

Name: _____

Period: Filled in Notes

SM2H Unit 6 Circle Notes

o.1 Circles

Objectives:

- Sketch the graph of a circle, given the equation.
- Given the general equation of a circle, complete the square to find the center.

Circles

Circles with Center (h, k)	
General Equation	$x^2 + y^2 + Ax + By + C = 0$
Radius	r

Circle: The set of all points in the xy -plane that are a fixed distance r , called the **radius**, from a fixed point (h, k) , called the **center** of the circle.

Standard Form of the Equation of a Circle with radius r and center (h, k) :

$$(x-h)^2 + (y-k)^2 = r^2$$

General Form of the Equation of a Circle:

$$x^2 + y^2 + Ax + By + C = 0$$

To find the standard form of the equation of a circle when you know the general form, **complete the square** for both x and y .

Distance Formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example: Find the distance between the two given points.

1. $P_1 = (-2, 10)$ and $P_2 = (5, 4)$

$$D = \sqrt{(-2-5)^2 + (10-4)^2}$$

$$D = \sqrt{49 + 36}$$

$$D = \sqrt{85}$$

2. $P_1 = (3, -6)$ and $P_2 = (-5, -7)$

$$D = \sqrt{(3+5)^2 + (-6+7)^2}$$

$$D = \sqrt{64 + 1}$$

$$D = \sqrt{65}$$

Example: Find the midpoint of the given points.

3. $P_1 = (3, -6)$ and $P_2 = (-5, 8)$

$$\left(\frac{3+(-5)}{2}, \frac{-6+8}{2} \right) = (-1, 1)$$

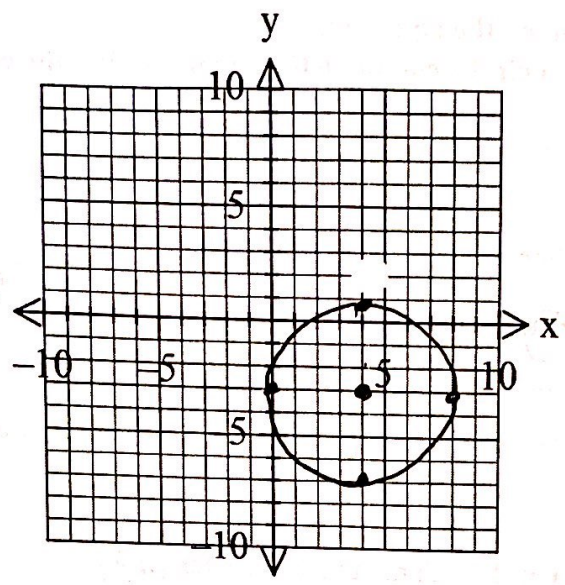
4. $P_1 = (-3, 5)$ and $P_2 = (1, 8)$

$$\left(\frac{-3+1}{2}, \frac{5+8}{2} \right) = \left(-1, \frac{13}{2} \right) \text{ or } (-1, 6.5)$$

5. **Example:** Write the standard form of the equation of the circle with radius $r = 4$ and center $(h, k) = (4, -3)$.
 Then graph the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

Equation: $(x-4)^2 + (y+3)^2 = 4^2$ or $(x-4)^2 + (y+3)^2 = 16$

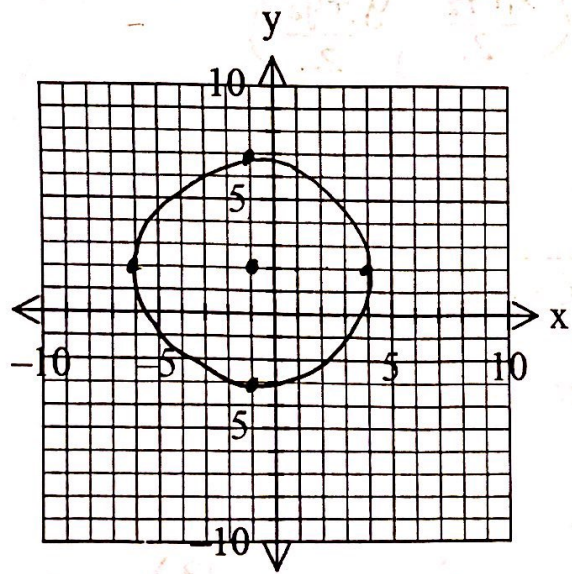


Examples: Find the center (h, k) and radius r of each circle, graph the circle.

6. $(x+1)^2 + (y-2)^2 = 25$

center: $(-1, 2)$

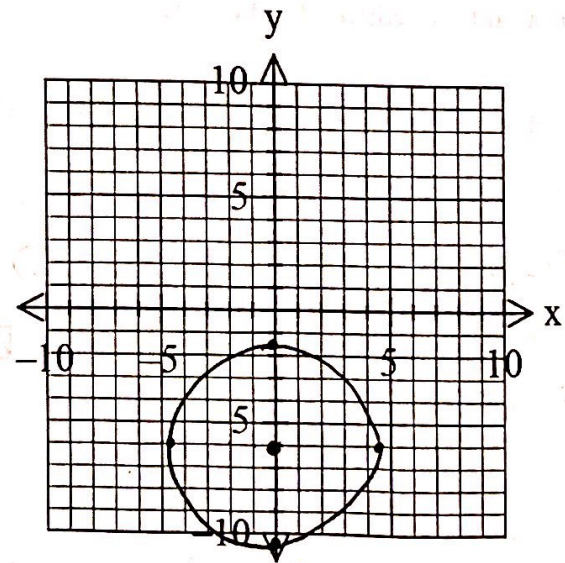
radius: 5



7. $x^2 + (y+6)^2 = 20$

center: $(0, -6)$

radius: $\sqrt{20} \approx 4.5$



Examples: Complete the square.

8. $x^2 - 8x + 7 = 0$

$$x^2 - 8x = -7$$

$$x^2 - 8x + 4^2 = -7 + 4^2$$

$$(x - 4)^2 = 9$$

Examples: Find the center (h, k) and radius r of each circle, graph the circle.

8. $x^2 + y^2 - 6x + 2y + 9 = 0$

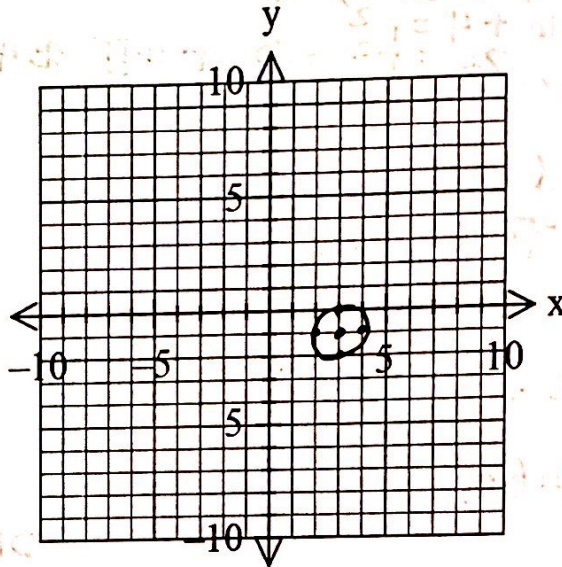
center: $(3, -1)$

radius: 1

$$x^2 - 6x + y^2 + 2y = -9$$

$$x^2 - 6x + 3^2 + y^2 + 2y + 1^2 = -9 + 3^2 + 1^2$$

$$(x - 3)^2 + (y + 1)^2 = 1$$



Examples: Find the radius if the diameter is given.

9. $d=8$

$r = 4$

10. $d=7$

$r = 3.5$

11. $d = \sqrt{6} \approx 2.4$

$r = 1.2$

Examples: Find the standard form of the equation of each circle.

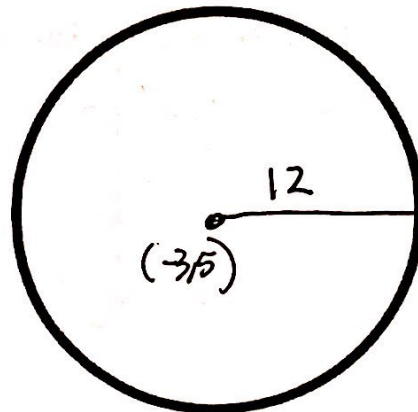
9. Center at $(-3, 5)$ and a diameter of 24.

$r = 24 \div 2$
 $r = 12$

Center: $(-3, 5)$

Radius: 12

Equation: $(x + 3)^2 + (y - 5)^2 = 144$



10. Center at $(1,0)$ and containing the point $(-3,2)$.

Center: $(1,0)$

Radius: $\sqrt{20}$ or 4.5
 $r^2 = (\sqrt{20})^2 = 20$

Equation: $(x-1)^2 + y^2 = 20$

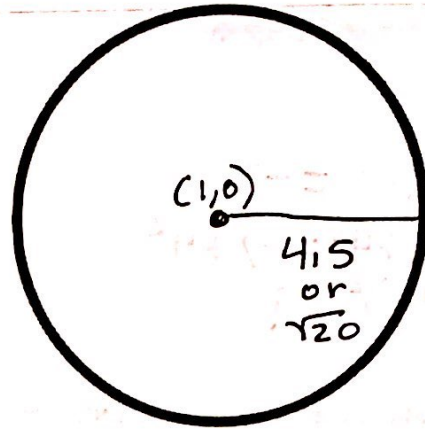
$$(x-1)^2 + (y-0)^2 = r^2$$

$$(-3-1)^2 + (2-0)^2 = r^2$$

$$16 + 4 = r^2$$

$$20 = r^2 \Rightarrow$$

$$r = \sqrt{20} \approx 4.5$$



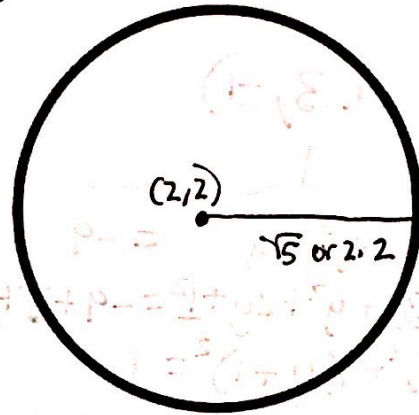
11. Endpoints of a diameter at $(4,3)$ and $(0,1)$.

Center: $(2,2)$

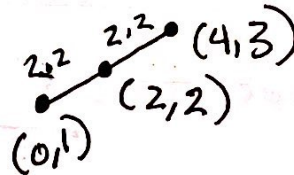
Radius: $\sqrt{5}$ or 2.2

Equation: $(x-2)^2 + (y-2)^2 = 5$

$$r^2 = (\sqrt{5})^2 = 5$$



Diameter



midpoint
is center
of diameter

$$\left(\frac{4+0}{2}, \frac{3+1}{2} \right)$$

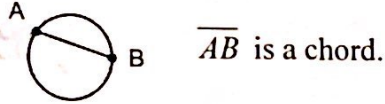
$$r = \sqrt{\frac{(4-2)^2}{4} + \frac{(3-2)^2}{1}} \quad (2,2)$$

$$r = \sqrt{5} \text{ or } 2.2$$

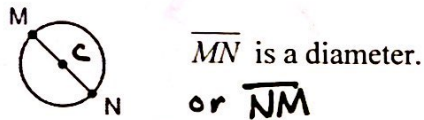
6.2 Circle Vocabulary, Arc and Angle Measures

Circle: All points in a plane that are the same distance from a given point, called the *center* of the circle.

Chord: A segment with both endpoints on a circle.



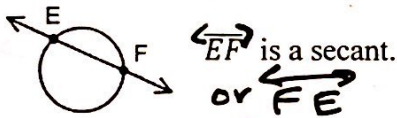
Diameter: A chord that passes through the center of a circle.



Radius: A segment with one endpoint on the circle and one endpoint at the center of the circle.

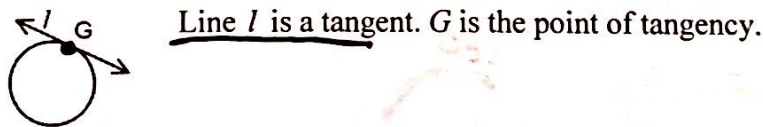


Secant: A line that intersects a circle at two points.

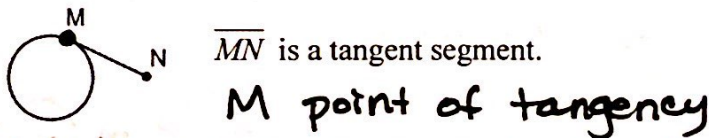


Tangent: A line in the plane of the circle that intersects a circle at exactly one point.

Point of Tangency: The point where a tangent intersects a circle.



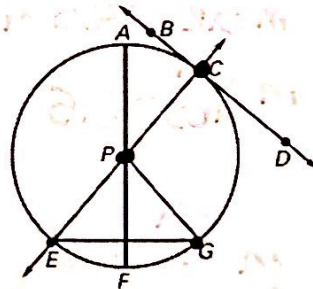
Tangent Segment: A segment that touches a circle at one of its endpoints and lies in the line that is tangent to the circle at that point.



Example: In circle P , name the term that best describes the given line, segment, or point.

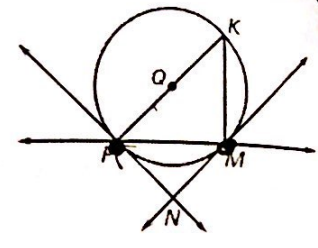
\overline{AF} Diameter
 \overline{EG} chord
 \overline{PF} radius
 \overline{PG} radius

C Point of tangency
 \overline{CE} Secant
 \overline{BD} tangent
 P center



Example: In circle Q : identify a chord, a diameter, two radii, a secant, two tangents, and two points of tangency.

chord: $\overline{PM}, \overline{PK}, \overline{EM}$ secant: \overleftrightarrow{PM}
 diameter: \overline{PE} tangents: \overleftrightarrow{PN} \overleftrightarrow{NM}
 radii: \overline{QP} point of tangency: P, M
 \overline{QE}

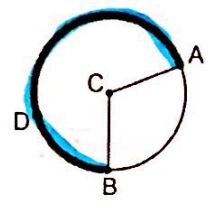
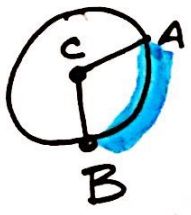


Minor Arc: All the points on a circle that lie in the interior of a central angle whose measure is less than 180° .

Major Arc: All the points on a circle that do not lie on the corresponding minor arc. *bigger than 180°*

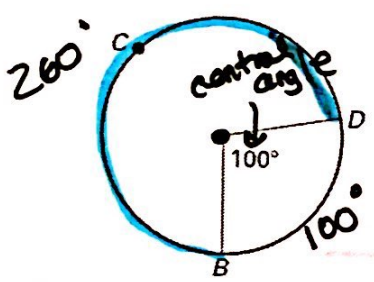
\widehat{AB} is a minor arc.

\widehat{ADB} is a major arc.



Measure of a Minor Arc: The measure of its central angle.

Measure of a Major Arc: 360° minus the measure of the minor arc.

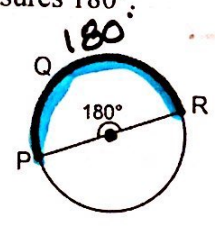


$$m\widehat{BD} = 100^\circ$$

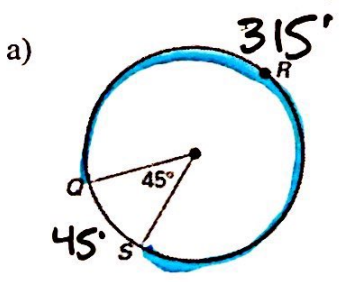
$$m\widehat{BCD} = 360^\circ - 100^\circ = 260^\circ$$

Semicircle: An arc whose central angle measures 180° .

\overline{PR} is a diameter. $m\widehat{PQR} = 180^\circ$

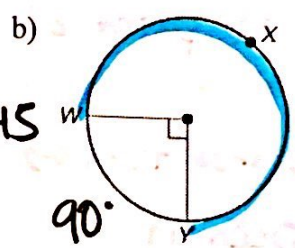


Examples: Name the major and minor arcs. Find the measure of each.



$$m\widehat{RS} = 360 - 45$$

$$m\widehat{RS} = 315^\circ$$



$$m\widehat{YX} = 360 - 90 = 270^\circ$$

Major arc:
 $m\widehat{RS} =$

Minor arc:
 $m\widehat{QS} = 45^\circ$
measure

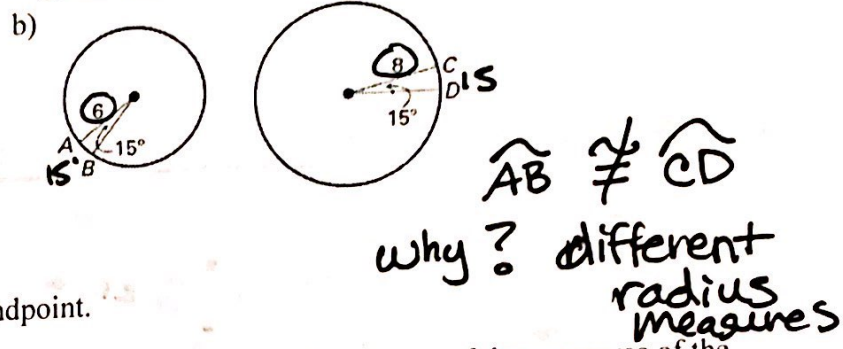
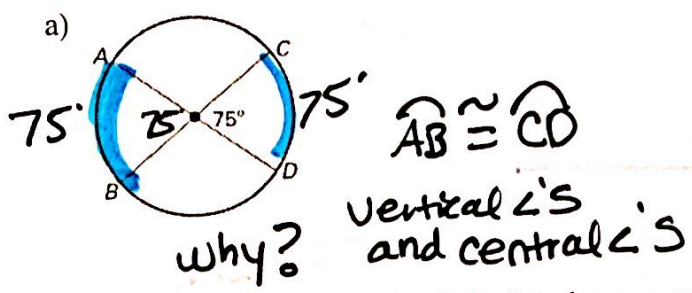
Major arc:
 $m\widehat{YX} = 270^\circ$

Minor arc:
 $m\widehat{XY} = 90^\circ$

Congruent Circles: Two circles that have the same radius. *same size/same shape*

Congruent Arcs: Two arcs of the *same circle* or of *congruent circles* that have the same measure. *and circles must have same radius.*

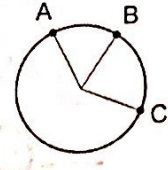
Examples: Are arcs \widehat{AB} and \widehat{CD} congruent? Explain your reasoning.



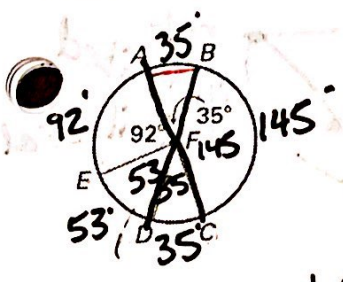
Adjacent Arcs: Two arcs of a circle that share an endpoint.

Arc Addition Postulate: The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

\widehat{AB} and \widehat{BC} are adjacent arcs.
 $m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$



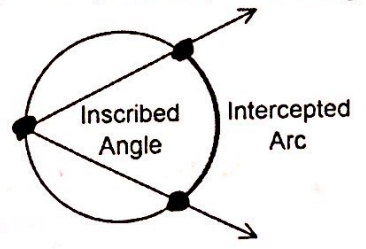
Examples: \overline{AC} and \overline{BD} are diameters. Find the indicated measures. What theorem did you use to find it?



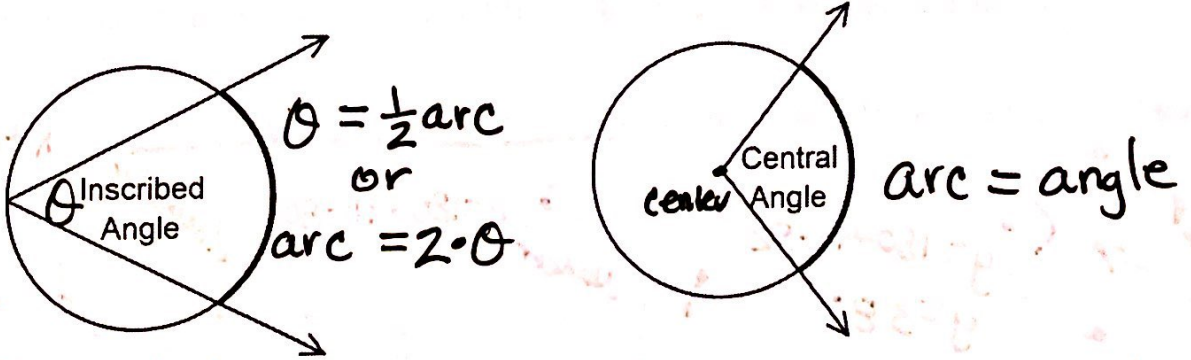
- a) $m\widehat{DC} = 35^\circ$ *why? vertical \angle central \angle*
- b) $m\widehat{BC} = 145^\circ$ *why? Supplementary \angle 's* $180 - 35 = 145$
- c) $m\widehat{EDC} = 88^\circ$ *why? Arc Addition Postulate* $53 + 35 = 88$
- d) $m\widehat{ED} = 53^\circ$ *why? supplementary \angle 's* $180 - 92 = 53$
- e) $m\widehat{ABE} = 268^\circ$ *Arc Addition Postulate* $360 - 92 = 268$
- f) $m\widehat{ABD} = 215^\circ$ *Arc Addition Postulate* $35 + 145 + 35 = 215$

Inscribed Angle: An angle whose vertex is on a circle and whose sides contain chords of the circle.

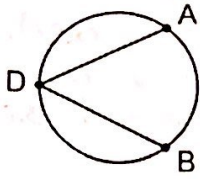
Intercepted Arc: An arc that lies in the interior of an inscribed angle and has endpoints on the sides of the angle.



WARNING: Don't get *inscribed* angles and *central* angles mixed up!



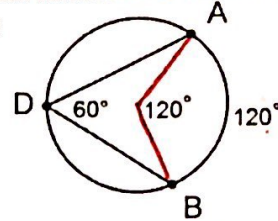
Theorem: If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



$$m \angle ADB = \frac{1}{2} m \widehat{AB}$$

$$m \widehat{AB} = 2 m \angle ADB$$

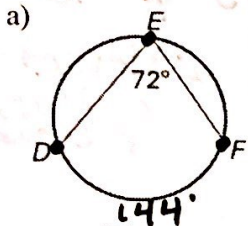
Example:



$$m \angle ADB = 60^\circ$$

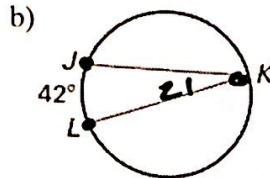
$$m \widehat{AB} = 120^\circ$$

Examples: Find the measure of the inscribed angle or the intercepted arc.



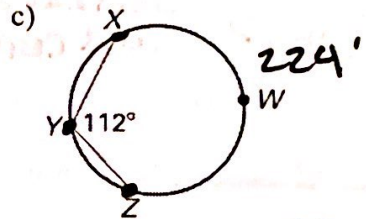
$$m \widehat{DF} = 72 \cdot 2$$

$$m \widehat{DF} = 144^\circ$$



$$m \angle K = 42 \div 2$$

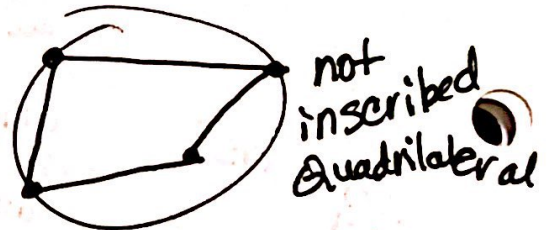
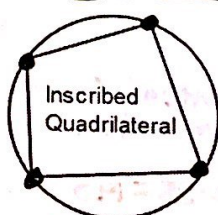
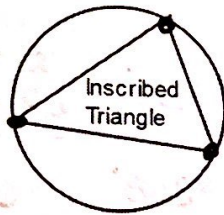
$$m \angle K = 21^\circ$$



$$m \widehat{XWZ} = 112 \cdot 2$$

$$m \widehat{XWZ} = 224^\circ$$

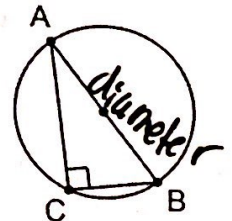
Inscribed Polygon: A polygon whose vertices all lie on a circle.



Theorems: **Inscribed Δ Thm.**

- If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

If ΔABC is a right triangle with hypotenuse \overline{AB} , then \overline{AB} is a diameter of the circle.

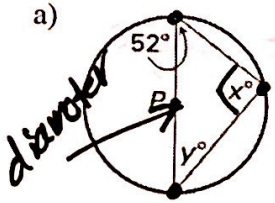


Converse

- If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.

If \overline{AB} is a diameter of the circle, then ΔABC is a right triangle with \overline{AB} as hypotenuse.

Examples: Find the values of x and y in $\odot P$. Explain which rule or theorem you used.

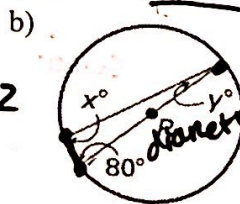


$$y = 180 - 90 - 52$$

$$y = 38^\circ$$

$$x = 90^\circ \quad y = 38^\circ$$

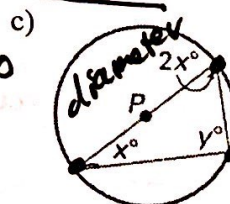
why? Inscribed Δ Thm.



$$y = 180 - 90 - 80$$

$$x = 90^\circ \quad y = 10^\circ$$

why? Inscribed Δ Thm



$$x = 30^\circ \quad y = 90^\circ$$

Inscribed Δ Thm.

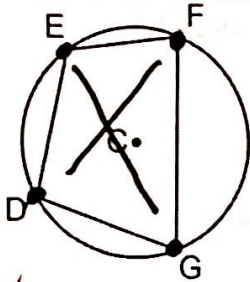
$$x + 2x + 90 = 180$$

$$3x = 90$$

$$x = 30^\circ$$

Theorem: Incribed Quadrilateral Thm.

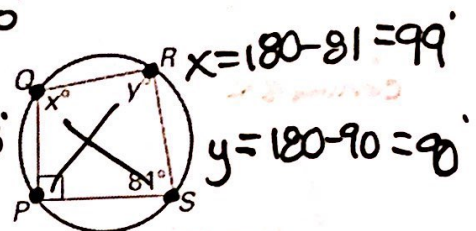
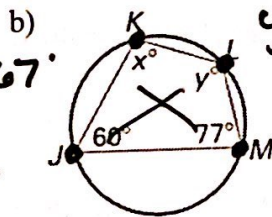
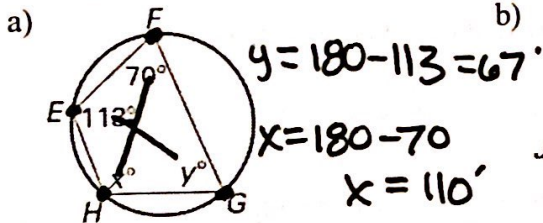
- If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.



$$m\angle D + m\angle F = 180^\circ$$

$$m\angle E + m\angle G = 180^\circ$$

Examples: Find the values of x and y .



$x = 110$ $y = 67$

$x = 103$ $y = 120$

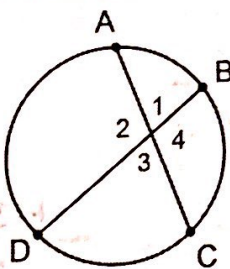
$x = 99$ $y = 90$

why? Incribed Quad Thm for a, b & c

Theorem:

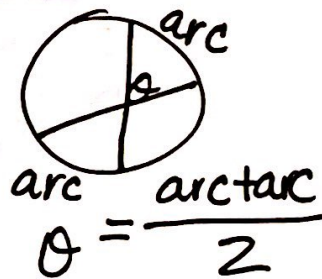
Chord Chord Angle Thm.

- If two chords intersect inside a circle, then the measure of each angle formed is the average of the measures of the arcs intercepted by the angle and its vertical angle.

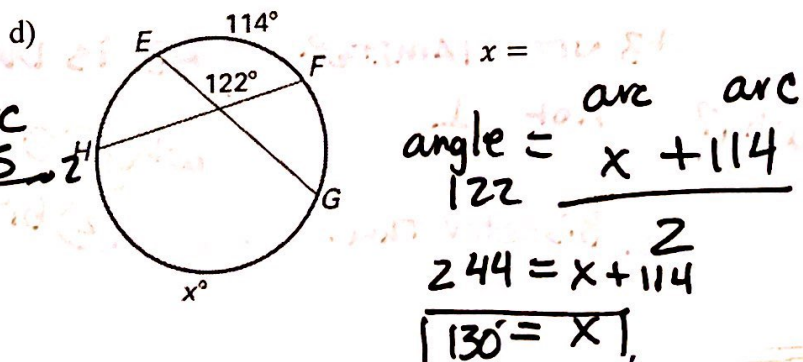
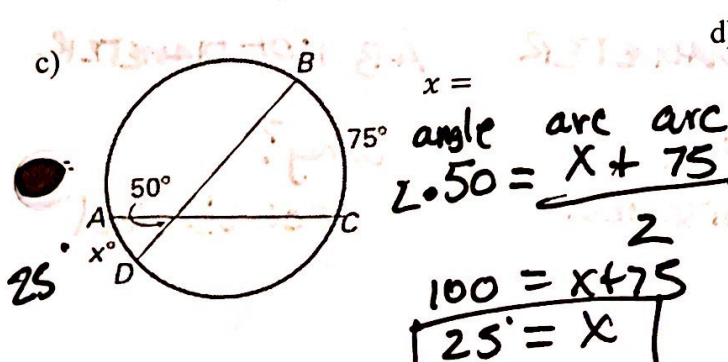
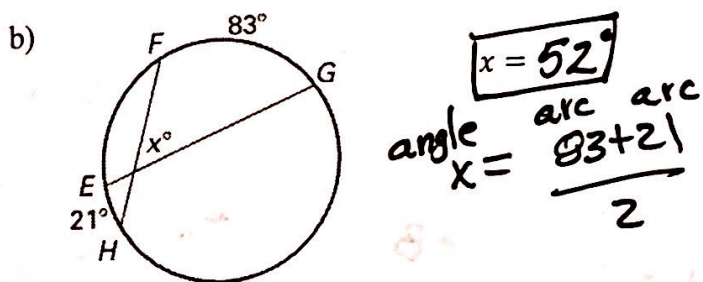
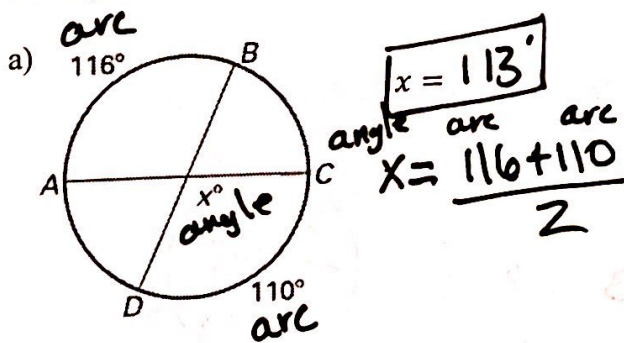


$$m\angle 1 = m\angle 3 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$$

$$m\angle 2 = m\angle 4 = \frac{1}{2}(m\widehat{BC} + m\widehat{AD})$$



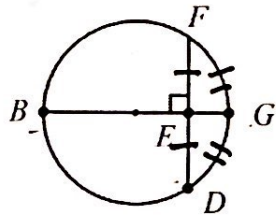
Examples: Find the value of x . Which Theorem or Rule did you use?



SM2H 6.3 Inscribed Angles, Chord, Tangent and Secant Theorems Notes.

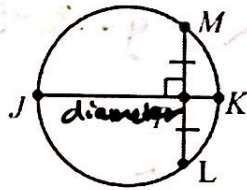
Theorems about Chords: Diameter/Chord Thm ⊥ bisector

- If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.



If $\overline{BG} \perp \overline{FD}$, then $\overline{ED} \cong \overline{EF}$ and $\widehat{DG} \cong \widehat{GF}$

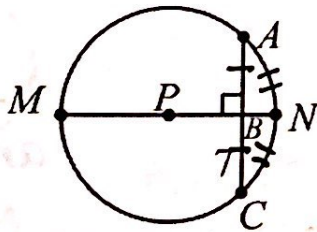
- If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. **converse**



If $\overline{JK} \perp \overline{ML}$ and $\overline{MP} \cong \overline{PL}$, then \overline{JK} is a diameter.

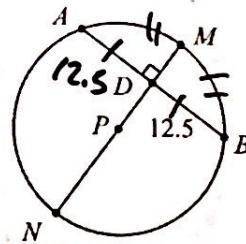
Examples: MN is a diameter of circle P. Find the length of AB

a)



$\overline{AB} = 7$

b)

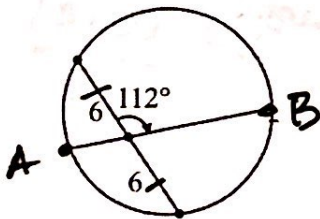


$\overline{AB} = 25$

$$\begin{array}{r} 12.5 \\ +12.5 \\ \hline 25 \end{array}$$

Examples: Determine whether \overline{AB} is a diameter of the circle. Which Theorem or rule did you use?.

a)



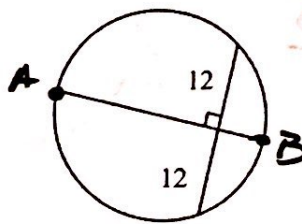
AB NOT DIAMETER

why?

not ⊥

⊥ Bisector Thm.

b)

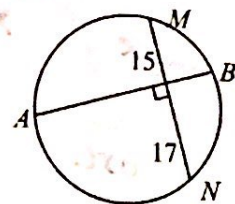


AB IS DIAMETER

why? ① ⊥

② bisected

c)



AB NOT DIAMETER

why?

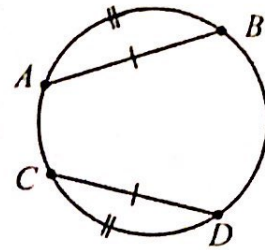
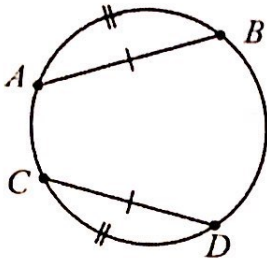
not bisected

If $\text{arc} \cong \text{arc}$ then $\text{chord} \cong \text{chord}$

Converse

- In the same circle, or in congruent circles, if two chords are congruent, then their corresponding minor arcs are congruent.

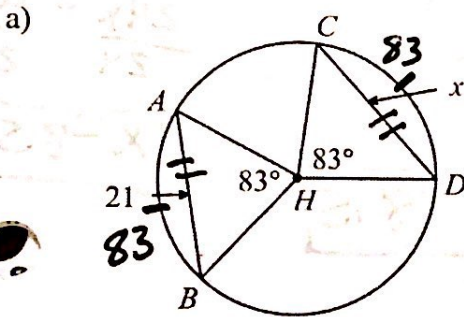
- In the same circle, or in congruent circles, if two minor arcs are congruent, then their corresponding chords are congruent.



If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{DC}$

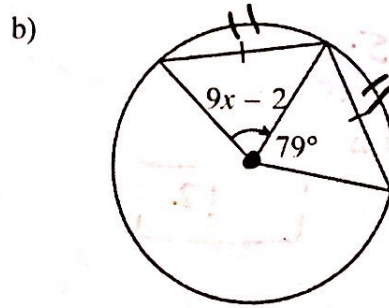
If $\widehat{AB} \cong \widehat{DC}$, then $\overline{AB} \cong \overline{CD}$

Examples: Find the value of x . Explain which theorem you used.



$x = 21$

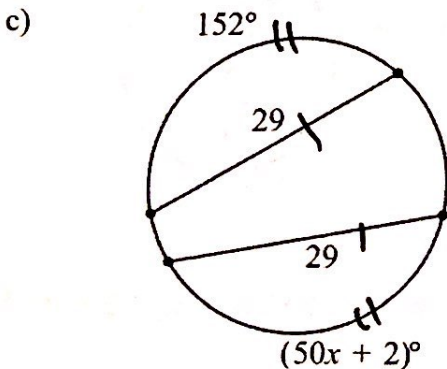
Theorem: central \angle 's
 $\text{arc} \cong \text{arc} / \text{chord} \cong \text{chord}$
 What is true about $\triangle AHB$ and $\triangle CHD$?



$x = 9$

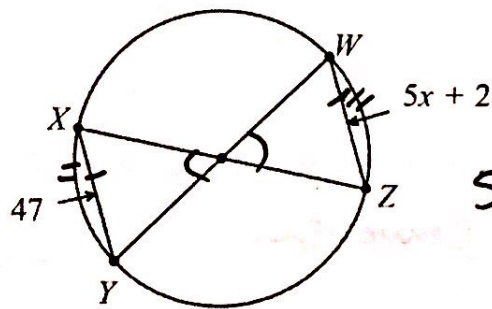
Why? $\text{chord} \cong \text{chord}$ then $\text{arc} \cong \text{arc}$
 central \angle 's

$$\begin{aligned} 9x - 2 &= 79 \\ 9x &= 81 \\ x &= 9 \end{aligned}$$



$$\begin{aligned} 5x + 2 &= 152 \\ 5x &= 150 \\ \boxed{x = 3} \end{aligned}$$

d) \overline{XZ} and \overline{YW} are diameters



$x = 9$

vertical \angle 's \cong
 and $\text{arc} \cong \text{arc}$

$$\begin{aligned} 5x + 2 &= 47 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

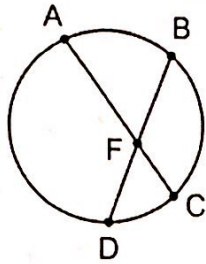
Which Theorem did you use?

$\text{chord} \cong \text{chord}$

Why?

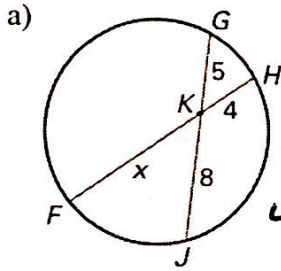
Theorem: Intersecting chord Thm.

- If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$\overline{AF} \cdot \overline{FC} = \overline{BF} \cdot \overline{FD}$$

Examples: Find the value of x .

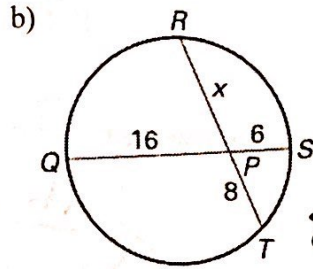


$$4x = 5 \cdot 8$$

$$4x = 40$$

$$x = 10$$

$$x = 10$$

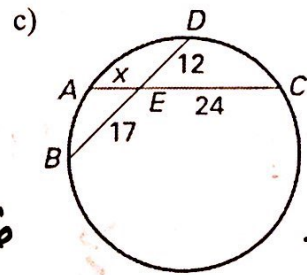


$$8x = 16 \cdot 6$$

$$8x = 96$$

$$x = 12$$

$$x = 12$$



$$24x = 17 \cdot 12$$

$$24x = 204$$

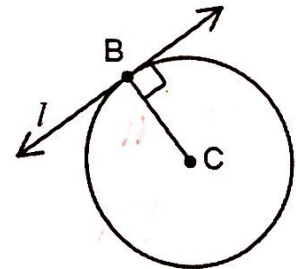
$$\frac{204}{24} = \frac{x}{1}$$

$$x = 8.5$$

$$x = 8.5$$

Theorems About Tangents: Tangent \perp radius

- If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency

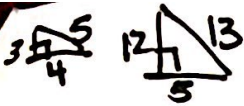


If line l is tangent to circle C at B then $l \perp \overline{CB}$

Converse

- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

If $l \perp \overline{CB}$: then line l is tangent to circle C at B .

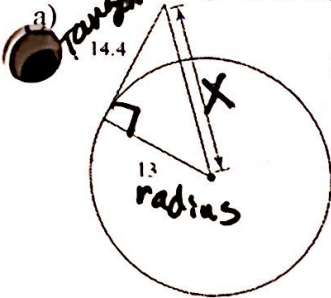


Common RT Δ's

Pythag. Thm (rt. Δ's)

$$a^2 + b^2 = c^2 \quad a \perp b \quad c$$

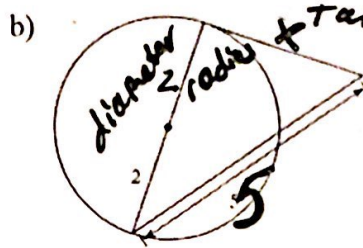
Examples: Find the length of the missing segment. Assume that segments which appear to be tangent to the circle are tangent to the circle.



$$14.4^2 + 13^2 = x^2$$

$$\sqrt{376.36} = \sqrt{x^2}$$

$$19.4 = x$$

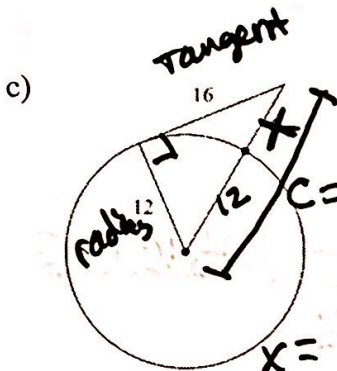


$$x^2 + 4^2 = 5^2$$

$$x^2 + 16 = 25$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = 3$$



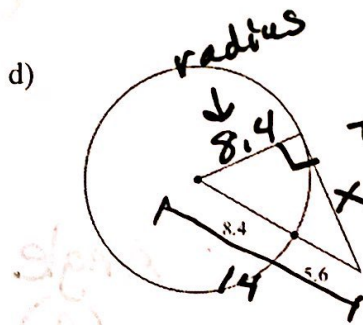
$$16^2 + 12^2 = c^2$$

$$\sqrt{400} = \sqrt{c^2}$$

$$20 = c$$

$$x = 20 - 12 = 8$$

$$x = 8$$

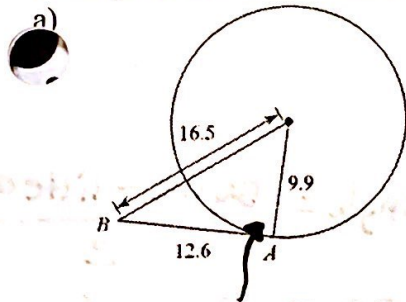


$$x^2 + 8.4^2 = 14^2$$

$$\sqrt{x^2} = \sqrt{125.44}$$

$$x = 11.2$$

Examples: Determine whether \overline{AB} is tangent to the circle. Explain your reasoning.

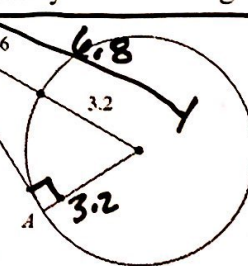


b) $9.9^2 + 12.6^2 \stackrel{?}{=} 16.5^2$

$$256.77 \neq 272.25$$

not RT Δ

$$\text{NOT TANGENT}$$



$$3.2^2 + 3.6^2 \stackrel{?}{=} 6.8^2$$

$$46.24 = 46.24$$

IS RT Δ

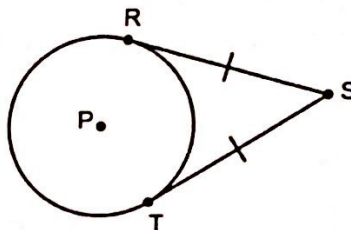
$$\text{TANGENT}$$

Is this a RT angle?
Does the Pythag. Thm work?

HAT THEOREM

Theorem: If two segments from the same point outside a circle are both tangent to the circle, then they are congruent.

If \overline{SR} and \overline{ST} are tangent to $\odot P$ at points R and T , then $\overline{SR} \cong \overline{ST}$.



Examples: \overline{DE} and \overline{DF} are both tangent to $\odot C$. Find the value of x .

a)

$x + 10$ Tangent
17 Tangent

$$x + 10 = 17$$

$$x = 7$$

b)

$6x - 2$ Tangent
4 Tangent

$$6x - 2 = 4$$

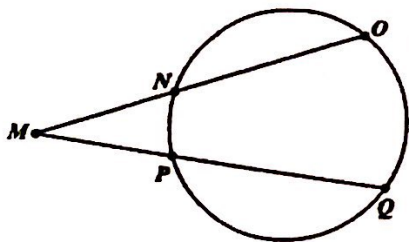
$$6x = 6$$

$$x = 1$$

Theorems About Secants:

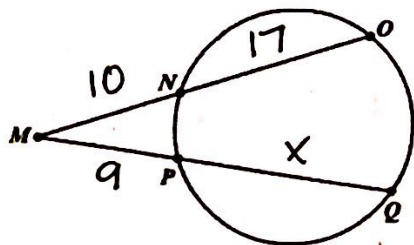
Intersecting Secants Theorem

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



In the circle, \overline{MO} and \overline{MQ} are secants that intersect at point M .
So, $\underline{MN} \cdot \underline{MO} = \underline{MP} \cdot \underline{MQ}$.

Examples:

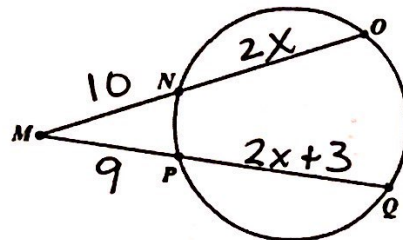


$$10 \cdot 27 = 9(9 + x)$$

$$270 = 81 + 9x$$

$$189 = 9x$$

$$21 = x$$



$$10(2x + 10) = 9(9 + 2x + 3)$$

$$10(2x + 10) = 9(12 + 2x)$$

$$20x + 100 = 108 + 18x$$

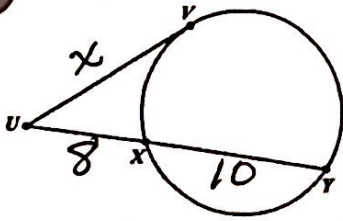
$$2x = 8 \quad x = 4$$

a) In the circle shown, if $MN=10$, $NO=17$, $MP=9$, then find the length of PQ .

b) In the circle shown, if $NO=2x$, $MN=10$, $PQ=2x+3$, $MP=9$, then solve for x .

Intersecting Secant-Tangent Theorem

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



$$x^2 = 8 \cdot 18$$

$$x^2 = 144$$

$$x = 12$$

In the circle, \overline{UV} is a tangent and \overline{UY} is a secant. They intersect at point U .
So $(UV)^2 = UX \cdot UY$.

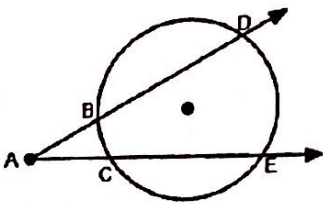
Example:

In the circle shown above, if $UX=8$, and $XY=10$, then find the length of UV .

Theorem

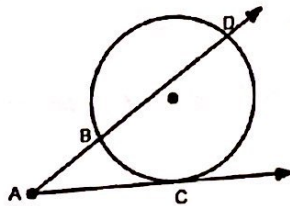
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

Two Secants



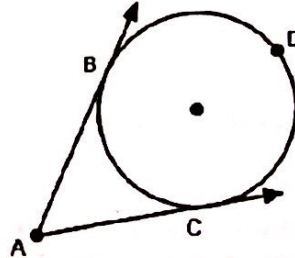
$$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$$

Secant and Tangent



$$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$$

Two Tangents



$$m\angle A = \frac{1}{2}(m\widehat{CDB} - m\widehat{BC})$$

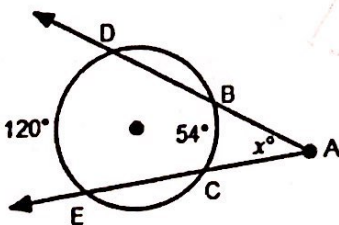
all three Theorems



$$\text{Angle} = \frac{\text{Big Arc} - \text{Little Arc}}{2}$$

Find the Value of x .

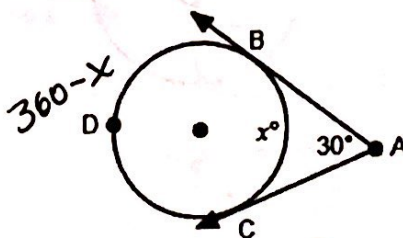
1.



$$x = \frac{120 - 54}{2} = \frac{66}{2} = 33^\circ$$

Why? $\frac{\text{Secant Thm or big arc - little arc}}{2}$

2.



$$2 \cdot 30 = \frac{360 - x - x}{2}$$

$$60 = 360 - x - x$$

$$60 = 360 - 2x$$

$$2x = 300$$

$$x = 150$$

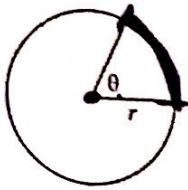
Why? $\frac{\text{Tangent Thm or big arc - little arc}}{2}$

Area of circle = $\pi \cdot r^2$

Circumference of circle = $2\pi r$

SM2H 6.3 Arc Length and Sector Area Notes

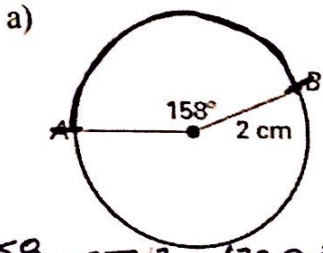
Arc Length: Arc Length = $\frac{\theta}{360^\circ} \cdot \text{circumference of circle} = \frac{\theta}{360^\circ} \cdot 2\pi r$ *circumference*



Arc Length

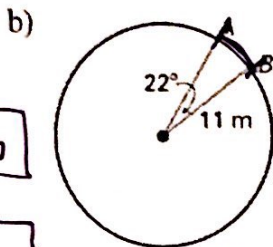
Examples: Find the length of \widehat{AB} . Write your answers in terms of π and as decimals rounded to the nearest hundredth.

2 answers



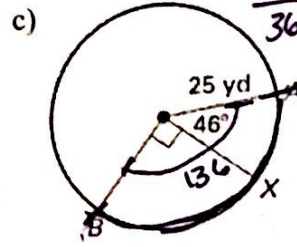
5.52 cm

$$\frac{158}{360} \cdot \frac{2\pi \cdot 2}{1} = \frac{632\pi}{360} = \frac{79\pi}{45} \text{ cm}$$



4.22 m

$$\frac{22}{360} \cdot \frac{2\pi \cdot 11}{1} = \frac{484\pi}{360} = \frac{121\pi}{90} \text{ m}$$

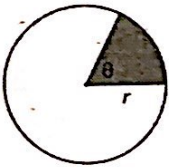


$\frac{170\pi}{9} \text{ yd}$

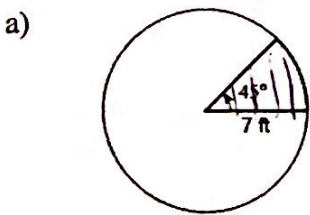
59.34 yd

$$\frac{136}{360} \cdot \frac{2\pi \cdot 25}{1} = \frac{6800\pi}{360}$$

Sector Area: Sector Area = $\frac{\theta}{360^\circ} \cdot \text{area of circle} = \frac{\theta}{360^\circ} \cdot \pi r^2$ *area*



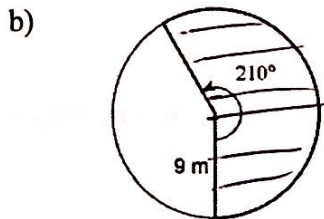
Examples: Find the area of each sector. Write your answers in terms of π and as decimals rounded to the nearest tenth.



Sector Area =

$$\frac{45}{360} \cdot \frac{\pi \cdot 7^2}{1} = \frac{2205\pi}{360}$$

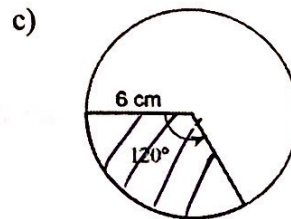
$\frac{49\pi}{8} \text{ ft}^2$
19.2 ft²



Sector Area =

$$\frac{210}{360} \cdot \frac{\pi \cdot 9^2}{1} = \frac{1701\pi}{360}$$

$\frac{189\pi}{4} \text{ m}^2$
148.4 m²



Sector Area =

$$\frac{120}{360} \cdot \frac{\pi \cdot 6^2}{1} = \frac{4320\pi}{360}$$

$12\pi \text{ cm}^2$
or
37.7 cm²