CONIC SECTIONS Unit 5 Notes

Parabolas

Parabolas with Vertex (h, k)		
Transformation Equation	$(x-h)^2 = 4a(y-k)$	$(y-k)^2 = 4a(x-h)$
Opens	Down if a < 0 Up if a > 0	Left if a < 0 Right if a > 0
Axis of Symmetry	x = h	y = k

Circles

Circles with Center (h, k)	
General Equation	$(x-h)^2 + (y-k)^2 = r^2$
Radius	r

Ellipses

$a^2 - b^2 = c^2$

Ellipses with Center (h, k) a > b		
Standard Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Radii	horizontal = a vertical = b	horizontal = b vertical = a

Hyperbolas

$a^2 + b^2 = c^2$

Hyperbolas with center (h, k) a is always first		
Standard equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Vertices	(h ± a, k)	(h, k ± a)
Asymptotes from the center	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$
Opens	Left and Right	Up and Down

Classifying Conic Sections

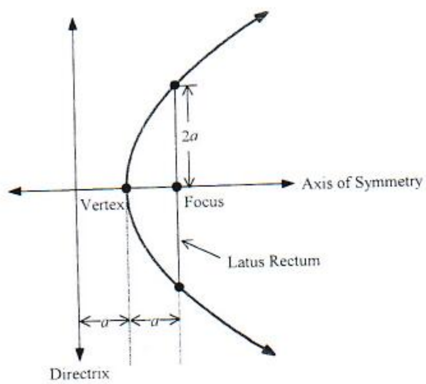
Equations	Circles	Parabolas	Ellipse	Hyperbola
$Ax^2 + Cy^2 + Dx + Ey + F = 0$	$A = C$	$AC = 0$, both are not 0	$AC > 0$	$AC < 0$

Parabolas

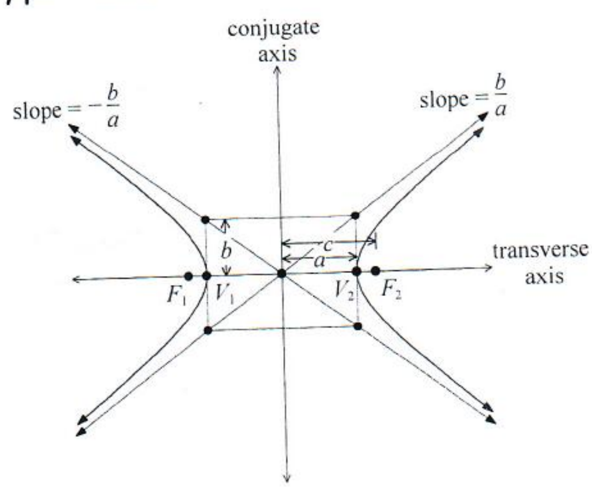
General Forms of the Equation of a Parabola with Vertex (h,k)

a = Distance from Focus to Vertex **a = Distance from Vertex to Directrix**

Equation	Description	Picture
$(y-k)^2 = 4a(x-h)$	Opens Right, Axis of Symmetry parallel to x-axis	
$(y-k)^2 = -4a(x-h)$	Opens Left, Axis of Symmetry parallel to x-axis	
$(x-h)^2 = 4a(y-k)$	Opens Up, Axis of Symmetry parallel to y-axis	
$(x-h)^2 = -4a(y-k)$	Opens Down, Axis of Symmetry parallel to y-axis	

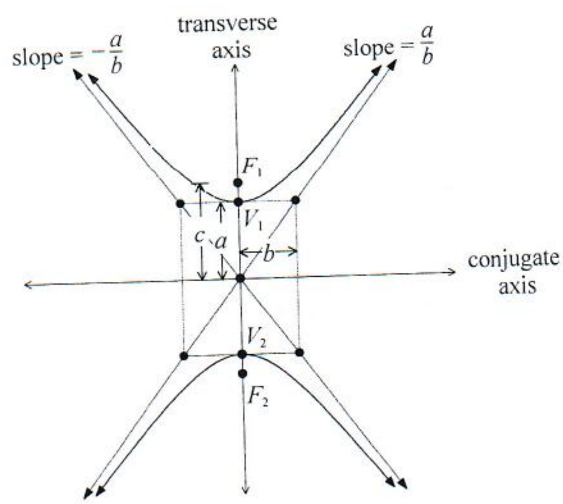


Hyperbola



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$a^2 + b^2 = c^2$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$a^2 + b^2 = c^2$$

a = distance from center to vertices

c = distance from center to foci

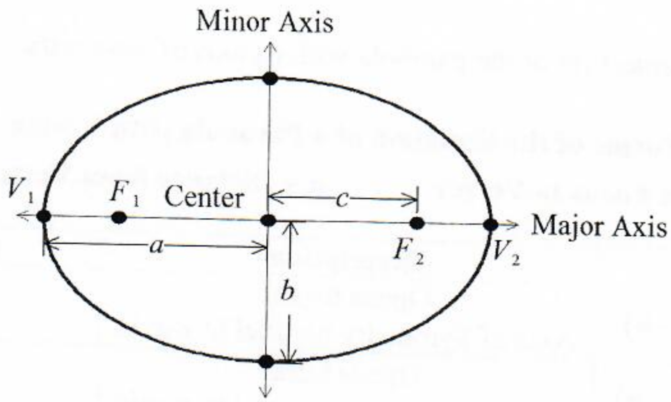
b used to find the width of branches and slope of asymptotes

Ellipse

Standard Form of the Equation of an Ellipse with Center at (h,k)

Equation	Description	Picture
$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to x-axis	
$\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to y-axis	

a **= Distance from center to vertices**
b **= Distance from center to covertices**
c **= Distance from center to foci**



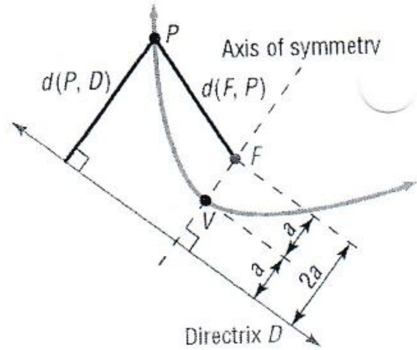
Distance Formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint Formula

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

The Parabola



Objectives:

- **Find the equation of a parabola given the focus and the directrix; where the directrix is parallel to either of the coordinate axes.**
- **Sketch the graph of a parabola given the equation.**

Parabola: The collection of all points P in the plane that are the same distance from a fixed point F, called the **focus** of the parabola, as they are from a fixed line D, called the **directrix** of the parabola.

Axis of Symmetry: The line through the focus F and perpendicular to the directrix D.

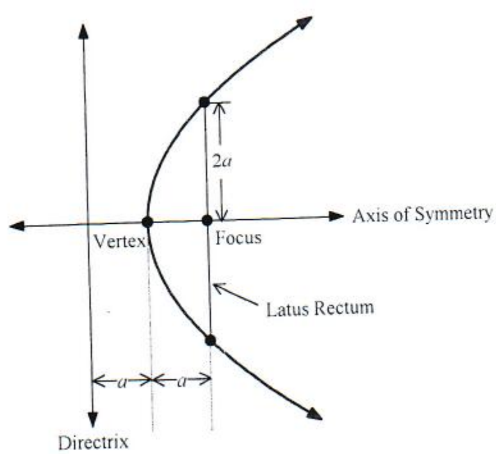
Vertex: The point of intersection of the parabola with its axis of symmetry.

General Forms of the Equation of a Parabola with Vertex (h,k)

a = Distance from Focus to Vertex **a = Distance from Vertex to Directrix**

Equation	Description	Picture
$(y-k)^2 = 4a(x-h)$	Opens Right, Axis of Symmetry parallel to x-axis	
$(y-k)^2 = -4a(x-h)$	Opens Left, Axis of Symmetry parallel to x-axis	
$(x-h)^2 = 4a(y-k)$	Opens Up, Axis of Symmetry parallel to y-axis	
$(x-h)^2 = -4a(y-k)$	Opens Down, Axis of Symmetry parallel to y-axis	

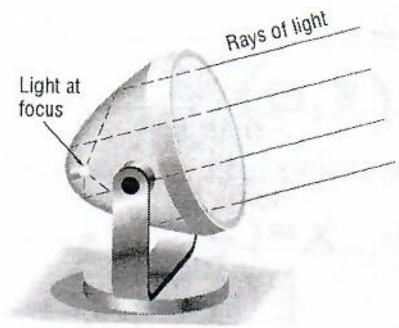
Latus Rectum: The line segment with endpoints on the parabola that passes through the focus and is perpendicular to the axis of symmetry. Each of the endpoints is at a distance of $2a$ from the focus.

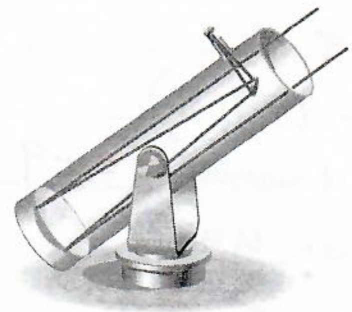


Paraboloid of Revolution: A surface formed by rotating a parabola about its axis of symmetry.

Suppose a mirror is shaped like a paraboloid of revolution. If a light (or other radiation source) is placed at the focus of the parabola, all the rays emanating from the light will reflect off the mirror in lines parallel to the axis of symmetry.

Conversely, when rays of light from a distant source strike the surface of a parabolic mirror, they are reflected to a single point at the focus. This fact is used in the design of telescopes and other optical devices.





Review Problems

Complete the square and write in factored form.

1. $y^2 - 14y +$ 49

-7

$(y-7)^2$

2. $y^2 + \frac{2}{3}y +$ $\frac{1}{9}$

$\frac{2}{6} = \frac{1}{3}$

$\left(y - \frac{1}{3}\right)^2$

Write the equations of the parabola in standard form by completing the square.

3. $x = y^2 - 8y + 5$

$x - 5 = y^2 - 8y + 16$

$+16 \qquad -4$

$x + 11 = (y-4)^2$

or

$(y-4)^2 = 1(x+11)$

4. $y = x^2 - 5x + \frac{1}{4}$

$y - \frac{1}{4} = x^2 - 5x + \frac{25}{4}$

$+\frac{25}{4} \qquad -\frac{5}{2}$

$y + \frac{24}{4} = \left(x - \frac{5}{2}\right)^2$

$y + 6 = \left(x - \frac{5}{2}\right)^2$

or

$\left(x - \frac{5}{2}\right)^2 = 1(y+6)$

Determine the direction of opening, vertex, the focal width, focus, directrix, value of a and the axis of symmetry.

5. $(x+9)^2 = 16(y-4)$

Direction of opening up

Vertex (-9, 4)

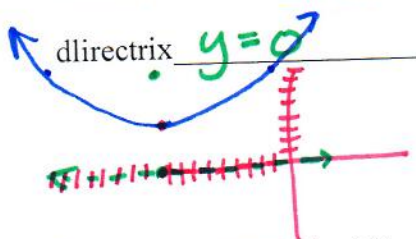
Focal Width 16

a= 4

focus (-9, 8)

axis of symmetry x = -9

dlirectrix y = 0



68. $y^2 = -8(x-10)$

Direction of opening left

Vertex (10, 0)

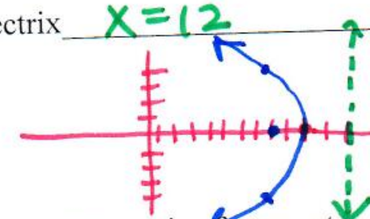
Focal Width 8

a= -2

focus (8, 0)

axis of symmetry y = 0

dlirectrix x = 12



Examples: Graph the following parabolas. State the vertex, focus, axis of symmetry, directrix, length of latus rectum (focal width), and direction of opening.

7. $x^2 = 8(y-2)$

Direction of opening up

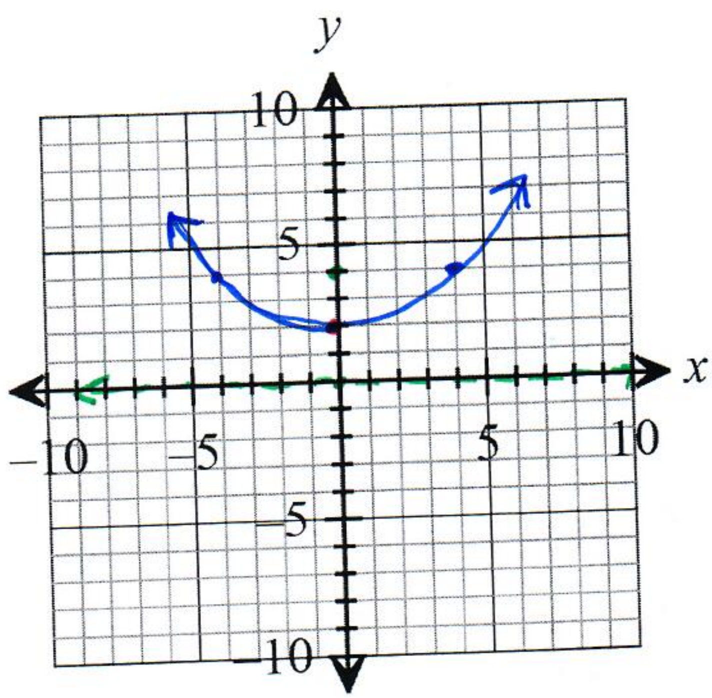
Vertex (0, 2)

Focal Width 8

a= 2

Focus (0, 4)

Directrix y = 0



8. $(y+1)^2 = -16(x-4)$

Direction of opening left

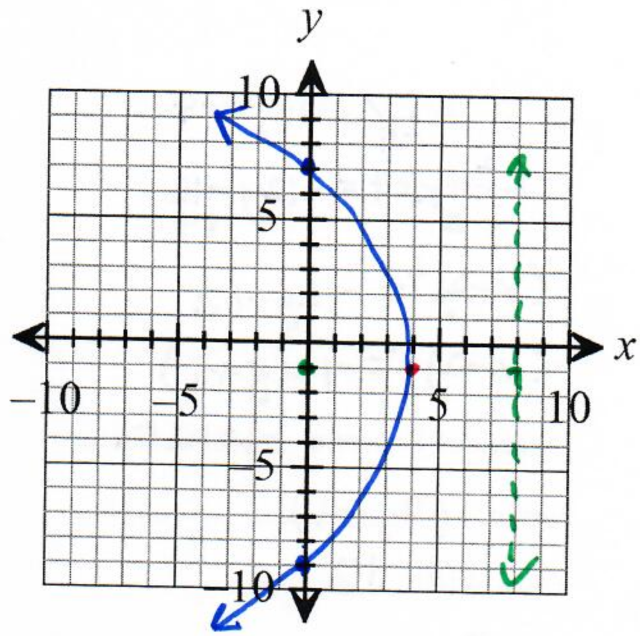
Vertex (4, -1)

Focal Width 16

a= 4

Focus (0, -1)

Directrix x=8



9. $(y+3)^2 = 4(x-4)$

Direction of opening right

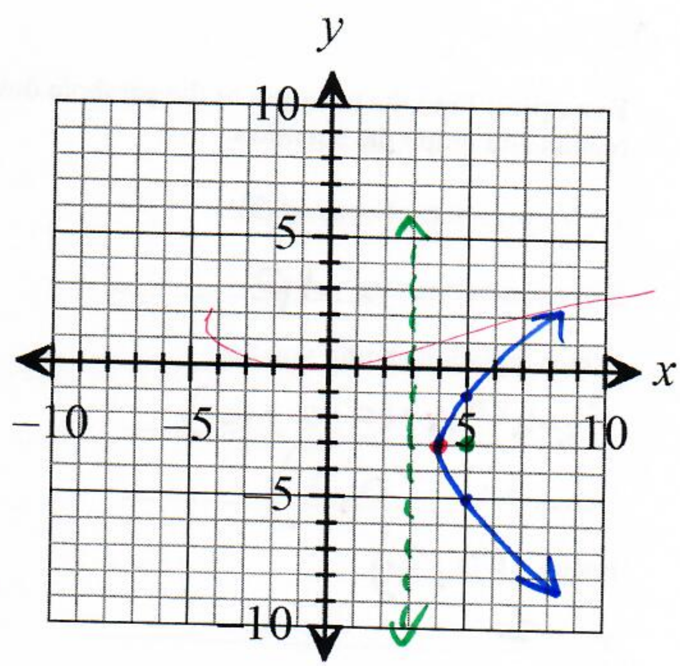
Vertex (4, -3)

Focal Width 4

a= 1

Focus (5, -3)

Directrix x= 3



10. $x^2 - 4x \cancel{+4} = -12y - 12 - 4$
-4

$x^2 - 4x + 4 = -12y - 16$
$-2 \qquad\qquad\qquad +4$

$(x-2)^2 = -12y - 12$

$(x-2)^2 = -12(y+1)$

Direction of opening down

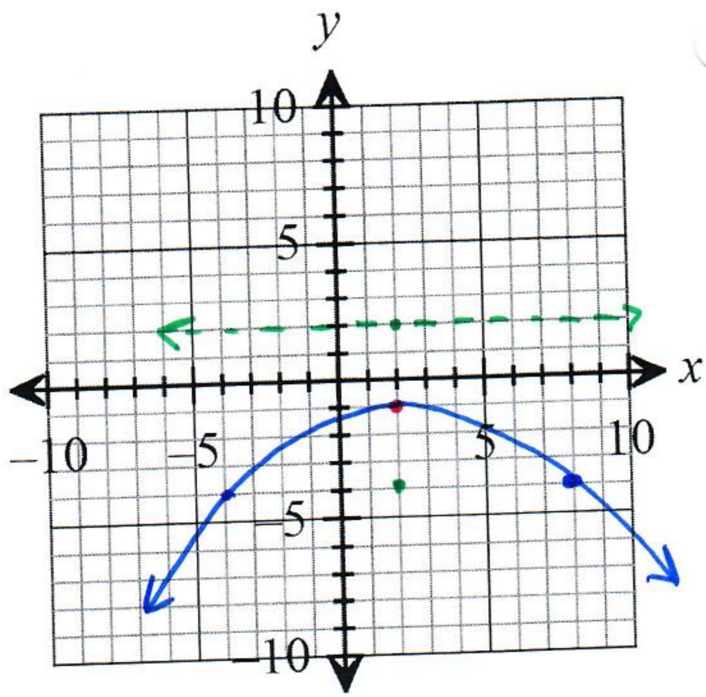
Vertex (2, −1)

Focal Width 12

a= −3

Focus (2, −4)

Directrix y =



Examples: Find the equation of the parabola described. Find the two points that define the latus rectum and graph the equation.

11. Vertex: (0,0); Focus: (0,2)

Direction of opening up

Which equation should you use

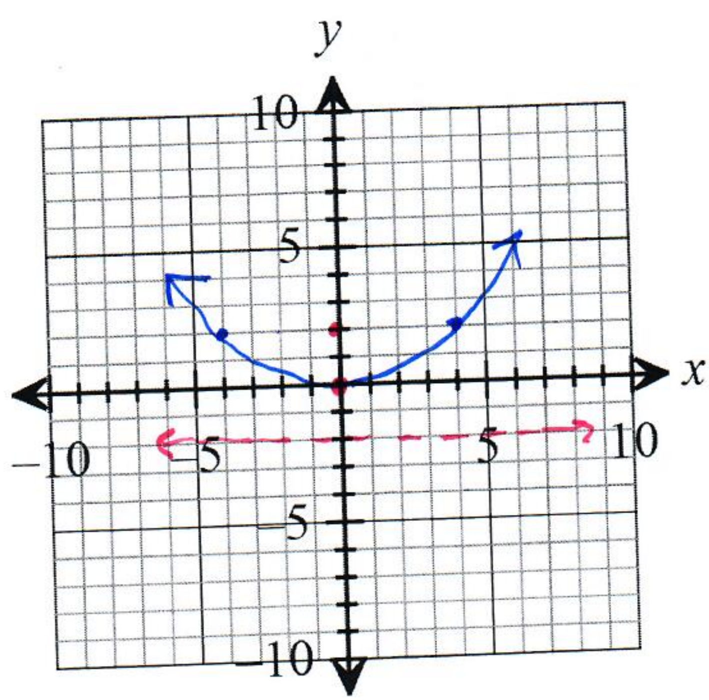
$(x-h)^2 = 4a(y-k)$

Vertex (h,k) (0, 0)

Focus (0, 2)

a= 2

Focal Width 8



Equation: $(x-0)^2 = 8(y-0)$

$x^2 = 8y$

12. Vertex: (3,-2); Focus: (3,2)

Direction of opening up

Which equation should you use

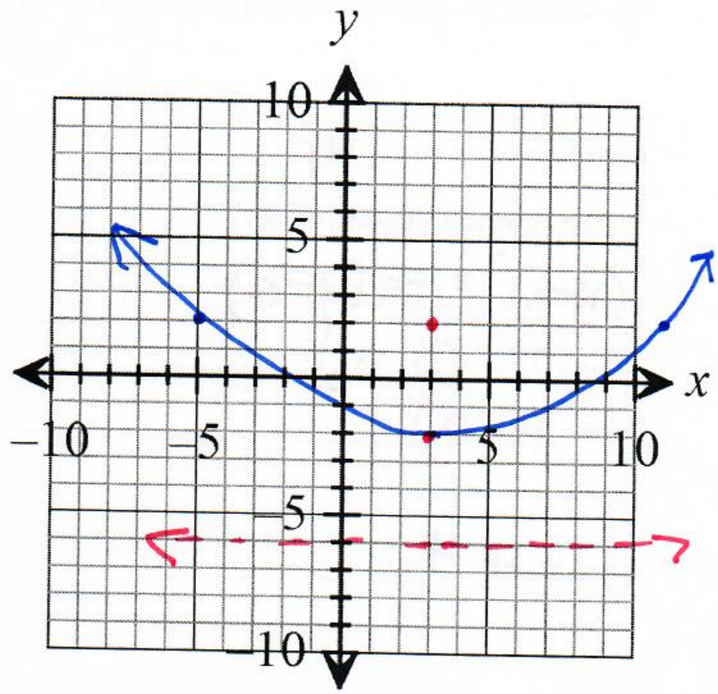
$(x-h)^2=4a(y-k)$

Vertex (h,k) (3, -2)

Focus (3, 2)

a= 4

Focal Width 16



Equation: $(x-3)^2=16(y+2)$

13. Vertex: (-1,4); Directrix: $x=1$

Direction of opening left

Which equation should you use

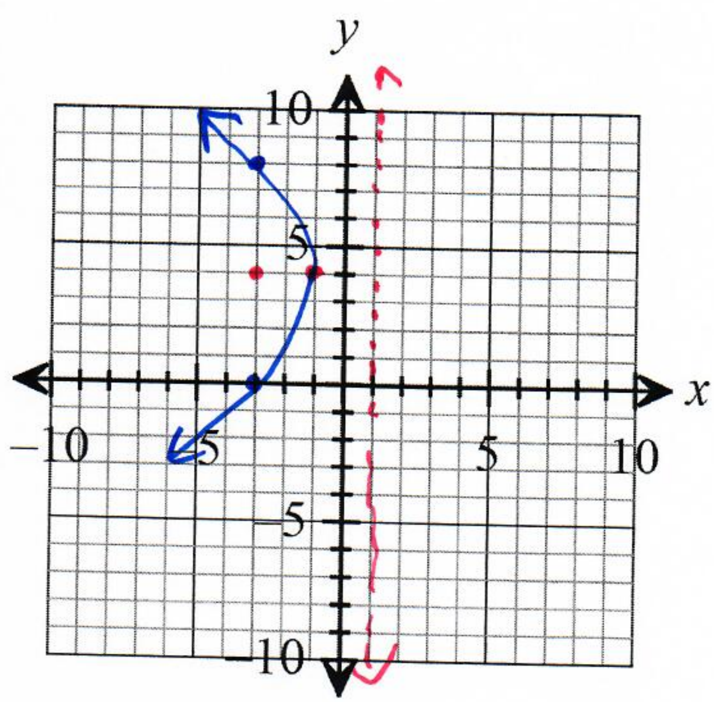
$(y-k)^2=4a(x-h)$

Vertex (h,k) (-1, 4)

Focus (-3, 4)

a= -2

Focal Width 8



Equation: $(y-4)^2=-8(x+1)$

14. Vertex: (0,-1); Axis of Symmetry: y-axis; Contains the point (2,0)

Direction of opening up

Which equation should you use

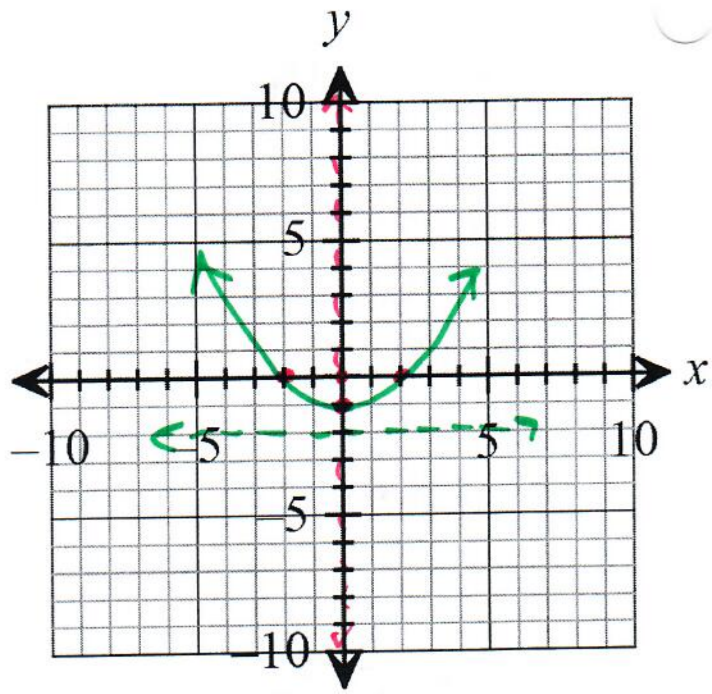
$(x-h)^2=4a(y-k)$

Vertex (h,k) (0,-1)

Focus (0,0)

a= 1

Focal Width 4



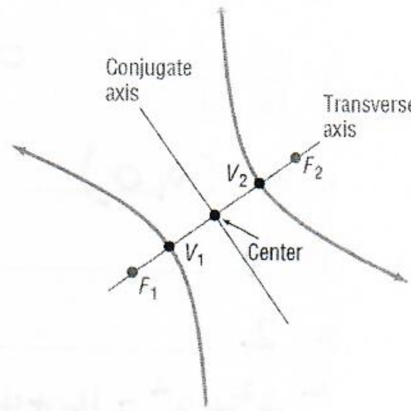
Equation: $(x-0)^2=4(y+1)$

$x^2=4(y+1)$

The Hyperbola

Objectives:

- **Find the equation of a hyperbola given the foci and the vertices**
- **Sketch the graph of a hyperbola, given the equation.**



Hyperbola: The collection of all points in the plane, the difference of whose distances from two fixed points, called the **foci,** is a constant.

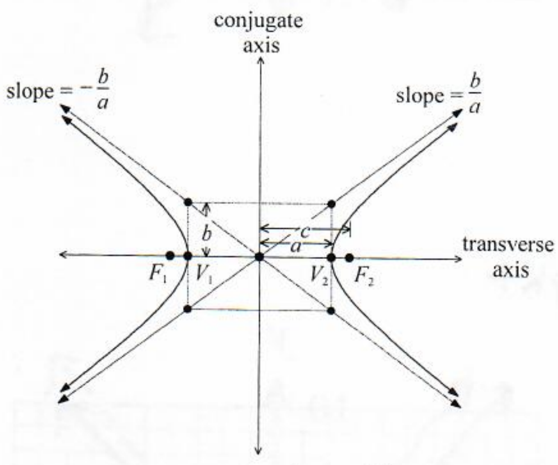
Transverse Axis: The line containing the foci.

Center: The midpoint of the line segment joining the foci.

Conjugate Axis: The line through the center and perpendicular to the transverse axis.

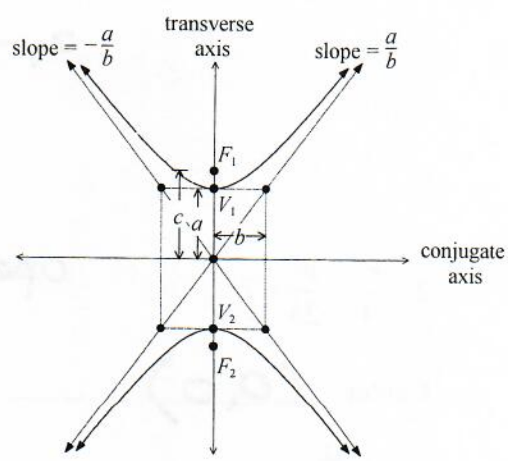
Branches: The separate curves of the hyperbola. They are symmetric with respect to the transverse axis, conjugate axis, and center.

Vertices: The points of intersection of the hyperbola and the transverse axis.



$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$$a^2+b^2=c^2$$



$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$$a^2+b^2=c^2$$

a = distance from center to vertices

c = distance from center to foci

b used to find the width of branches and slope of asymptotes

When finding the equations of the asymptotes, remember that $m=\dfrac{\text{change in } y}{\text{change in } x}$, then use point slope form $y-y_1=m(x-x_1)$ with the center (h,k) as (x_1,y_1).

$a^2+b^2=c^2$

Examples: Find the center, transverse axis, vertices, foci, and the slope of the asymptotes. Graph each equation.

1. $\frac{y^2}{16}-\frac{x^2}{4}=1$ opens up/down

Center: $(0,0)$

a= 4

b= 2

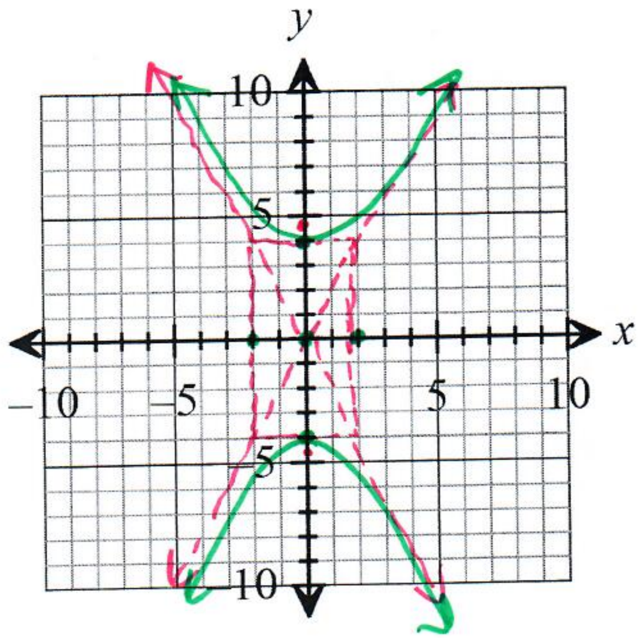
$c^2= 4^2+2^2=16+4=20$ $c=\sqrt{20}$ 4.47

Vertices: $(0,4);(0,-4)$

Foci: $(0,\sqrt{20});(0,-\sqrt{20})$

Slope of the Asymptotes: $\frac{4}{2}, -\frac{4}{2}$

$2, -2$



2. $\frac{x^2}{9}-\frac{y^2}{25}=1$ opens left/right

Center: $(0,0)$

a= 3

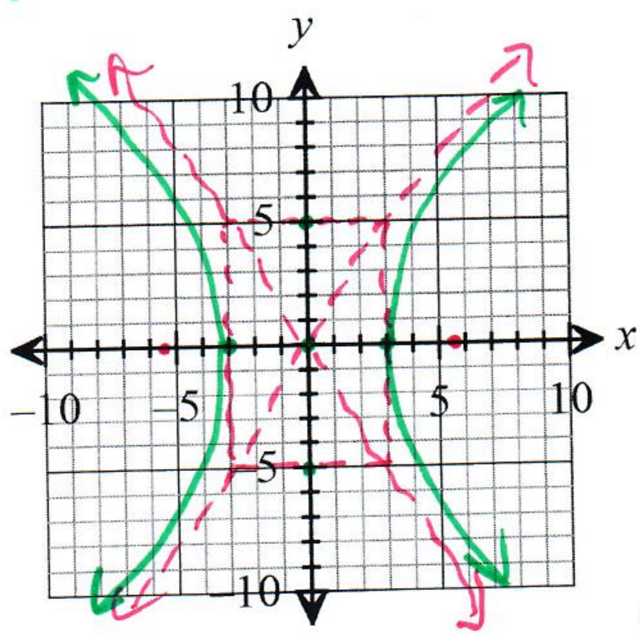
b= 5

$c^2= 3^2+5^2=9+25=34$ $c=\sqrt{34}$ 5.83

Vertices: $(3,0);(-3,0)$

Foci: $(\sqrt{34},0);(-\sqrt{34},0)$

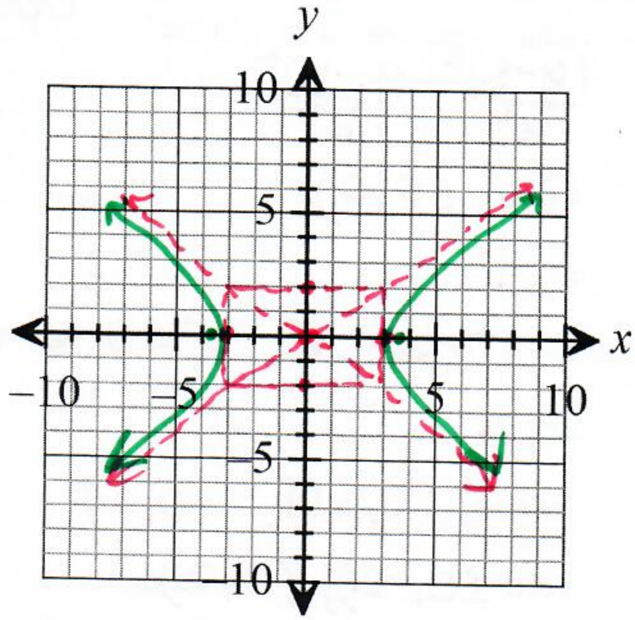
Slope of the Asymptotes: $\frac{5}{3}, -\frac{5}{3}$



Sec Math 2H – 5.3 Notes

The Hyperbola

3. $\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$ $\frac{x^2}{9} - \frac{y^2}{4} = 1$ opens left/right



Center: (0, 0)

a= 3

b= 2

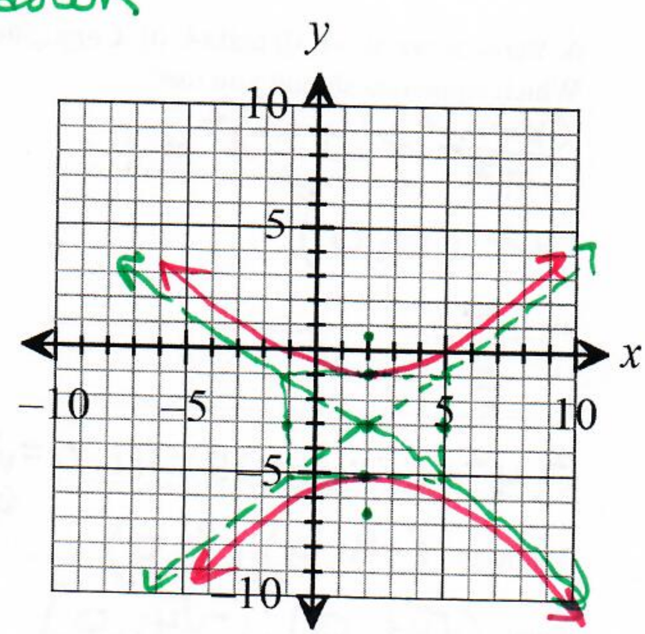
c^2= $3^2+2^2=9+4=13$ $c=\sqrt{13}$ 3.61

Vertices: (3, 0) ; (−3, 0)

Foci: $(\sqrt{13}, 0)$; $(-\sqrt{13}, 0)$

Slope of the Asymptotes: $\frac{2}{3}$, $-\frac{2}{3}$

4. $\frac{(y+3)^2}{4} - \frac{(x-2)^2}{9} = 1$ opens up/down



Center: (2, −3)

a= 2

b= 3

c^2= $2^2+3^2=4+9=13$ $c=\sqrt{13}$ 3.61

Vertices: (5, −3) ; (−1, −3)

Foci: $(2, -3+\sqrt{13})$; $(2, -3-\sqrt{13})$

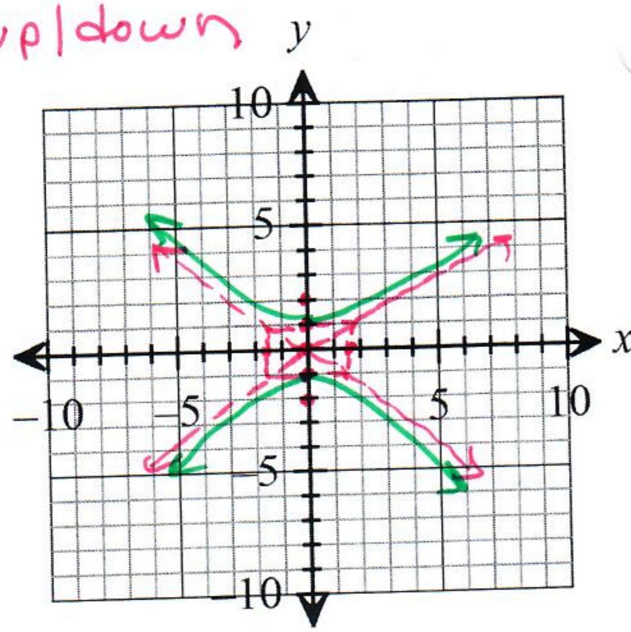
Slope of the Asymptotes: $\frac{2}{3}$, $-\frac{2}{3}$

Examples: Write an equation in standard form of the hyperbola described.

5. Foci are at (0, -2) and (0, 2); Vertices are at (0,-1) and (0, 1)

Which equations should you use? opens up/down

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$



Center: $(0,0)$

a= 1

b= $1^2+b^2=2^2 \quad b^2=3 \quad b=\sqrt{3}$ 1.73

c= 2

Vertices: $(0,-1)$; $(0,1)$

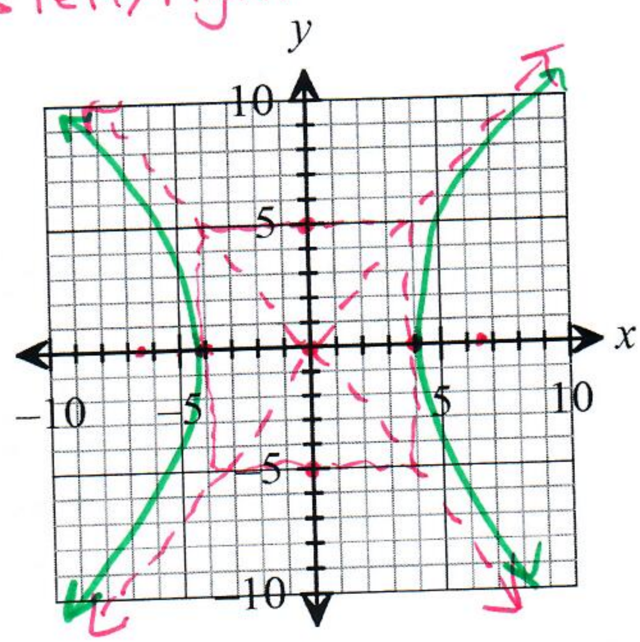
Foci: $(0,-2)$; $(0,2)$

Slope of the Asymptotes: $\frac{1}{\sqrt{3}}$, $-\frac{1}{\sqrt{3}}$

Equation $\frac{y^2}{1} - \frac{x^2}{3} = 1$

6. Vertices are at (-4, 0) and (4, 0); Conjugate axis length is 10.
Which equations should you use? opens left/right

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$



Center: $(0,0)$

a= 4

b= 5

$c^2=$ $4^2+5^2=16+25=41 \quad c=\sqrt{41}$ 6.40

Vertices: $(-4,0)$; $(4,0)$

Foci: $(\sqrt{41},0)$; $(-\sqrt{41},0)$

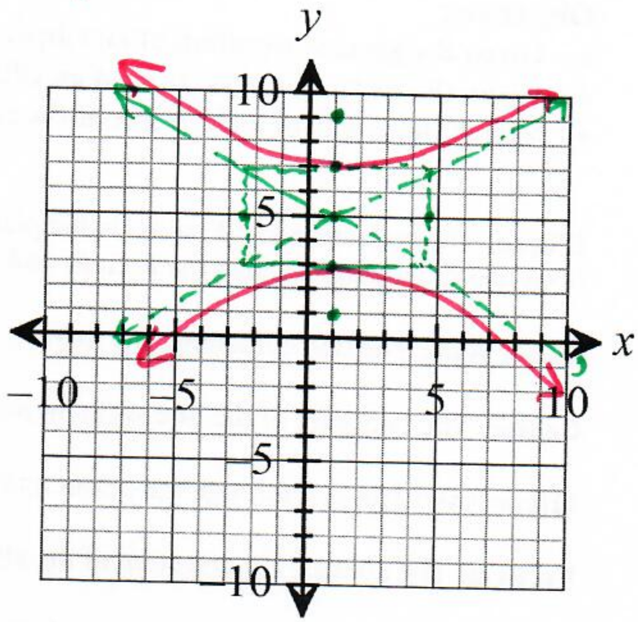
Slope of the Asymptotes: $\frac{5}{4}$, $-\frac{5}{4}$

Equation $\frac{x^2}{16} - \frac{y^2}{25} = 1$

7. Foci are at (1, 9) and (1, 1); Vertices are at (1, 7) and (1, 3)

Opens up/down

Which equations should you use? $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$



Center: $(1, 5)$

a= 2

b= $2^2+b^2=4^2 \quad b^2=12 \quad b=\sqrt{12}$ 3.46

c= 4

Vertices: $(1, 7); (1, 3)$

Foci: $(1, 9); (1, 1)$

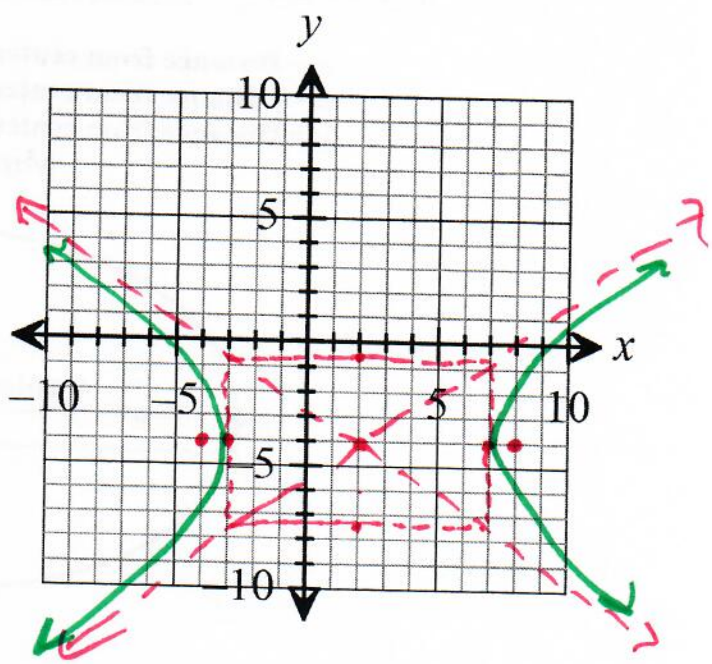
Slope of the Asymptotes: $\frac{2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}$

Equation $\frac{(y-5)^2}{4}-\frac{(x-1)^2}{12}=1$

8. Foci are at (8, -4) and (-4, -4); Vertices are at (7,-4) and (-3, -4)

opens left/right

Which equations should you use? $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$



Center: $(2, -4)$

a= 5

b= $5^2+b^2=6^2 \quad b^2=11 \quad b=\sqrt{11}$ 3.32

c= 6

Vertices: $(7, -4); (-3, -4)$

Foci: $(8, -4); (-4, -4)$

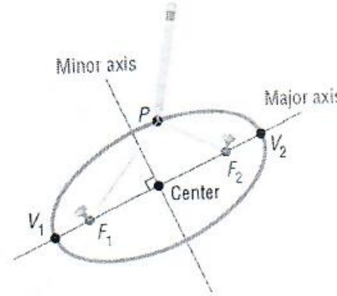
Slope of the Asymptotes: $\frac{\sqrt{11}}{5}, \frac{-\sqrt{11}}{5}$

Equation $\frac{(x-2)^2}{25}-\frac{(y+4)^2}{11}=1$

The Ellipse

Objectives:

- **Given the general equation of an ellipse, identify the foci and vertices.**
- **Given the foci and the vertices of an ellipse, write an equation for the ellipse.**
- **Sketch the graph of a circle, given the equation.**



Ellipse: The collection of all points in the plane, the sum of whose distances from two fixed points, called the **foci**, F_1 and F_2, is a constant.

Major Axis: The line containing the foci.

Center: The midpoint of the line segment joining the two foci.

Minor Axis: The line through the center and perpendicular to the major axis.

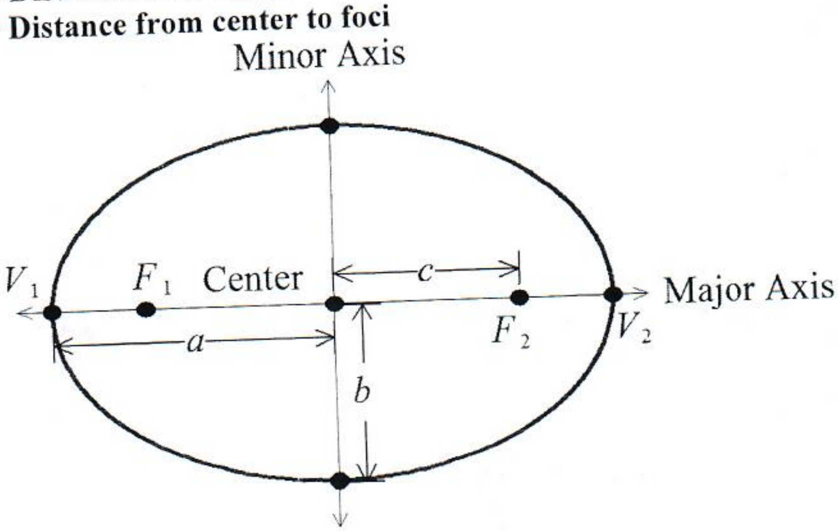
Vertices: The points of intersection of the ellipse and the major axis.

Covertices: The points of intersection of the ellipse and the minor axis.

Standard Form of the Equation of an Ellipse with Center at (h,k)

Equation	Description	Picture
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to x-axis	
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to y-axis	

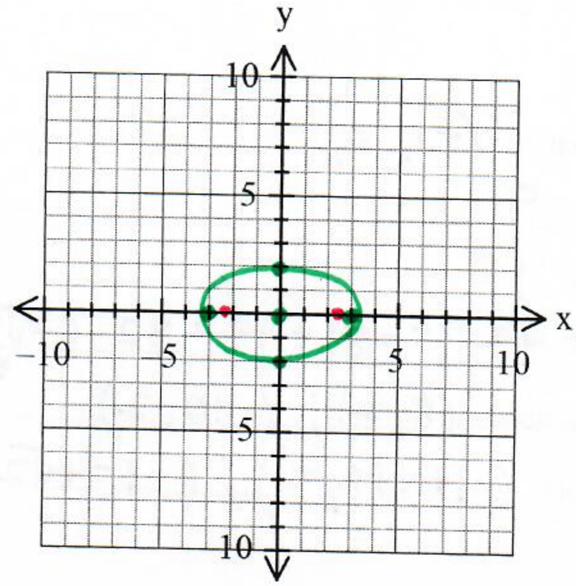
a = Distance from center to vertices
b = Distance from center to covertices
c = Distance from center to foci



Examples: Find the center, foci, and vertices of each ellipse. Graph each equation.

$a^2 - b^2 = c^2$

1. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$



Center: $(0,0)$

a= 3

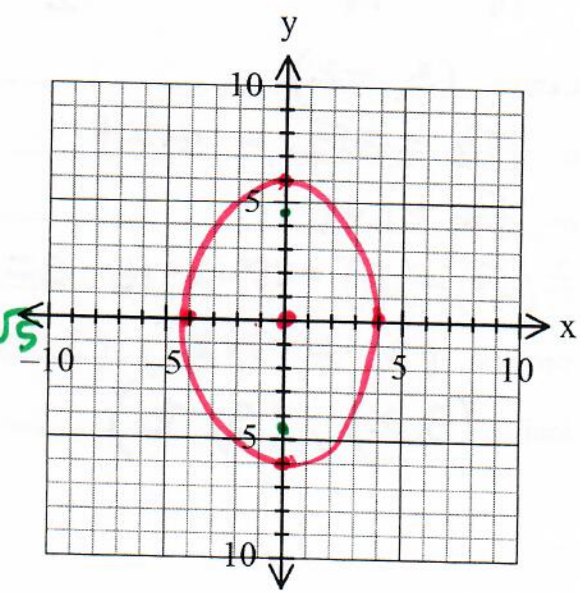
b= 2

c^2= $3^2 - 2^2 = 9 - 4 = 5$ $\quad c = \sqrt{5}$ $\quad 2.24$

vertices: $(3,0)$; $(-3,0)$

foci: $(\sqrt{5},0)$; $(-\sqrt{5},0)$

2. $\dfrac{x^2}{16} + \dfrac{y^2}{36} = 1$



Center: $(0,0)$

a= 6

b= 4

c^2= $6^2 - 4^2 = 36 - 16 = 20$ $\quad c = \sqrt{20} = 2\sqrt{5}$ $\quad 4.47$

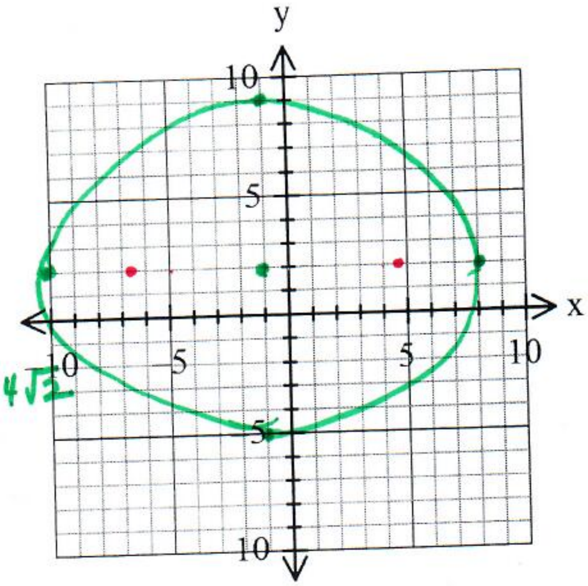
vertices: $(0,6)$; $(0,-6)$

foci: $(0,\sqrt{20})$; $(0,-\sqrt{20})$

or: $(0,2\sqrt{5})$; $(0,-2\sqrt{5})$

$a^2-b^2=c^2$

3. $\frac{(x+1)^2}{81}+\frac{(y-2)^2}{49}=1$



Center: $(-1, 2)$

a= 9

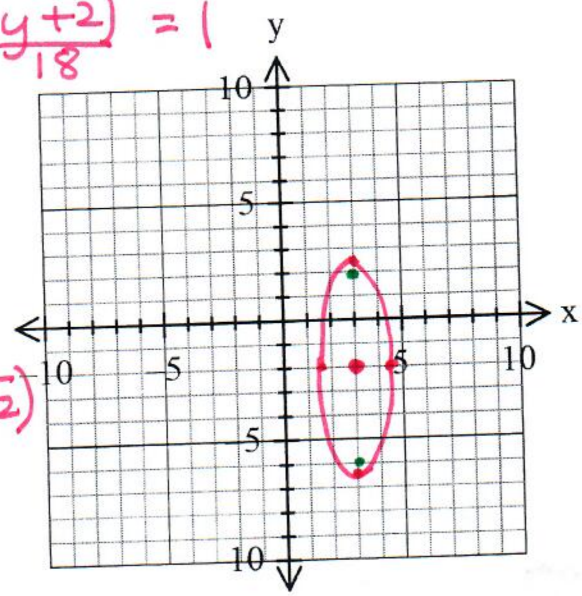
b= 7

c^2= $9^2-7^2=81-49=32$ $c=\sqrt{32}=4\sqrt{2}$ 5.66

vertices: $(8,2)$; $(-10,2)$

foci: $(-1+4\sqrt{2},2)$; $(-1-4\sqrt{2},2)$

4. $\frac{9(x-3)^2}{18}+\frac{(y+2)^2}{18}=\frac{18}{18}$ $=\frac{(x-3)^2}{2}+\frac{(y+2)^2}{18}=1$



Center: $(3, -2)$

a= $\sqrt{18}=3\sqrt{2}=4.24$

b= $\sqrt{2}=1.41$

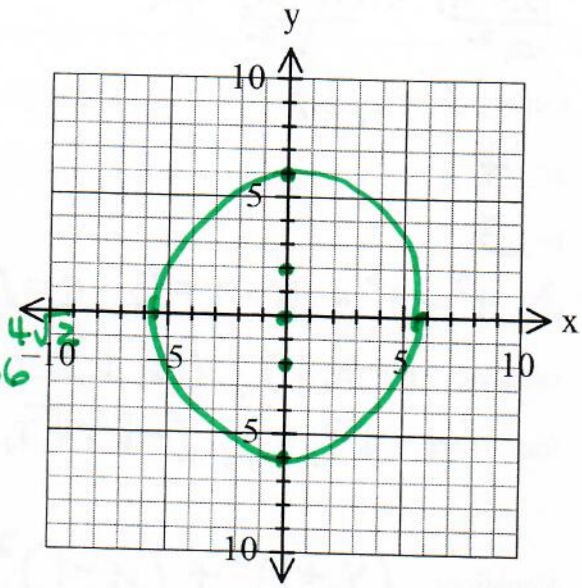
c^2= $(\sqrt{18})^2-(\sqrt{2})^2=18-2=16$ $c=4$

vertices: $(3,-2+3\sqrt{2})$; $(3,-2-3\sqrt{2})$

foci: $(3,2)$; $(3,-6)$

Examples: Write the standard equation of the ellipse having the given characteristics.

5. Foci at (0,-2) and (0, 2); Vertices at (0,–6) and (0, 6)



Which equation should you use? $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

Center: $(0,0)$

a= 6

b= $2^2 = 6^2 - b^2$ $32 = b^2$ $b = \sqrt{32} = 4\sqrt{2}$
5.66

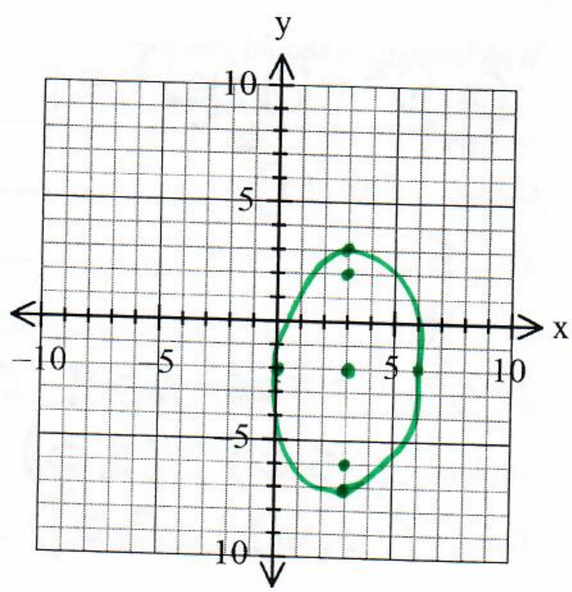
c= 2

vertices: $(0,-6)$; $(0,6)$

foci: $(0,-2)$; $(0,2)$

Equation: $\dfrac{x^2}{32} + \dfrac{y^2}{36} = 1$

6. Foci at (3, -6) and (3, 2); Vertices at (3, -7) and (3, 3)



Which equation should you use? $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

Center: $(3,-2)$

a= 5

b= $5^2 - b^2 = 4^2$ $b^2 = 9$ $b = 3$

c= 4

vertices: $(3,-7)$; $(3,3)$

foci: $(3,-6)$; $(3,2)$

Equation: $\dfrac{(x-3)^2}{9} + \dfrac{(y+2)^2}{25} = 1$

7. Vertices: (-5, 1) and (3, 1) ; Minor axis length is 6.

Which equation should you use?

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Center: $(-1,1)$

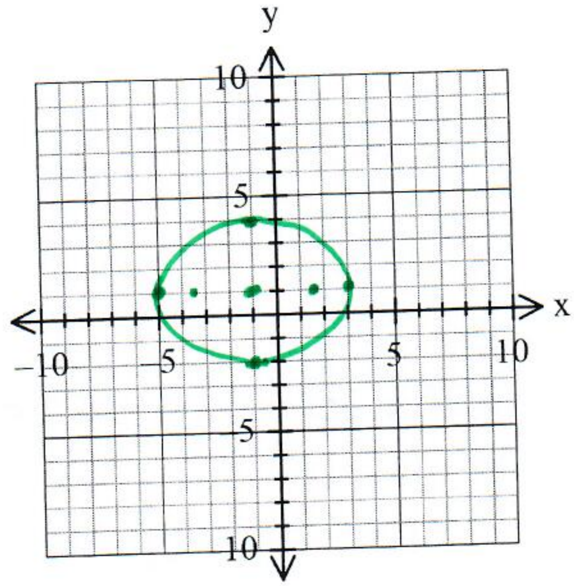
a= 4

b= 3

$c^2 = 4^2-3^2=16-9=7 \quad c=\sqrt{7}$ 2.65

vertices: $(-5,1)$; $(3,1)$

foci: $(-1-\sqrt{7},1)$; $(-1+\sqrt{7},1)$



Equation: $\frac{(x+1)^2}{16}+\frac{(y-1)^2}{9}=1$

8. Endpoints of axes are: (-5, 0), (5, 0), (0, -4) and (0, 4)

Which equation should you use?

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Center: $(0,0)$

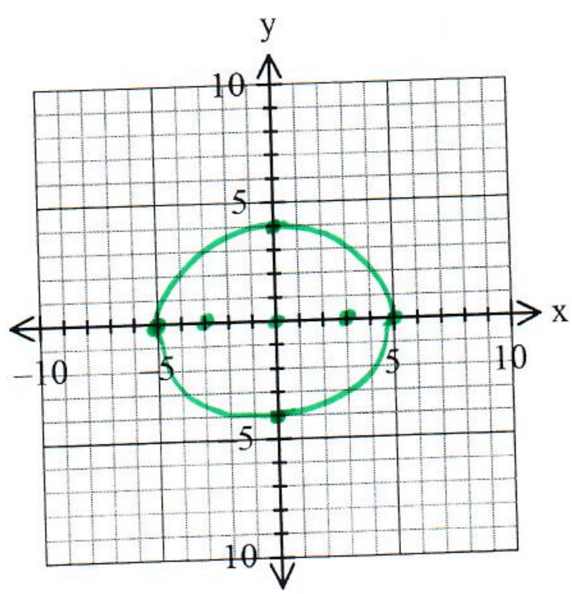
a= 5

b= 4

$c^2 = 5^2-4^2=25-16=9 \quad c=3$

vertices: $(-5,0)$; $(5,0)$

foci: $(-3,0)$; $(3,0)$



Equation: $\frac{x^2}{25}+\frac{y^2}{16}=1$

Circles

Objectives:

- **Sketch the graph of a circle, given the equation.**
- **Given the general equation of a circle, complete the square to find the center.**

Circles

Circles with Center (h, k)	
General Equation	$(x-h)^2+(y-k)^2=r^2$
Radius	r

Circle: The set of all points in the *xy*-plane that are a fixed distance *r*, called the **radius**, from a fixed point (h,k), called the **center** of the circle.

Standard Form of the Equation of a Circle with radius *r* and center (h,k):

$$(x-h)^2+(y-k)^2=r^2$$

General Form of the Equation of a Circle:

$$x^2+y^2+ax+by+c=0$$

To find the standard form of the equation of a circle when you know the general form, **complete the square** for both *x* and *y*.

Distance Formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint Formula

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Example: Find the distance between the two given points.

1. $P_1=(-2,\ 10)$ and $P_2=(5,\ 4)$

$d=\sqrt{(5--2)^2+(4-10)^2}$

$=\sqrt{7^2+(-6)^2}=\sqrt{49+36}$

$=\sqrt{85}=9.22$

2. $P_1=(3,\ -6)$ and $P_2=(-5,\ -7)$

$d=\sqrt{(-5-3)^2+(-7--6)^2}$

$=\sqrt{(-8)^2+(-1)^2}=\sqrt{64+1}$

$=\sqrt{65}=8.06$

Example: Find the midpoint of the given points.

3. $P_1=(3,\ -6)$ and $P_2=(-5,\ 8)$

$\left(\frac{3+-5}{2},\frac{-6+8}{2}\right)=\left(-\frac{2}{2},\frac{2}{2}\right)$

$=(-1,1)$

4. $P_1=(-3,\ 5)$ and $P_2=(1,\ 8)$

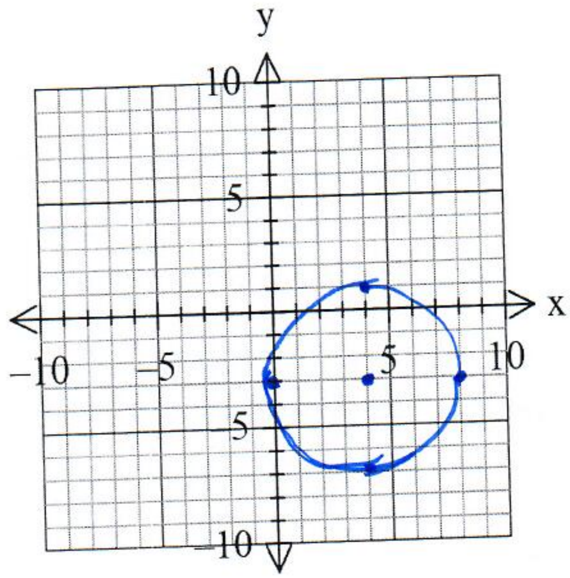
$\left(\frac{-3+1}{2},\frac{5+8}{2}\right)=\left(-\frac{2}{2},\frac{13}{2}\right)$

$=\left(-1,\frac{13}{2}\right)$

5. **Example:** Write the standard form of the equation of the circle with radius $r = 4$ and center $(h, k) = (4, -3)$. Then graph the circle.

$(x-h)^2 + (y-k)^2 = r^2$

Equation: $(x-4)^2 + (y+3)^2 = 16$

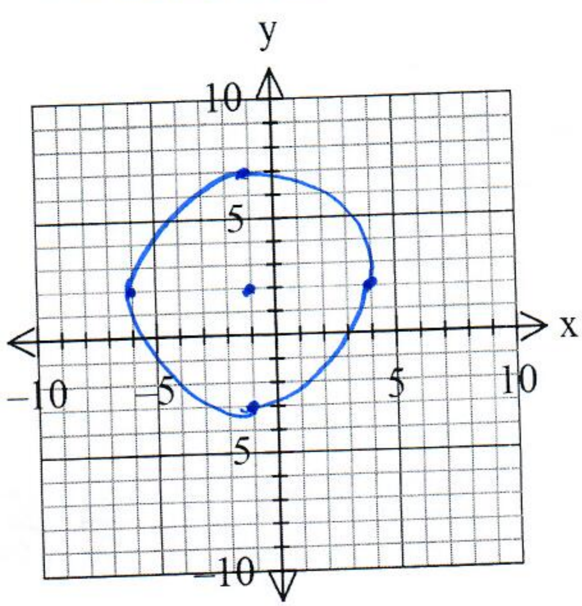


Examples: Find the center (h, k) and radius r of each circle, graph the circle.

6. $(x+1)^2 + (y-2)^2 = 25$

center: $(-1, 2)$

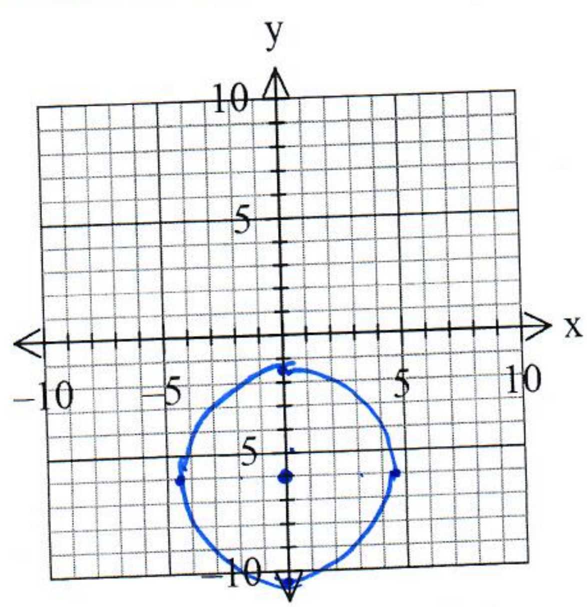
radius: 5



7. $x^2 + (y+6)^2 = 20$

center: $(0, -6)$

radius: $\sqrt{20} = 4.47$



Examples: Complete the square.

8. $x^2 - 8x + 7 = 0$

$x^2 - 8x + 16 = -7 + 16$
-4

$(x-4)^2 = 9$

Examples: Find the center (h, k) and radius r of each circle, graph the circle.

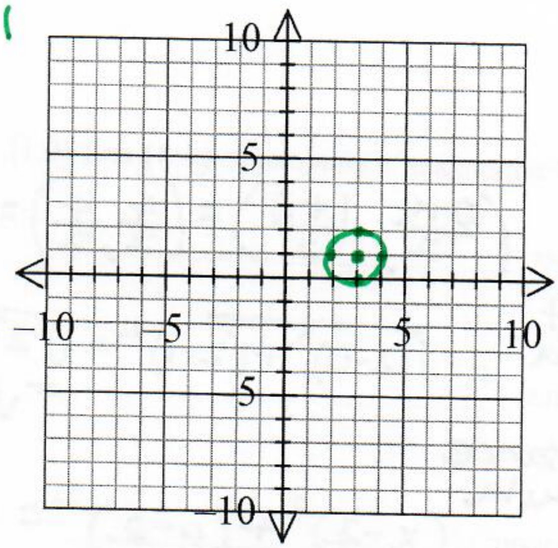
$(x-3)^2 + (y-1)^2 = 1$

8. $x^2 + y^2 - 6x + 2y + 9 = 0$

$x^2 - 6x + 9 + y^2 + 2y + 1 = -9 + 9 + 1$
$-3 \qquad -1$

center: $(3, 1)$

radius: 1



Examples: Find the radius if the diameter is given.

9. d=8

$r = 4$

10. d=7

$r = 3.5$

11. $d = \sqrt{6}$

$r = \frac{\sqrt{6}}{2} = 1.22$

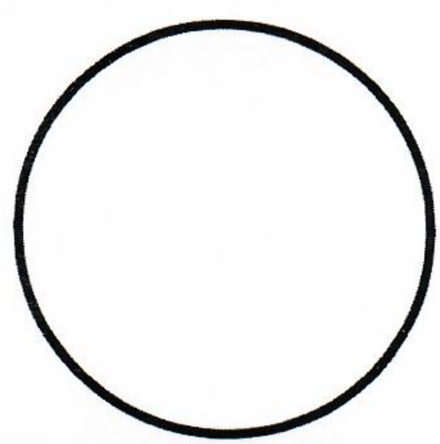
Examples: Find the standard form of the equation of each circle.

9. Center at $(-3, 5)$ and a diameter of 24.

Center: $(-3, 5)$

Radius: 12

Equation: $(x+3)^2 + (y-5)^2 = 144$



10. Center at $(1,0)$ and containing the point $(-3,2)$.

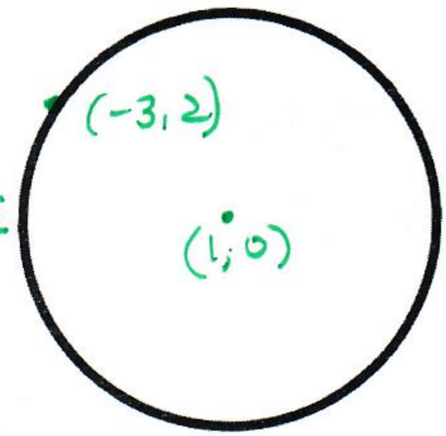
Center: $(1,0)$

Radius: $d = \sqrt{(-3-1)^2 + (2-0)^2} = \sqrt{(-4)^2 + 2^2}$

use distance formula

$= \sqrt{16+4} = \sqrt{20}$

Equation: $(x-1)^2 + y^2 = 20$



11. Endpoints of a diameter at $(4,3)$ and $(0,1)$.

Center: $\left(\frac{0+4}{2}, \frac{1+3}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2,2)$

use midpoint formula

Radius: $d = \sqrt{(2-0)^2 + (2-1)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$

use distance formula

Equation: $(x-2)^2 + (y-2)^2 = 5$