10.4N – Modeling Periodic Behavior

$f(x) = a\sin(b(x-c)) + d$

$f(x) = a\cos(b(x-c)) + d$

A. Write an equation for the sine curve that has the given information.

Period $= \frac{2\pi}{b}$

1. Amplitude = 4 Vertical Shift = down 2 Period = π

$f(x) = 4\sin(2x) - 2$

Period $= \frac{2\pi}{b}$

$\frac{\pi}{1} = \frac{2\pi}{b}$ $\quad \frac{b\pi}{\pi} = \frac{2\pi}{\pi}$ $\quad b = 2$

2. Amplitude = 3 Phase Shift = right $\frac{\pi}{3}$ Period = $\frac{\pi}{4}$

$f(x) = 3\sin(8(x - \frac{\pi}{3}))$ ← opposite

Period $= \frac{2\pi}{b}$

$\frac{\pi}{4} = \frac{2\pi}{b}$ $\quad \frac{b\pi}{\pi} = \frac{8\pi}{\pi}$ $\quad b = 8$

B. Write an equation for the cosine curve that has the given information.

1. Amplitude = 1 Vertical Shift = up $\frac{5}{8}$ Period = $\frac{\pi}{6}$

$f(x) = 1\cos(12(x)) + \frac{5}{8}$

$\frac{\pi}{6} = \frac{2\pi}{b}$

$b\pi = 12\pi$

$b = 12$

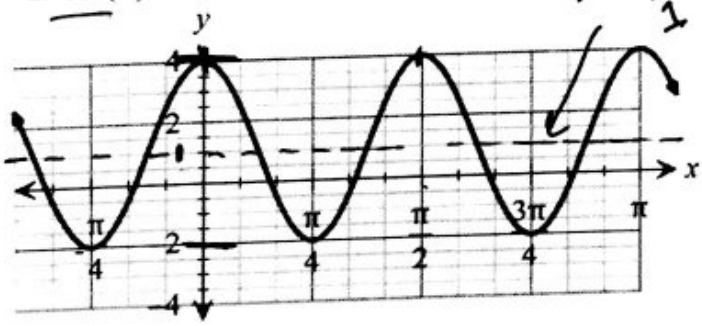
2. Amplitude = 3 Phase Shift = left $\frac{\pi}{6}$ Period = 2π

$f(x) = 3\cos(x + \frac{\pi}{6})$

$\frac{2\pi}{1} = \frac{2\pi}{b}$ $\quad \frac{b2\pi}{2\pi} = \frac{2\pi}{2\pi}$ $\quad b = 1$

C. Given the graph, write either a sine or cosine equation.

1. $\cos(\theta)$



* amplitude is distance from midline to max or minpoint
* vertical shift is the midline
* Period – how long it takes graph to complete one cycle

V.S = 1 outside parentheses

$a = \frac{6}{2} = 3$

Period is $\frac{\pi}{2}$ (one cycle)

form pd of

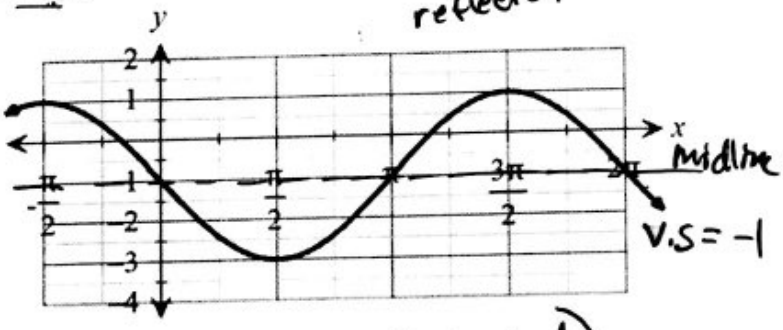
$\frac{\pi}{2} = \frac{2\pi}{b}$

$\pi b = 4\pi$

$b = 4$

$f(x) = 3\cos 4(x) + 1$

2. $\sin(\theta)$



$a = 2$ (reflected)

Period $= 2\pi$ (one cycle)

$2\pi = \frac{2\pi}{b}$

$b = 1$

$f(x) = -2\sin(x) - 1$

Period is Time

D. Read each story and write the appropriate trigonometric function to model each periodic situation below.

cosine ∩ sine ∩

1. At the Bay of Fundy, low tide is at 11:30 am and high tide is at 5:30 pm. The water level varies 50 feet between low and high tide. Write a cosine equation that represents this function.

Range

6 hrs. 1/2 cycle

Period is 12 hrs.

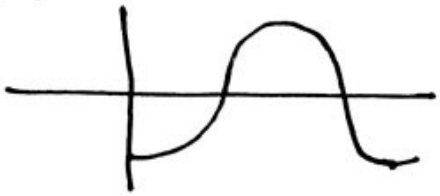
$a = \frac{\text{Range}}{2} = \frac{50}{2} = 25$ ft

Reflected started low

$\frac{12}{1} = \frac{2\pi}{b}$

$\frac{12b}{12} = \frac{2\pi}{12}$

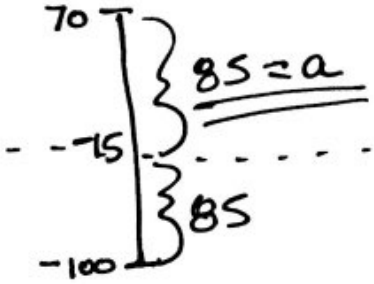
$b = \pi/6$



$f(x) = -25\cos \pi/6 x$

2. On Mars at the equator, the temperature varies from 70° F to -100° F in a single day. Write a sine equation that represents this function.

1 Day = Period



midline is Vertical shift

midline is the average

$\frac{70 + -100}{2} = \frac{-30}{2} = -15$

$\frac{1}{1} = \frac{2\pi}{b}$

$b = 2\pi$

V.S = -15

$f(x) = 85 \sin 2\pi(x) - 15$

3. A Ferris wheel 100 feet in diameter makes one revolution every 60 seconds. The center of the wheel is 50 above the ground. People load at the bottom of the Ferris wheel. Write a cosine function to model the height of a car on the Ferris wheel at any time *t*.

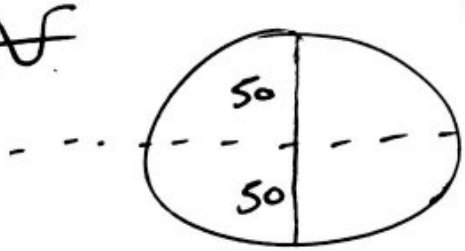
V.S.

load a ferris wheel start low

opposite of cosine curve - reflected.

∩

V.S. = 50



$f(x) = -50\cos \frac{\pi}{30}(x) + 50$

$a = 50$

Period = 60

$\frac{60}{1} = \frac{2\pi}{b}$

$\frac{60b}{60} = \frac{2\pi}{60}$

$b = \frac{\pi}{30}$

4. A greater wax moth has hearing capable of sensing high-frequency sounds up to 300,000 cycles per second. Write a sine function representing the sound wave of the pitch. (Amplitude is 1.)

Freq. is reciprocal of period.

$\text{Frequency} = \frac{300{,}000 \text{ cycles}}{1 \text{ min}}$

$\text{Period} = \frac{1}{300{,}000}$

$\frac{1}{300{,}000} = \frac{2\pi}{b}$

$b = 600000\pi$

$f(x) = \sin 600{,}000\pi x$