[3]{8})^4$ $\cancel{2\cdot2\cdot2}$ $2^4 = 16$

B. Write numbers as exponents.

Example: $9 = 3^2$ Hint: They all have more than one answer.	1. $4 = 2^2$	2. $16 = 4^2$ $16 = 2^4$	3. $32 = 2^5$ $32 = \frac{1}{2^{-5}} = 2^5$	4. $27 = 3^3$ $27 = \frac{1}{3^{-3}}$	5. $243 = 3^5$ or $\frac{1}{3^{-5}}$
	6. $\frac{1}{25} = \frac{1}{5^2}$ $\frac{1}{25} = 5^{-2}$	7. $\frac{1}{2} = 2^{-1}$	8. $\frac{1}{6^x} = 6^{-x}$	9. $81 = 9^2$ $81 = \frac{1}{9^{-2}}$ $81 = 3^4$	10. $\frac{1}{7} = 7^{-1}$

C. Same base

- In the expression, 5^2: **5** is the base and **2** is the exponent.
- If the *bases* of both sides of an exponential equation are the same:

$$B^m = B^n$$

then

the *exponents* are *equal*: $m = n$

example

$$2^5 = 2^5$$

If the bases are the same the exponents must be the same.

D. Steps to Solve by changing the base

$5^{3x} = \frac{1}{125}$	Given
$5^{3x} = \frac{1}{5^3}$	Express the denominator of the right side with a base of 5. We have $125 = 5^3$.
	Apply the Negative Exponent Property.
$5^{3x} = 5^{-3}$ exponents same so bases same	At this point, the bases are the same. Set the exponents equal to each other.
$3x = -3$	Solve for x.
$\frac{\cancel{3}x}{\cancel{3}} = \frac{-3}{3}$	
$x = -1$	To solve x, divide both sides by 3. That's it.

E. Examples Trying to get same base & set exp. equal and solve

1. $4^5 = 4^x$

$\boxed{5 = x}$

bases are same so exp. are same.

2. $7^{-3x-5} = 7^{2x}$ bases same

$-3x - 5 = 2x$
$+3x \quad +3x$
$\frac{-5}{5} = \frac{5x}{5}$
$\boxed{-1 = x}$

3. $3^{-3n} = 243$

$3^{-3n} = 3^{5}$
$\frac{-3n}{-3} = \frac{5}{-3}$
$\boxed{n = \frac{-5}{3}}$

4. $5^{-3x-3} = \frac{1}{625}$

$5^{-3x-3} = \frac{1}{5^4}$
$5^{-3x-3} = 5^{-4}$
$-3x - 3 = -4$ solve
$+3 \quad +3$
$\frac{-3x}{-3} = \frac{-1}{-3}$
$\boxed{x = \frac{1}{3}}$

5. $16^{m+1} = 64$ change both #'s so same

$4^{2(m+1)} = 4^3$
$2(m+1) = 3$
$2m + 2 = 3$
$\frac{2m}{2} = \frac{1}{2}$
$\boxed{m = 1/2}$

6. $81^{m+2} = \frac{1}{9}$

$9^{2(m+2)} = 9^{-1}$
$2(m+2) = -1$
$2m + 4 = -1$
$-4 \quad -4$
$2m = -5$
$\boxed{m = -5/2}$

7. $\left(\frac{1}{9}\right)^{-3r-2} = 27^r$

$(3^{-2})^{-3r-2} = 3^{3r}$
$-2(-3r-2) = 3r$
$6r + 4 = 3r$
$\frac{3r}{3} = \frac{-4}{3}$
$\boxed{r = -\frac{4}{3}}$

8. $\frac{4^{-x}}{4^{5x-2}} = 32$ simplify change signs

$4^{-x-5x+2} = 32$
$2^{2(-6x+2)} = 2^5$
$2(-6x+2) = 5$
$-12x + 4 = 5$
$-12x = 1$
$\boxed{x = -1/12}$

9. $\frac{16}{2^{2n+1}} = 8$

$\frac{2^4}{2^{2n+1}} = 2^3$
$2^{4-2n-1} = 2^3$
$4 - 2n - 1 = 3$
$3 - 2n = 3$
$-2n = 0$
$\boxed{n = 0}$