**Name:**_______________________________ **Period:** ___________

# SM2H Unit 6 Circle Notes

## 6.1 Circles

**Objectives:**

- **Sketch the graph of a circle, given the equation.**
- **Given the general equation of a circle, complete the square to find the center.**

## Circles

| Circles with Center (h, k) | |
|---|---|
| General Equation | $(x-h)^2+(y-k)^2=r^2$ |
| Radius | r |

**Circle:** The set of all points in the *xy*-plane that are a fixed distance *r*, called the **radius**, from a fixed point $(h,k)$, called the **center** of the circle.

**Standard Form of the Equation of a Circle** with radius *r* and center $(h,k)$:

$$(x-h)^2+(y-k)^2=r^2$$

**General Form of the Equation of a Circle:**

$$x^2+y^2+ax+by+c=0$$

To find the standard form of the equation of a circle when you know the general form, **complete the square** for both *x* and *y*.

## Distance Formula

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

## Midpoint Formula

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

**Example:** Find the distance between the two given points.

1. $P_1=(-2,\ 10)$ and $P_2=(5,\ 4)$

2. $P_1=(3,\ \text{-}6)$ and $P_2=(-5,\ \text{-}7)$

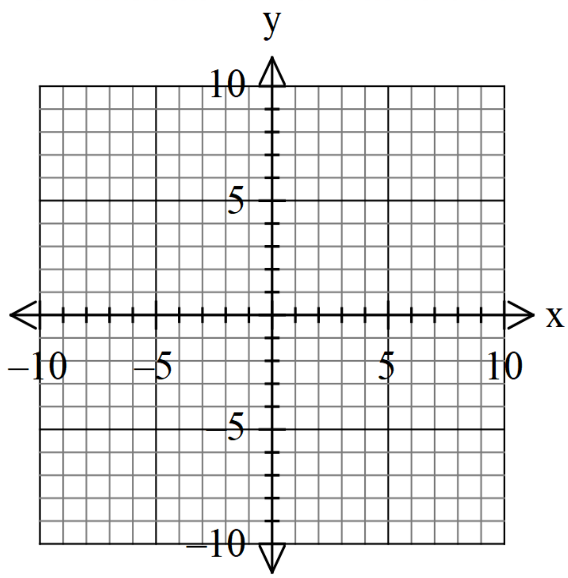
**Example:** Find the midpoint of the given points.

3. $P_1=(3,\ \text{-}6)$ and $P_2=(-5,\ 8)$

4. $P_1=(-3,\ 5)$ and $P_2=(1,\ 8)$

5. **Example:** Write the standard form of the equation of the circle with radius $r = 4$ and center $(h, k) = (4, -3)$. Then graph the circle.

Equation: ______________________________________________

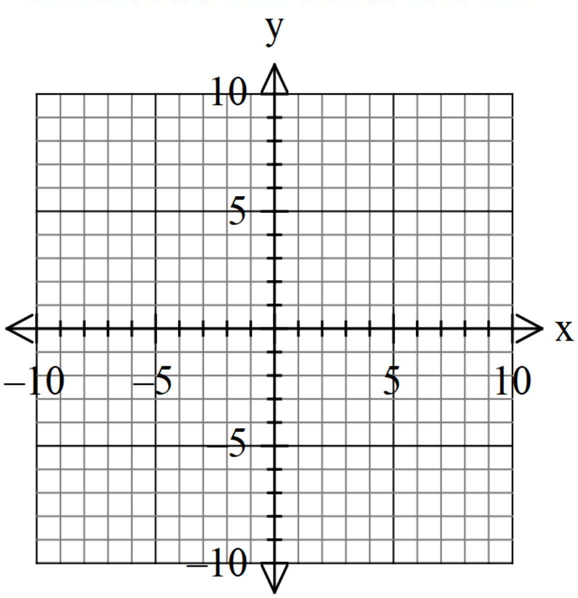


**Examples:** Find the center $(h, k)$ and radius $r$ of each circle, graph the circle.

6. $(x+1)^2 + (y-2)^2 = 25$

center:________________________________

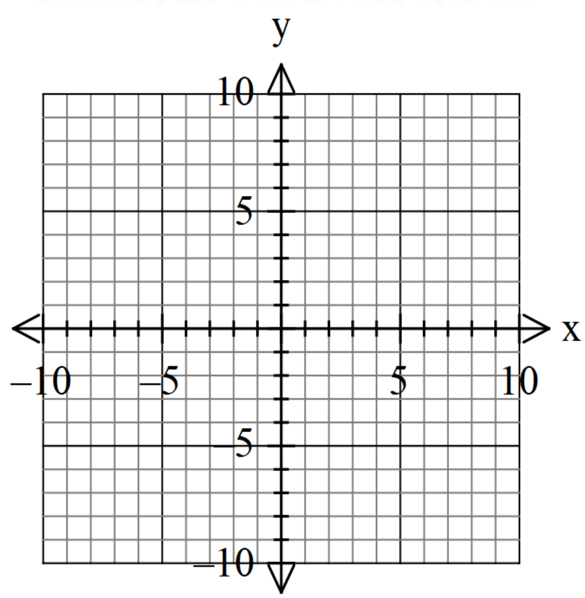
radius:________________________________



7. $x^2 + (y+6)^2 = 20$

center:________________________________

radius:________________________________



**Examples:** Complete the square.

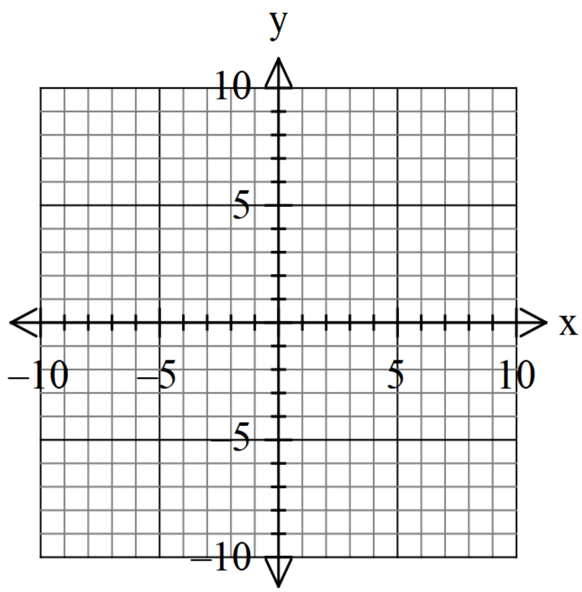
8. $x^2 - 8x + 7 = 0$

**Examples:** Find the center $(h, k)$ and radius $r$ of each circle, graph the circle.

9. $x^2 + y^2 - 6x + 2y + 9 = 0$

center:______________________________

radius:______________________________




**Examples:** Find the radius if the diameter is given.

10. d=8

11. d=7

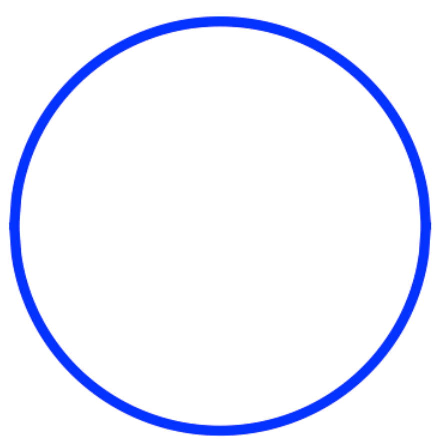
12. $d = \sqrt{6}$

**Examples:** Find the standard form of the equation of each circle.

13. Center at $(-3, 5)$ and a diameter of 24.

Center: ______________________________

Radius: ______________________________



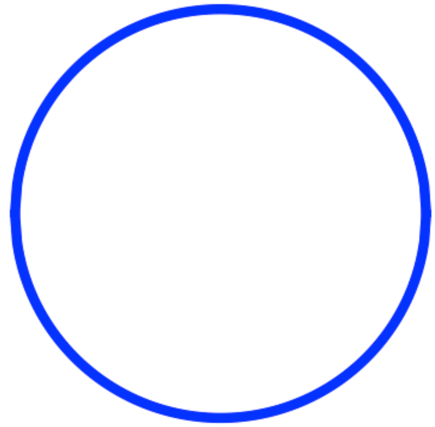
Equation: ______________________________

14. Center at $(1,0)$ and containing the point $(-3,2)$.

Center: ______________________________

Radius: ______________________________

Equation: ______________________________

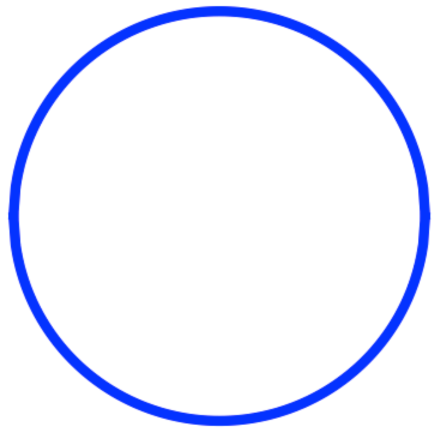


15. Endpoints of a diameter at $(4,3)$ and $(0,1)$.

Center: ______________________________

Radius: ______________________________

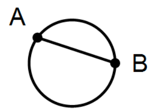
Equation: ______________________________



## 6.2 Circle Vocabulary, Arc and Angle Measures

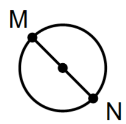
***Circle:*** All points in a plane that are the same distance from a given point, called the ***center*** of the circle.

***Chord:*** A segment with both endpoints on a circle.



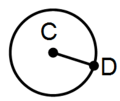

$\overline{AB}$ is a chord.

***Diameter:*** A chord that passes through the center of a circle.



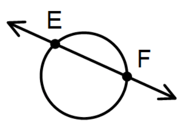

$\overline{MN}$ is a diameter.

***Radius:*** A segment with one endpoint on the circle and one endpoint at the center of the circle.




$\overline{CD}$ is a radius.

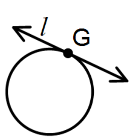
***Secant:*** A line that intersects a circle at two points.




$\overleftrightarrow{EF}$ is a secant.

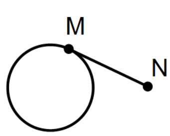
***Tangent:*** A line in the plane of the circle that intersects a circle at exactly one point.

***Point of Tangency:*** The point where a tangent intersects a circle.




Line $l$ is a tangent. $G$ is the point of tangency.

***Tangent Segment:*** A segment that touches a circle at one of its endpoints and lies in the line that is tangent to the circle at that point.




$\overline{MN}$ is a tangent segment.

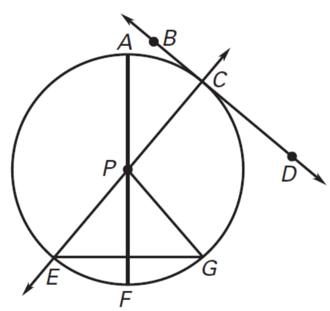
**Example:** In circle *P*, name the term that best describes the given line, segment, or point.

$\overline{AF}$ $\qquad$ $C$

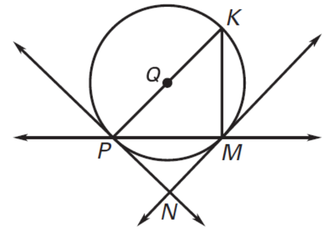
$\overline{EG}$ $\qquad$ $\overleftrightarrow{CE}$

$\overline{PF}$ $\qquad$ $\overleftrightarrow{BD}$

$\overline{PG}$ $\qquad$ $P$



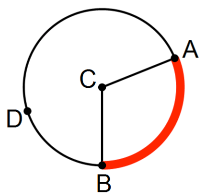
**Example:** In $\odot Q$, identify a chord, a diameter, two radii, a secant, two tangents, and two points of tangency.



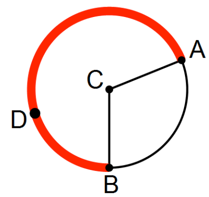

***Minor Arc:*** All the points on a circle that lie in the interior of a central angle whose measure is less than 180°.

***Major Arc:*** All the points on a circle that do not lie on the corresponding minor arc.

$\overset{\frown}{AB}$ is a minor arc.

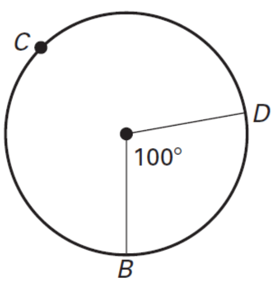



$\overset{\frown}{ADB}$ is a major arc.




***Measure of a Minor Arc:*** The measure of its central angle.

***Measure of a Major Arc:*** 360° minus the measure of the minor arc.

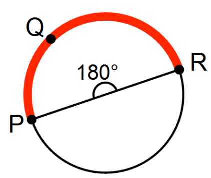



$m\overset{\frown}{BD} = 100°$

$m\overset{\frown}{BCD} = 360° - 100° = 260°$

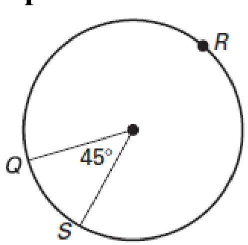
***Semicircle:*** An arc whose central angle measures 180°.

$\overline{PR}$ is a diameter. $m\overset{\frown}{PQR} = 180°$.




**Examples:** Name the major and minor arcs. Find the measure of each.

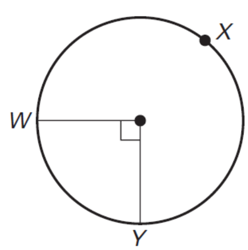
a)




major arc:

minor arc:

b)




major arc:

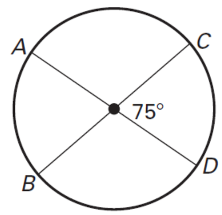
minor arc:

***Congruent Circles:*** Two circles that have the same radius.

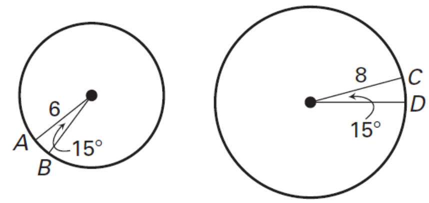
***Congruent Arcs:*** Two arcs of the *same circle* or of *congruent circles* that have the same measure.

**Examples:** Are arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ congruent? Explain your reasoning.

a)




b)

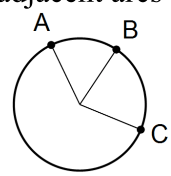



***Adjacent Arcs:*** Two arcs of a circle that share an endpoint.

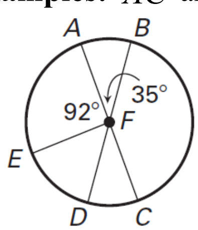
***Arc Addition Postulate:*** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are adjacent arcs.

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$




**Examples:** $\overline{AC}$ and $\overline{BD}$ are diameters. Find the indicated measures.




a) $m\overset{\frown}{DC}$

d) $m\overset{\frown}{DE}$

b) $m\overset{\frown}{BC}$

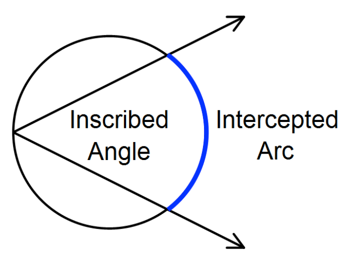
e) $m\overset{\frown}{ABE}$

c) $m\overset{\frown}{CDE}$

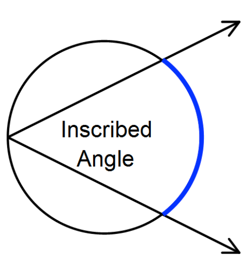
f) $m\overset{\frown}{ABD}$

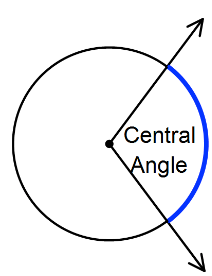
***Inscribed Angle:*** An angle whose vertex is on a circle and whose sides contain chords of the circle.

***Intercepted Arc:*** An arc that lies in the interior of an inscribed angle and has endpoints on the sides of the angle.

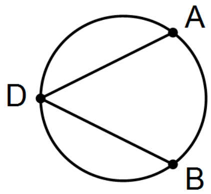



**WARNING:** Don't get ***inscribed*** angles and ***central*** angles mixed up!



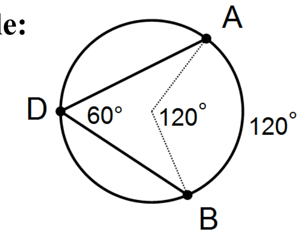
**Theorem:** If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.




$$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$$

$$m\overset{\frown}{AB} = 2m\angle ADB$$

**Example:**

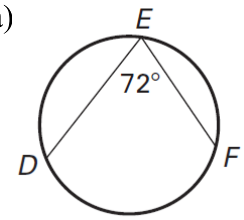



$$m\angle ADB = 60°$$

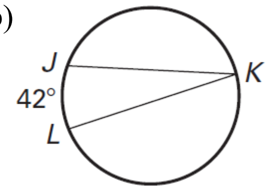
$$m\overset{\frown}{AB} = 120°$$

**Examples:** Find the measure of the inscribed angle or the intercepted arc.

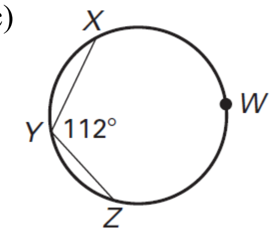
a)




b)

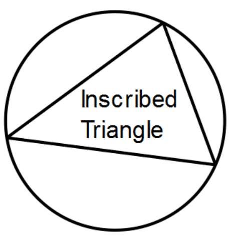


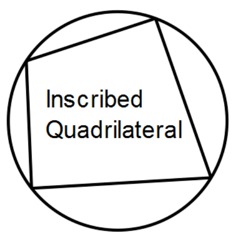

c)




$m\ \overset{\frown}{DF} =$ $\qquad$ $m\ \angle K =$ $\qquad$ $m\ \overset{\frown}{XWZ} =$

***Inscribed Polygon:*** A polygon whose vertices all lie on a circle.

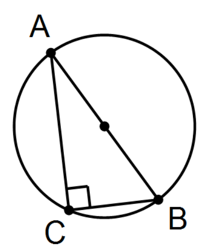



**Theorems:**

- If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

  If $\triangle ABC$ is a right triangle with hypotenuse $\overline{AB}$, then $\overline{AB}$ is a diameter of the circle.

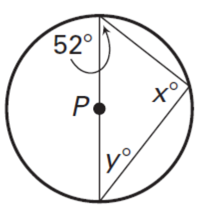



- If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.

  If $\overline{AB}$ is a diameter of the circle, then $\triangle ABC$ is a right triangle with $\overline{AB}$ as hypotenuse.

**Examples:** Find the values of $x$ and $y$ in $\odot P$.

a)

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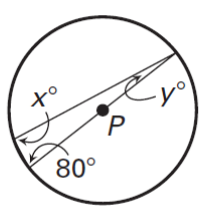


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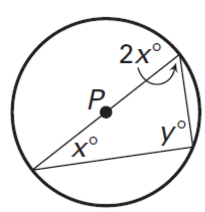
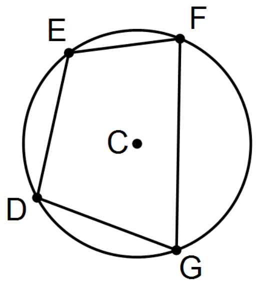



$x =$ $\qquad$ $y =$ $\qquad$ $x =$ $\qquad$ $y =$ $\qquad$ $x =$ $\qquad$ $y =$

**Theorem:**

**Theorem:**

- If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.

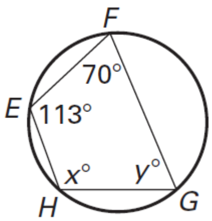



$$m\angle D + m\angle F = 180°$$

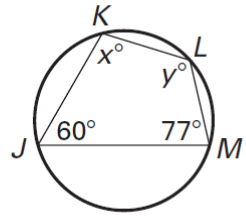
$$m\angle E + m\angle G = 180°$$

**Examples:** Find the values of $x$ and $y$.

a)




b)

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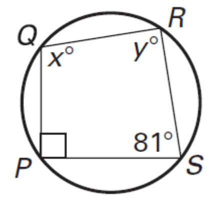
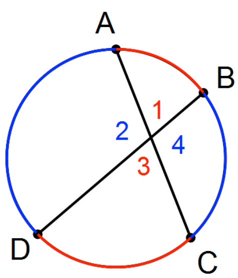



$x =$ $y =$ $x =$ $y =$ $x =$ $y =$

**Theorem:**

- If two chords intersect inside a circle, then the measure of each angle formed is the average of the measures of the arcs intercepted by the angle and its vertical angle.

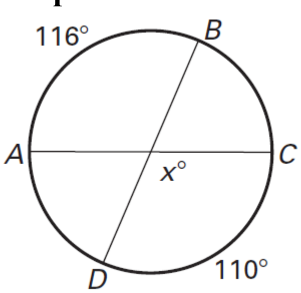



$$m\angle 1 = m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$$

$$m\angle 2 = m\angle 4 = \frac{1}{2}\left(m\overset{\frown}{BC} + m\overset{\frown}{AD}\right)$$

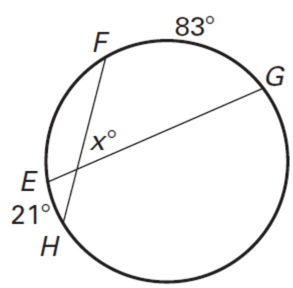
**Examples:** Find the value of $x$.

a)



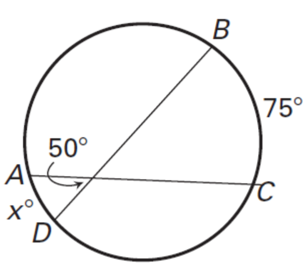

$x =$

b)



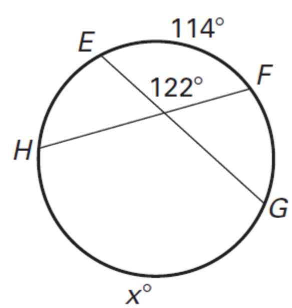

$x =$

c)




$x =$

d)

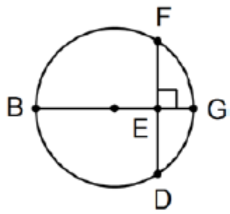



$x =$

# SM2H 6.3 Inscribed Angles, Chord, Tangent and Secant Theorems Notes

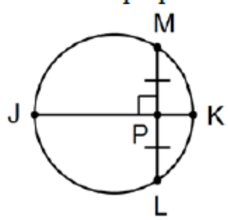
**Theorems about Chords:**

- If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.




If $\overline{BG} \perp \overline{FD}$, then $\overline{DE} \cong \overline{EF}$ and $\overset{\frown}{DG} \cong \overset{\frown}{GF}$

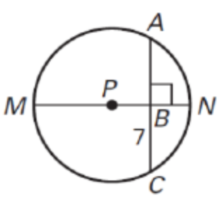
- If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



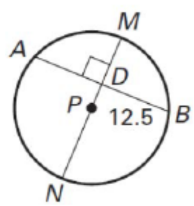

If $\overline{JK} \perp \overline{ML}$ and $\overline{MP} \cong \overline{PL}$, then $\overline{JK}$ is a diameter.

**Examples:** $\overline{MN}$ is a diameter of $\odot P$. Find the length of $\overline{AB}$.

a)

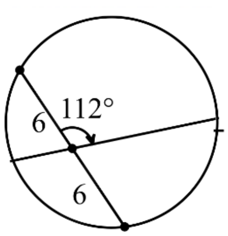



b)

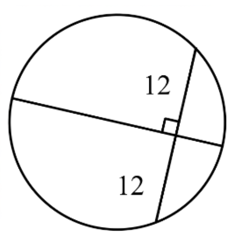



**Exanples**: Determine whether $\overline{AB}$ is a diameter of the circle. Which Theorem or rule did you use?.

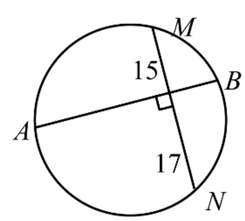
a)



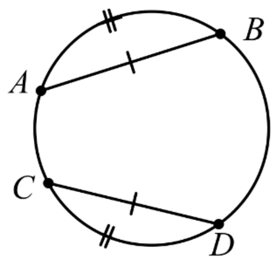

b)




c)

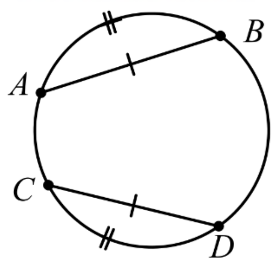


- In the same circle, or in congruent circles, if two chords are congruent, then their corresponding minor arcs are congruent.




If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{DC}$

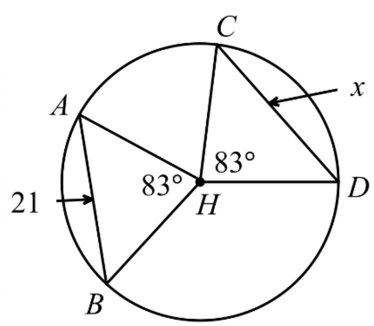
- In the same circle, or in congruent circles, if two minor arcs are congruent, then their corresponding chords are congruent.



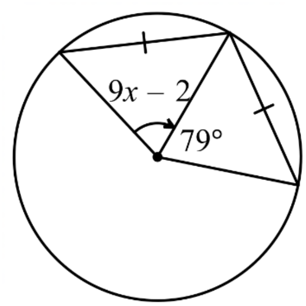

If $\widehat{AB} \cong \widehat{DC}$, then $\overline{AB} \cong \overline{CD}$

**Examples:** Find the value of $x$. Explain which theorem you used.

a)




b)




$x =$

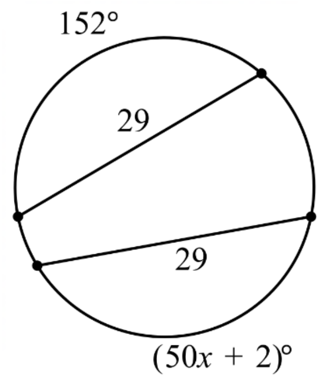
Theroem:

$x =$

Why?

What is true about $\Delta AHB$ and $\Delta CHD$?

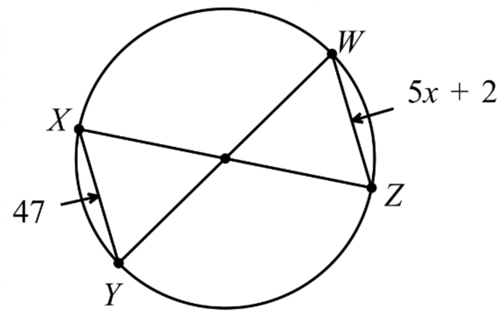
c)




$x =$

Which Theorem did you use?

d) $\overline{XZ}$ and $\overline{YW}$ are diameters

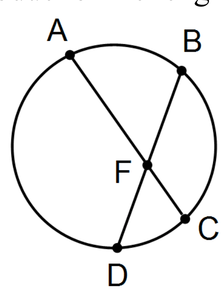



$x =$

Why?

**Theorem:**

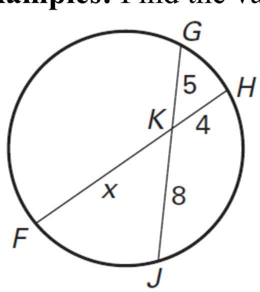
- If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.




$$AF \cdot FC = BF \cdot FD$$

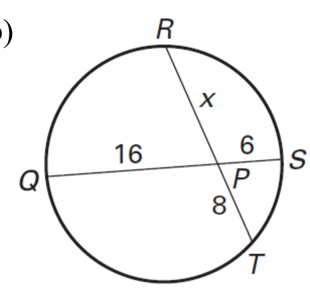
**Examples:** Find the value of $x$.

a)



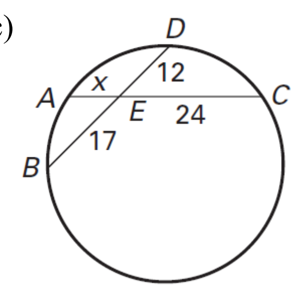

$x =$

b)




$x =$

c)

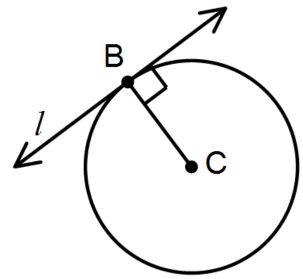



$x =$

***Theorems About Tangents:***

- If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency

If line $l$ is tangent to $\odot C$ at $B$, then $l \perp \overline{CB}$.

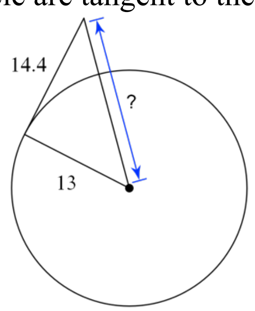



- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

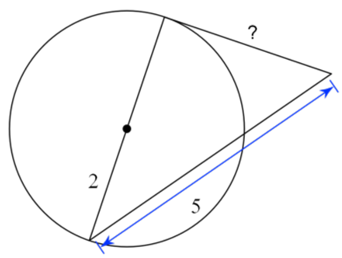
If $l \perp \overline{CB}$, then line $l$ is tangent to $\odot C$ at $B$.

**Examples:** Find the length of the missing segment. Assume that segments which appear to be tangent to the circle are tangent to the circle.

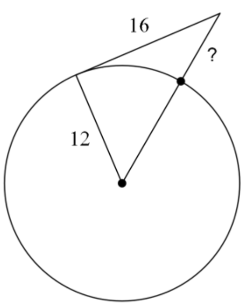
a)



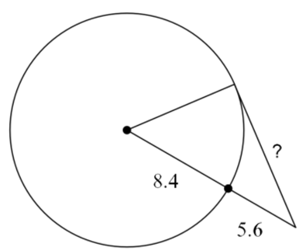

b)




c)

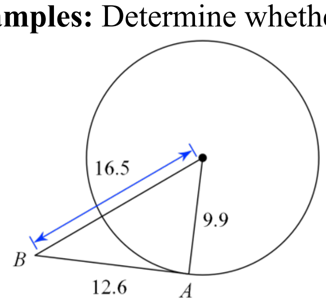



d)

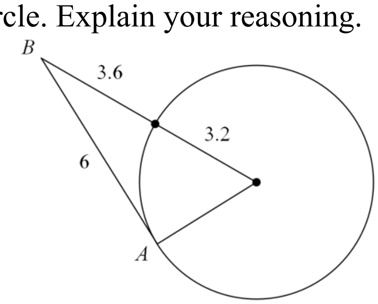



**Examples:** Determine whether $\overline{AB}$ is tangent to the circle. Explain your reasoning.

a)

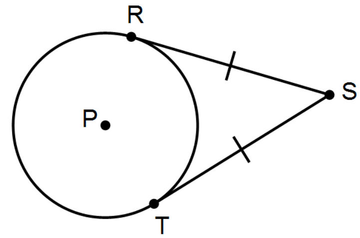



b)



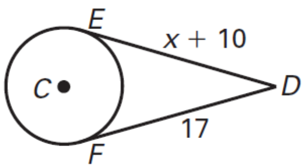
***Theorem:*** If two segments from the same point outside a circle are both tangent to the circle, then they are congruent.

If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$ at points $R$ and $T$, then $\overline{SR} \cong \overline{ST}$.

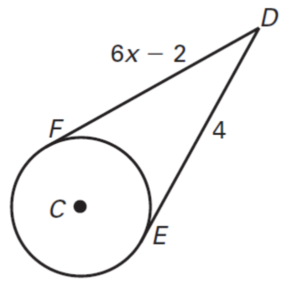



**Examples:** $\overline{DE}$ and $\overline{DF}$ are both tangent to $\odot C$. Find the value of $x$.

a)



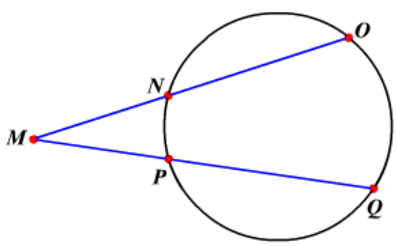

b)




***Theorems About Secants:***

**Intersecting Secants Theorem**

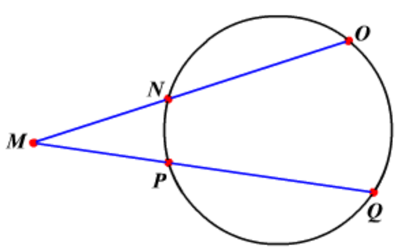
If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

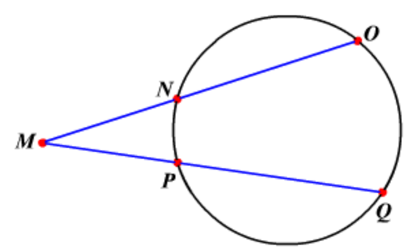



In the circle, $\overline{MO}$ and $\overline{MQ}$ are secants that intersect at point $M$.

So, $MN \cdot MO = MP \cdot MQ$.

**Examples:**



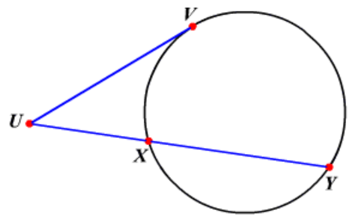




a) In the circle shown, if $MN$=10, $NO$=17, $MP$=9, then find the length of $PQ$.

b) In the circle shown, if $NO$=2$x$, $MN$=10, $PQ$=2$x$+3, $MP$=9, then solve for $x$.

**Intersecting Secant-Tangent Theorem**

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.




In the circle, $\overline{UV}$ is a tangent and $\overline{UY}$ is a secant. They intersect at point *U*.

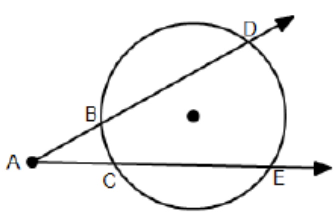
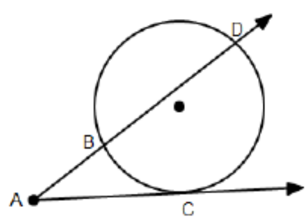
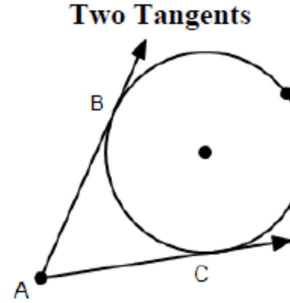
So $(UV)^2 = UX \cdot UY$ .

**Example:**

In the circle shown above, if *UX*=8, and *XY*=10, then find the length of *UV*.

**Theorem**

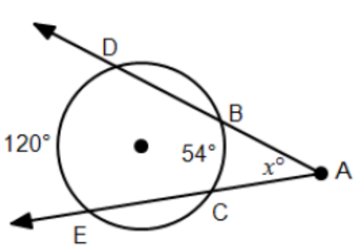
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

| Two Secants | Secant and Tangent | Two Tangents |
|---|---|---|
|  |  |  |
| $m\angle A = \frac{1}{2}(mDE - mBC)$ | $m\angle A = \frac{1}{2}(mDC - mBC)$ | $m\angle A = \frac{1}{2}(mCDB - mBC)$ |



Find the Value of x.

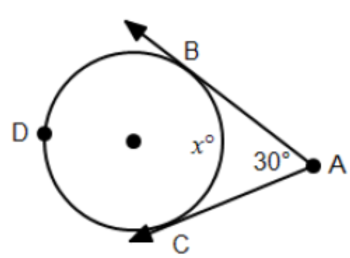
1.




$x =$

Why?

2.

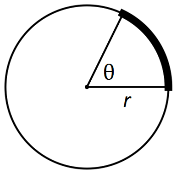



$x =$

Why?

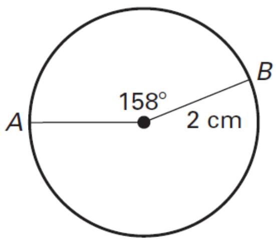
# SM2H 6.4 Arc Length and Sector Area Notes

***Arc Length:*** *Arc* Length = $\frac{\theta}{360°} \cdot$ circumference of circle = $\frac{\theta}{360°} \cdot 2\pi r$

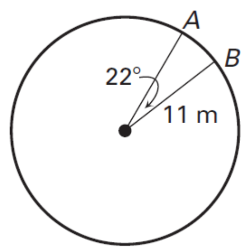



**Examples:** Find the length of $\overset{\frown}{AB}$. Write your answers in terms of π and as decimals rounded to the nearest hundredth.

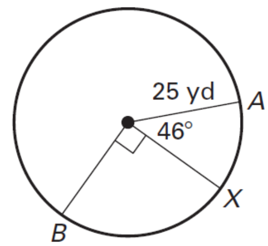
a)



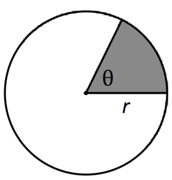

b)




c)

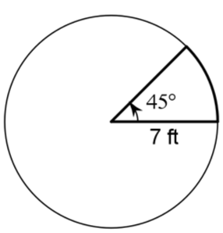



***Sector Area:*** Sector Area = $\frac{\theta}{360°} \cdot$ area of circle = $\frac{\theta}{360°} \cdot \pi r^2$

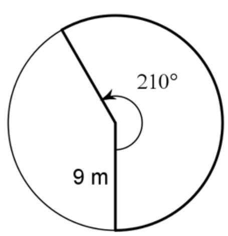



**Examples:** Find the area of each sector. Write your answers in terms of π and as decimals rounded to the nearest tenth.

a)




b)




c)