SM2H

Name:______________________________ Period: __________

Unit 1 – SM2H Unit 1 Analyzing Functions Notes

1.1 Functions

Function: When each domain element is paired with a unique range element.

- For every x-value there is only one y-value.
- For every input there is one and only one output.

x-values cannot repeat for the relation to be a function

Domain: The set of all inputs (the *x*-values) of a relation.

- If a relation is represented by a graph or ordered pairs, the domain is the set of all *x*-coordinates of points on the graph.

Range: The set of all outputs (the *y*-values) of a relation.

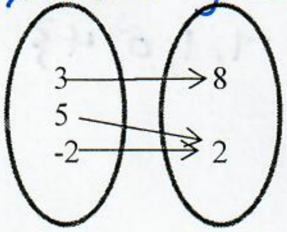
- If a relation is represented by a graph or ordered pairs, the range is the set of all *y*-coordinates of points on the graph.

relation - a relationship between two sets of values.

Examples: Decide whether each relation is a function. Then write the relation as a set of ordered pairs. Then find the domain and range.

1\.

input output
x-values y-values



Function: yes

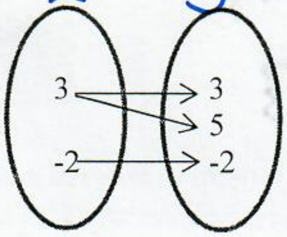
Ordered Pairs: {(3,8),(5,2),(-2,2)}

Domain: {3,5,-2}

Range: {8,2}

2\.

input output
x y



*ordering is not required. Don't duplicate repeated elements when writing the domain and range.

Function: no (because x-value of 3 repeats)

Ordered Pairs: {(3,3),(3,5),(-2,2)}

Domain: {3,-2}

Range: {3,5,-2}

3\.

x	y
-2	5
-1	7
0	9
1	-2
2	-2

Function: yes

Domain: {-2,-1,0,1,2}

range: {5,7,9,-2}

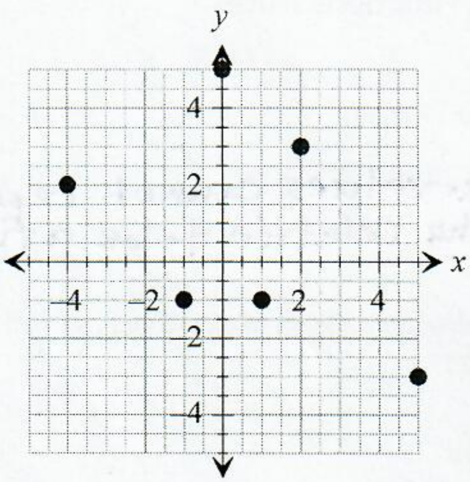
4\. $\{(4,-7),(-1,5),(8,2),(4,5)\}$

Function: no (because x-value of 4 repeats)

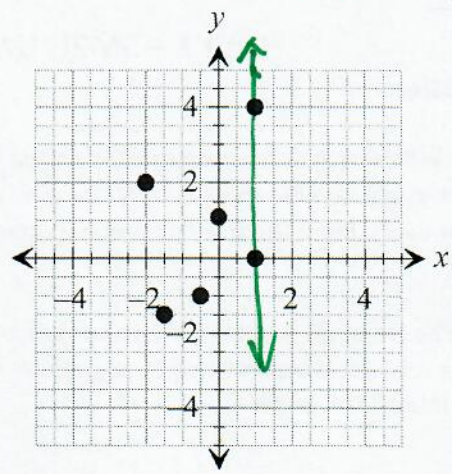
Domain: {4,-1,8}

Range: {-7,5,8}

5.



6. * vertical line test



Function: yes

Ordered Pairs:
{(-4, 2), (-2, -1), (0, 5), (1, -1), (2, 3), (5, -3)}

Domain:
{-4, -2, 0, 1, 2, 5}

Range:
{2, -1, 5, 3, -3}

Function: no

Ordered Pairs: {(-2, 2), (-1.5, -1.5), (-.5, -1), (0, 1), (1, 0), (1, 4)}

Domain: {-2, -1.5, -.5, 0, 1}

Range: {2, -1.5, -1, 1, 0, 4}

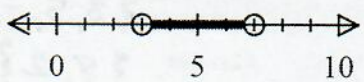
Interval Notation

Domain and range are often written in ***interval notation.***

- The numbers are where the interval starts and stops.
- If an endpoint is *included*, put it in a bracket [or]
- If an endpoint is *not included*, put it in parentheses (or)
- If the interval goes on forever, use –∞ or ∞. These always get put in parentheses (or).

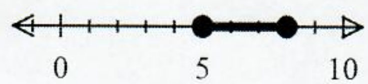
Examples:

- (3, 7) means everything between 3 and 7, not including either 3 or 7.



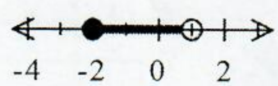
$3 < x < 7$

- [5, 8] means everything between 5 and 8, including both 5 and 8.



$5 \le x \le 8$

- [–2, 1) means everything between –2 and 1, including –2, but not including 1.

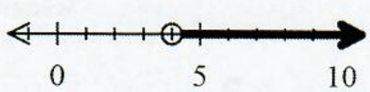


$-2 \le x < 1$

- (–∞, –3] means everything less than or equal to –3.

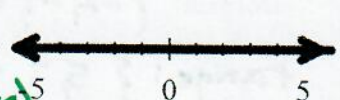
$x \le -3$

- (4, ∞) means everything greater than 4.



$x > 4$

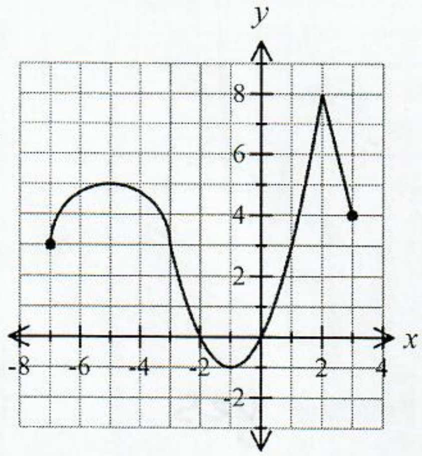
- (–∞, ∞) means all real numbers (goes on forever in both directions).



$-\infty < x < \infty$ ℝ (all real numbers) *use Vertical line test

Examples: Determine whether each graph represents a function. Then state the domain and range given the graph of the function. Write answers in interval notation.

a)

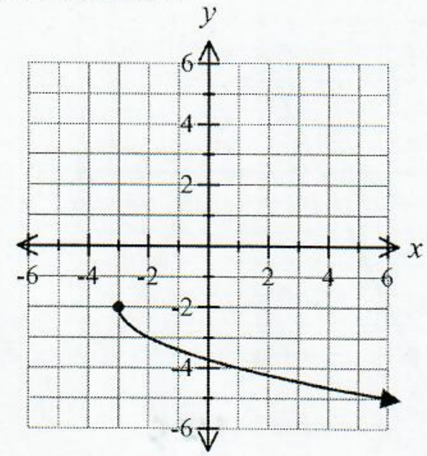


Function? yes

Domain: [-7, 3]

Range: [-1, 8]

b)

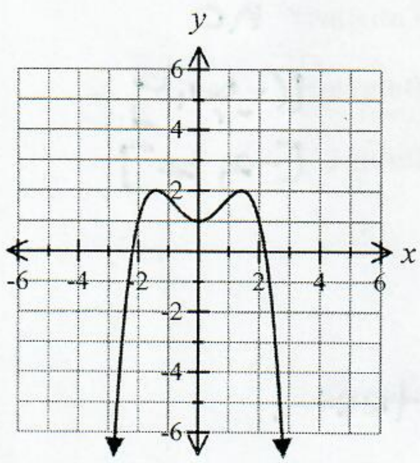


Function? yes

Domain: [-3, ∞)

Range: (-∞, -2]

c)

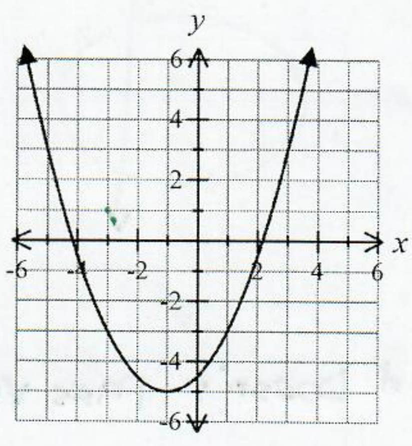


Function? yes

Domain: (-∞, ∞)

Range: (-∞, 2]

d)

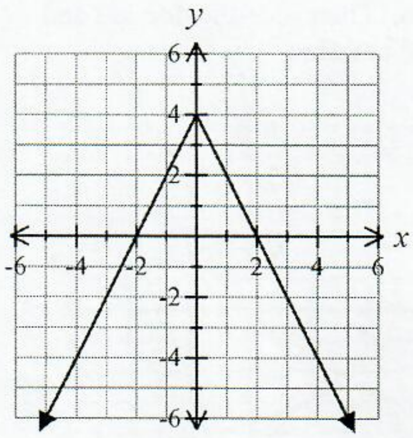


Function? yes

Domain: (-∞, ∞)

Range: [-5, ∞)

e)

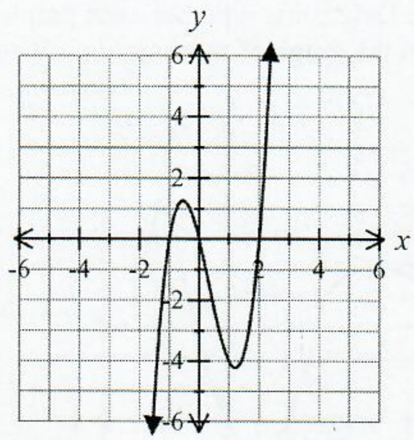


Function? yes

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

f)

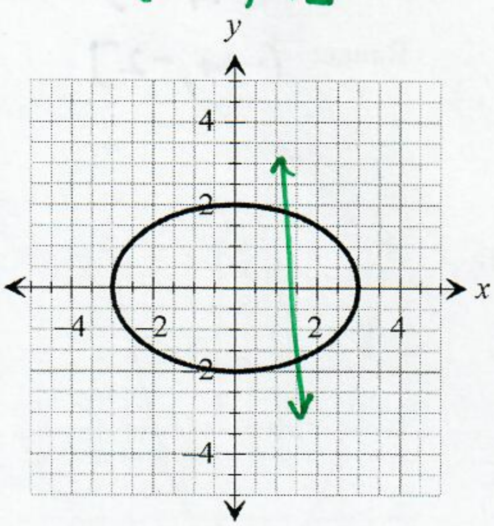


Function? yes

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

g)



Function? no

Domain: $[-3, 3]$

Range: $[-2, 2]$

* Doesn't pass vertical line test.

Evaluating a function or finding a value means substituting the given value for x in the equation. Evaluate the expression. You may use a calculator to evaluate the expression.

Examples: Find each value if $f(x) = x^2 - 2x + 3$, $g(x) = 3x - 5$, and $h(x) = \frac{x}{4-2x}$.

Leave your answers as simplified fractions, if necessary. Show all your work.

a) $f(2) = 2^2 - 2(2) + 3 = 4 - 4 + 3 = \boxed{3}$

b) $g(-1) = (-1)^2 - 2(-1) + 3 = 1 + 2 + 3 = \boxed{6}$

c) $h(4) = \frac{4}{4-2(4)} = \frac{4}{4-8} = \frac{4}{-4} = \boxed{-1}$

d) $g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) - 5 = 2 - 5 = \boxed{-3}$

e) $f(-5) = (-5)^2 - 2(-5) + 3 = 25 + 10 + 3 = \boxed{38}$

f) $h(-3) = \frac{-3}{4-2(-3)} = \frac{-3}{4+6} = \boxed{\frac{-3}{10}}$

Examples: The graph of $y = f(x)$ is shown below. Use it to answer the following questions.

a) Find $f(-4)$. 1

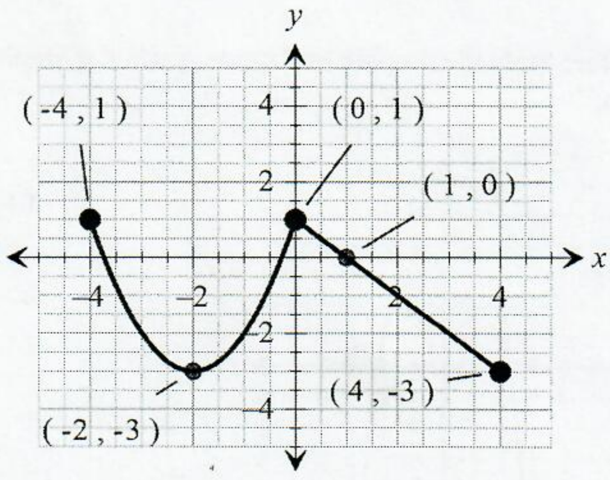
b) Find $f(0)$. 1

c) For what values of x is $f(x) = 0$? (y)
$-3.75, 1, -.25$

d) For what values of x is $f(x) = -3$?
$-2, 4$

e) What is the domain?
$[-4, 4]$

f) What is the range?
$[-3, 1]$



1.2 Analyzing Functions Notes

Domain: The set of all inputs (the x-values) of a relation.

- If a relation is represented by a graph, the domain is the set of all x-coordinates of points on the graph.
- When modeling a real-life situation, the domain is the set of x's that make sense in the problem.
- When given an equation, the domain is the set of x-values for which the equation is defined.

Range: The set of all outputs (the y-values) of a relation.

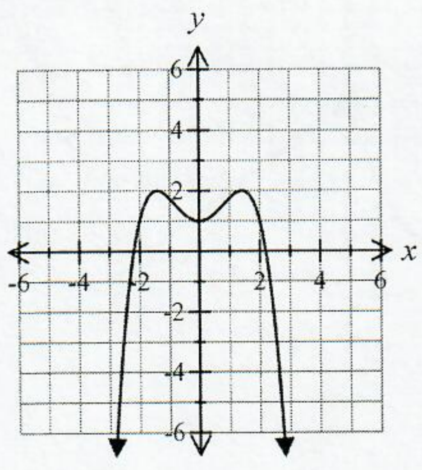
- If a relation is represented by a graph, the range is the set of all y-coordinates of points on the graph.
- When modeling a real-life situation, the range is the set of y's that make sense in the problem.

Interval Notation Tips

- Use parentheses (or) around numbers that *are not included* in the domain or range.
    - This happens when there is an ***open circle*** at a point or an ***asymptote*** (a line that the graph gets really close to, but never actually touches).
- Use brackets [or] around endpoints that *are included* in the domain or range.
    - It there is a point on the graph with the given x- or y-coordinate, use a bracket.
- Always use parentheses around $-\infty$ and ∞.
- ***Read the domain from left to right and the range from down to up.***
    - Write the lower value or $-\infty$ first and the higher value or ∞ last.

Reminder: State the domain and range given the graph of the function. Write answers in interval notation.

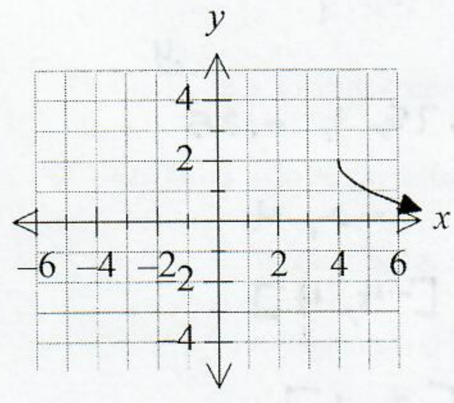
a)



Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

b)



Domain: $[4, \infty)$

Range: $(-\infty, 2]$

***Domain* - given the equation of a function.** When given an equation, think about what x-values will "work" as input into the equation. Are there any values of x for which the equation is not defined?

* Square root functions will have limited domains because under the square root cannot be negative.

Examples: Find the domain.

1. $f(x)=2x+6$

$(-\infty,\infty)$

2. $p(x)=2x^2-3x+6$

$(-\infty,\infty)$

3. $g(x)=\sqrt{x-5}$

$[5,\infty)$

$x-5\geq 0$
$+5\quad +5$
$x\geq 5$

4. $h(x)=|x|-2$

$(-\infty,\infty)$

5. $f(x)=7\sqrt{-x+2}-10$

$-x+2\geq 0$
$-2\quad -2$
$\frac{-x}{-1}\geq\frac{-2}{-1}$
$x\leq 2$

$(-\infty,2]$

Domain and Range in Real Life Situations: In real life situations, it's important to think through what values make sense in the problem. You also need to think about what *x* stands for and what *y* stands for.

Examples: Find the real world domain and range for each situation.

1. You are getting ready for the Homecoming dance. Your dad is going to let you borrow his new car, but you need to wash and fill it. The car wash costs \$5 and the gas costs \$3.89 per gallon. The car can hold 15 gallons of gas. What are the domain and range if the total cost is a function of the number of gallons and the car is completely empty when you pull into the gas station?

x: input - number of gallons of gas (domain)
y: output - total cost (range)

Write a function for the given situation: $f(x)=5+3.89x$
Is the domain the number of gallons of gas or total cost? number of gallons
Is the range the number of gallons of gas or total cost? total cost
What is the real world domain? $[0,15]$
What is the real world range? $f(0)=5+3.89(0)=5$ $\quad f(15)=5+3.89(15)=63.35$ $\quad$ [\$5, \$63.35]

2. Your cell phone plan charges a flat fee of \$10 for up to 1000 texts and \$0.10 per text over 1000.

Write a function for the given situation: $f(x)=10+.10(x-1000)$
Is the domain the number of texts or total cost? number of texts (over 1000)
Is the range the number of texts or total cost? cost
What is the real world domain? $[1000,\infty)$
What is the real world range? $[10,\infty)$

End Behavior

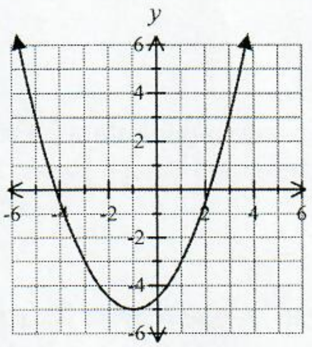
What happens to the y values of the graph at the tail ends of the graph. End behavior describes what is happening to the *y*-values of a graph when *x* goes to the far right (∞) or when *x* goes to the far left ($-\infty$).

Right End Behavior: $\lim_{x\to\infty} f(x) = c$ **Left End Behavior:** $\lim_{x\to-\infty} f(x) = c$

- If *y* is getting larger and larger (the graph is pointing up) with no upper limit, then the limit would be equal to ∞.
- If *y* is getting smaller and smaller (the graph is pointing down) with no lower limit, then the limit would be equal to $-\infty$.
- If the graph has an asymptote (gets closer and closer to a number, but never touches it), then the limit is equal to wherever the asymptote is.
- If there's an endpoint, the limit does not exist.

Examples: Describe the end behavior of each graph using limits. **If a limit does not exist, write DNE.**

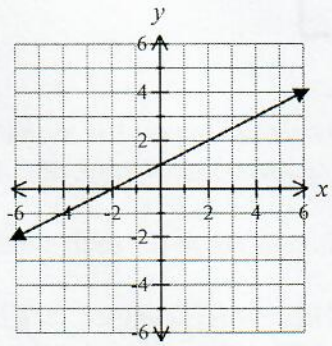
a)



Left: $\lim_{x\to-\infty} f(x) =$ ∞

Right: $\lim_{x\to\infty} f(x) =$ ∞

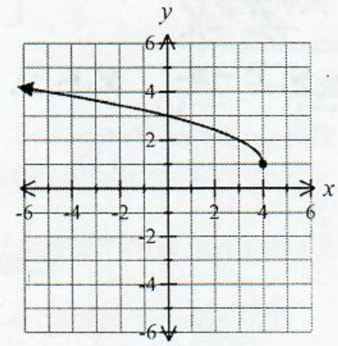
b)



Left: $\lim_{x\to-\infty} f(x) =$ $-\infty$

Right: $\lim_{x\to\infty} f(x) =$ ∞

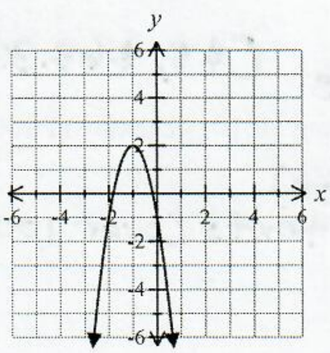
c)



Left: $\lim_{x\to-\infty} f(x) =$ ∞

Right: $\lim_{x\to\infty} f(x) =$ DNE

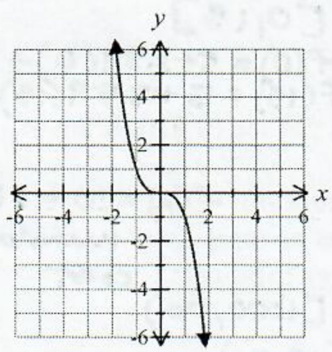
d)



Left: $\lim_{x\to-\infty} f(x) =$ $-\infty$

Right: $\lim_{x\to\infty} f(x) =$ $-\infty$

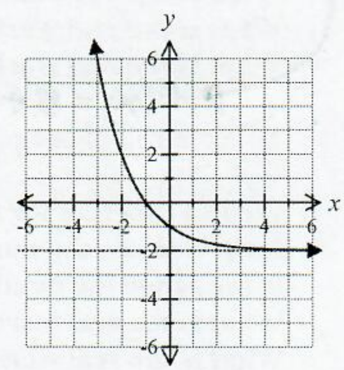
e)



Left: $\lim_{x\to-\infty} f(x) =$ ∞

Right: $\lim_{x\to\infty} f(x) =$ $-\infty$

f)



Left: $\lim_{x\to-\infty} f(x) =$ ∞

Right: $\lim_{x\to\infty} f(x) =$ -2

Find the end behavior graphically. Write answers as a limits.

a) $f(x)=|x-2|+3$

$\lim_{x\to-\infty} f(x)=\infty$

$\lim_{x\to\infty} f(x)=\infty$

b) $f(x)=-3x^3$

$\lim_{x\to-\infty} f(x)=\infty$

$\lim_{x\to\infty} f(x)=-\infty$

c) $f(x)=-2x^2+x$

$\lim_{x\to-\infty} f(x)=-\infty$

$\lim_{x\to\infty} f(x)=-\infty$

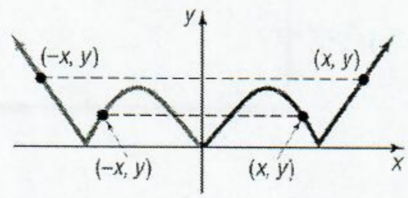
d) $f(x)=\sqrt{-x}+2$

$\lim_{x\to-\infty} f(x)=\infty$

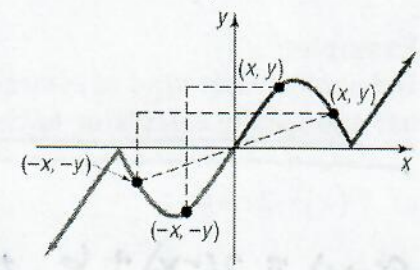
$\lim_{x\to\infty} f(x)=\text{DNE}$

Even or Odd Symmetry

- A function has ***even symmetry*** or ***y-axis symmetry*** if, for every point (x,y) on the graph, the point $(-x,y)$ is also on the graph. If you folded the graph along the *y*-axis, the two sides would overlap.

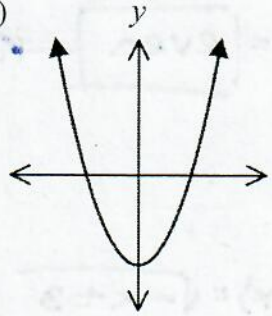


- A function has ***odd symmetry*** or ***origin symmetry*** if, for every point (x,y) on the graph, the point $(-x,-y)$ is also on the graph. If you rotated the graph 180°, it would end up in the same place it started.



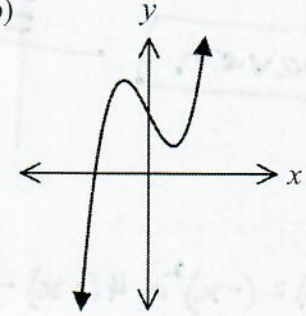
Examples: Determine what type of symmetry each function has, if any.

a)



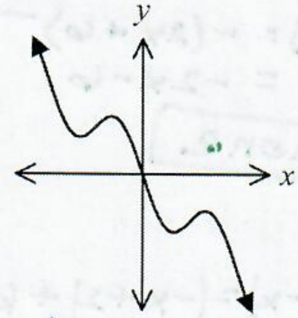
even

b)



none

c)



odd

Determine the Symmetry of a Function Algebraically

- A function is **symmetric with respect to the *y*-axis** or is an **even function**, if $f(-x) = f(x)$. In other words, if you substitute $-x$ in for every x, you end up with the original function. When looking at the graph, you could "fold" the graph along the *y-axis* and both sides are the same.
- A function is **symmetric with respect to the origin** or is an **odd function**, if $f(-x) = -f(x)$. In other words, if you substitute $-x$ in for every x, you end up with the opposite of the original function. When looking at the graph, there is a mirror image in Quadrants 1 & 3 or in Quadrants 2 & 4. The graph has rotational symmetry about the origin.
- A function has **no symmetry** if it is neither **even** nor **odd**. If you substitute $-x$ in for every x and you end up with something that is not the original function and is not its opposite, then there is no symmetry for this function.

SYMMETRY

Even or Odd?

To find algebraically, find *f(-x)*. Substitute *–x* for all *x*'s and simplify.

- *If f(-x) = f(x)* then it is **even**.
- If *f(-x) = -f(x)* then it is **odd**.

Examples:

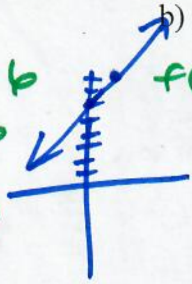
Determine what kind of symmetry (algebraically), if any, each function has. Show work. Then use a graphing calculator to check your answer, draw a sketch of the graph.

a) $f(x) = 2x + 6$

$f(-x) = 2(-x) + 6$
$= -2x + 6$

$-f(x) = -(2x + 6)$
$= -2x - 6$

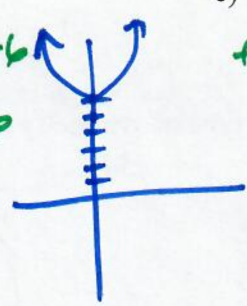
$\boxed{\text{none}}$



b) $f(x) = x^2 + 6$

$f(-x) = (-x)^2 + 6$
$= x^2 + 6$

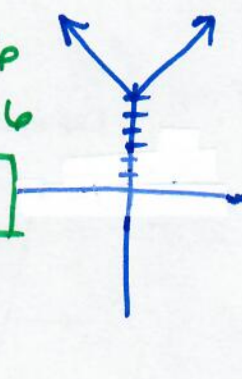
$\boxed{\text{even}}$



c) $f(x) = |x| + 6$

$f(-x) = |-x| + 6$
$= |x| + 6$

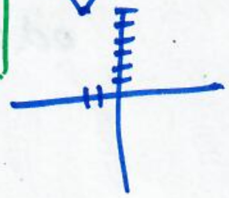
$\boxed{\text{even}}$



d) $f(x) = |x + 2| + 6$

$f(-x) = |-x + 2| + 6$

$\boxed{\text{none}}$

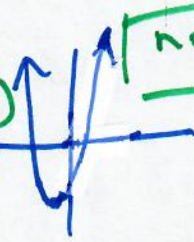


e) $f(x) = x^2 + 4x - 8$

$f(x) = (-x)^2 + 4(-x) - 8$
$= x^2 - 4x - 8$

$-f(x) = -(x^2 + 4x - 8)$
$= -x^2 - 4x + 8$

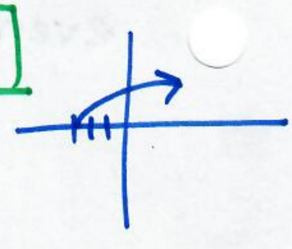
$\boxed{\text{none}}$



f) $f(x) = \sqrt{x + 3}$

$f(-x) = \sqrt{-x + 3}$

$\boxed{\text{none}}$



1.3 Analyzing Functions #2

Intercepts

x-Intercepts: The points where a graph crosses the x-axis. They have the form $(x, 0)$.

- **To find the x-intercept(s), set $y = 0$ and solve for x.**

y-Intercepts: The points where a graph crosses the y-axis. They have the form $(0, y)$.

- **To find the y-intercept(s), set $x = 0$ and solve for y.**

Examples: Find the intercepts using algebra. Write the intercepts as ordered pairs. Show all your work!

a) $f(x) = 2x + 6$

x-int: $0 = 2x + 6$, $-6 \quad -6$, $\frac{-6}{2} = \frac{2x}{2}$, $-3 = x$

y-int: $y = 2(0) + 6$, $y = 6$

x-intercept $(-3, 0)$

y-intercept $(0, 6)$

b) $f(x) = -3x + 2$

x-int: $0 = -3x + 2$, $-2 \quad -2$, $\frac{-2}{-3} = \frac{-3x}{-3}$, $\frac{2}{3} = x$

y-int: $y = -3(0) + 2$, $y = 2$

x-intercept $(\frac{2}{3}, 0)$

y-intercept $(0, 2)$

c) $3x + 2y = 12$

x-int: $3x + 2(0) = 12$, $\frac{3x}{3} = \frac{12}{3}$, $x = 4$

y-int: $3(0) + 2y = 12$, $\frac{2y}{2} = \frac{12}{2}$, $y = 6$

x-intercept $(4, 0)$

y-intercept $(0, 6)$

d) $x - 2y = 5$

x-int: $x - 2(0) = 5$, $x = 5$

y-int: $0 - 2y = 5$, $\frac{-2y}{-2} = \frac{5}{-2}$, $y = -5/2$

x-intercept $(5, 0)$

y-intercept $(0, -5/2)$

Relative Maxima and Minima: Where a graph changes from increasing to decreasing (or vice versa). When a point is higher that all the points near it, it is called a ***relative maximum.*** When a point is lower than all the points near it, it is called a ***relative minimum.*** **A function CAN have more than one relative maximum or relative minimum.**

- If you are asked for a ***maximum point or a minimum point,*** write the answer as an ordered pair.
- If you are asked for a ***maximum value or a minimum value,*** the answer is the ***y-coordinate.***

Absolute Maximum and Minimum
The largest (absolute maximum) or smallest (absolute minimum) value that a mathematical function can have over its entire curve.

★ **Infinity is NOT a maximum or a minimum.**

Example:
a) Find the relative maximum point and the relative maximum value.

point: $(-5, 5)$
value: 5

b) Find the relative minimum points and the relative minimum values.

points: $(-1, -1)$ and $(-7, 3)$
values: -1 and 3

c) Find the absolute maximum point and the absolute maximum value.

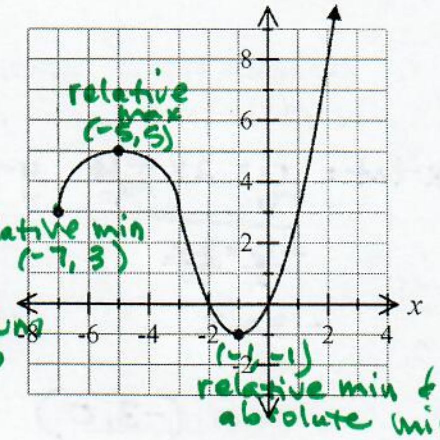
point: none
value: none

There is no absolute maximum because the graph continues to increase as $x \to \infty$

d) Find the absolute minimum points and the absolute minimum values.

point: $(-1, -1)$
value: -1

no absolute maximum



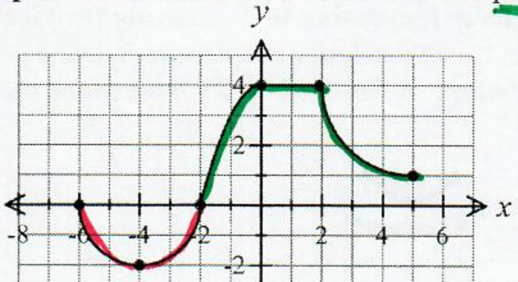
Positive and Negative

- A function is ***positive*** where the y-coordinates are positive. The graph is ***above the x-axis.***
- A function is ***negative*** where the y-coordinates are negative. The graph is ***below the x-axis.***

★ When you are asked to state where the graph is positive and negative, write the intervals of the **x-coordinates** from **left to right.**
★ Use (or) at the x-intercepts, where the graph crosses over from positive to negative. The y-coordinate is zero at the intercepts, so the graph is neither positive nor negative there. That means those points are not included in the interval.
★ Use [or] if the graph has an *endpoint* somewhere above or below the x-axis.

Example: Determine where the function is positive and where it is negative.

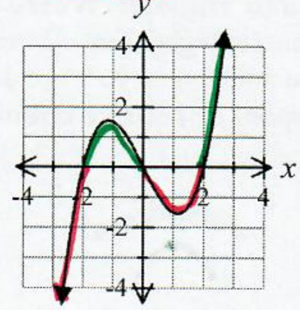
a)



Positive: $(-2, 5]$ ← use a bracket when the graph ends at a point with a positive or negative y-coordinate.

Negative: $(-6, -2)$

b)



Positive: $(-2, 0) \cup (2, \infty)$

Negative: $(-\infty, -2) \cup (0, 2)$

Increasing, Decreasing, and Constant

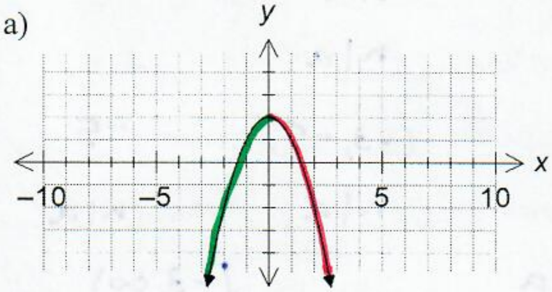
If you look from left to right along the graph of the function, you will notice parts are *rising*, parts are *falling* and parts are *horizontal*. In such cases, the function is described as *increasing*, *decreasing*, or *constant*, respectively.

Increasing **Decreasing** **Constant**

- ★ When you are asked to state where the graph is increasing, decreasing, and constant, write the intervals of ***x*-coordinates** from **left to right.**
- ★ Always use ( ) for increasing, decreasing, and constant. Never use [ ].

Example: Determine where each graph is increasing, decreasing, and constant.

a)

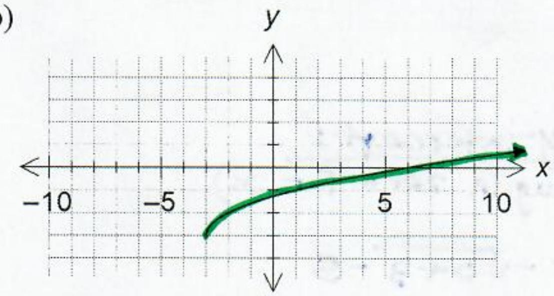


Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Constant: n/a

b)

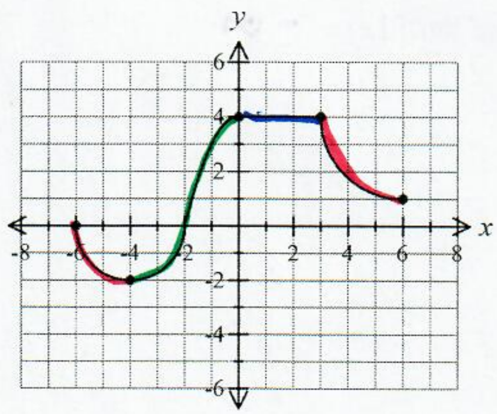


Increasing: $(-3, \infty)$

Decreasing: n/a

Constant: n/a

c)

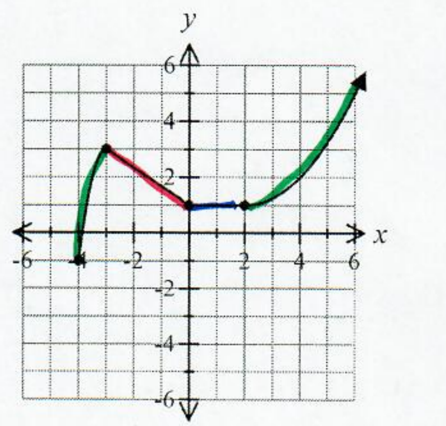


Increasing: $(-4, 0)$

Decreasing: $(-6, -4) \cup (3, 6)$

Constant: $(0, 3)$

d)



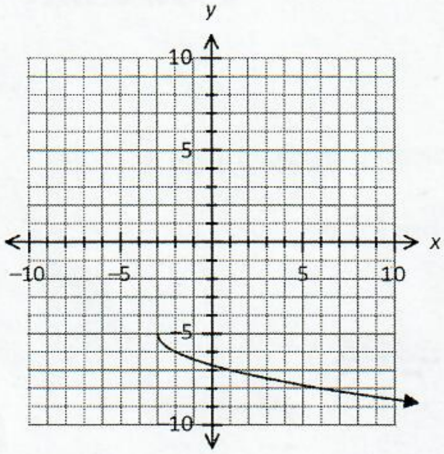
Increasing: $(-4, -3) \cup (2, \infty)$

Decreasing: $(-3, 0)$

Constant: $(0, 2)$

Fill in all requested information for each function. Write domain and range in interval notation. Write the intercepts as ordered pairs. Write the intervals in interval notation where the graph is positive and negative. Write the end behaviors in limit notation. If a limit does not exist, write DNE. If something is not applicable to the graph, write N/A.

a) $f(x) = -\sqrt{x+3} - 5$



Domain: $[-3, \infty)$ Range: $(-\infty, -5]$

x-intercept(s): n/a y-intercept: $(0, -6.73)$

Relative Maximum Point: $(-3, -5)$

Relative Maximum Value: -5

Relative Minimum Point: n/a

Relative Minimum Value: n/a

Absolute Maximum Point: $(-3, -5)$ Value: -5

Absolute Minimum Point: n/a Value: n/a

Positive: n/a Negative: $[-3, \infty)$

Incresing: n/a Decreasing: $(-3, \infty)$

Constant: n/a

Left End Behavior: $\lim_{x \to -\infty} f(x) =$ DNE

Right End Behavior: $\lim_{x \to \infty} f(x) = -\infty$

y-intercept:
(plug in zero for x)

$y = -\sqrt{0+3} - 5$
$= -\sqrt{3} - 5$
$\approx -1.73 - 5$
≈ -6.73

1.4 Transformations -Square Root Functions Notes

Types of transformations

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct "Order of Transformations."

1. **Reflection**:
   a. Vertical Reflection – graph is reflected over the x-axis
   b. Horizontal Reflection – graph is reflected over the y-axis
2. **Stretch/Compression (Shrink)**:
   a. Vertical Stretch – the y-coordinates are multiplied by a scalar that is greater than 1
   b. Vertical Compression/Shrink – the y-coordinates are multiplied by a scalar that is between 0 and 1
   c. Horizontal Stretch/Compression – the x-coordinates are multiplied by a scalar (reciprocal of b)
3. **Translation (or Shift)**:
   a. Horizontal Translation – graph is shifted to the left or right
   b. Vertical Translation – graph is shifted up or down

We will begin by looking at how transformations can be applied to the graph of a Square Root Function.

B. Parent Graph Analysis: Square Root Function

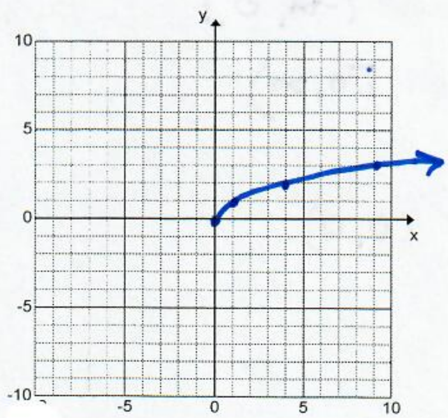
Fill in the table to find some **key points** for the parent functions. Use the table to generate ordered pairs for points on the graph, then sketch the graph.

$f(x) = \sqrt{x}$

What is the **endpoint** of the graph of the function? (0, 0)

Fill in the following information about the parent function.

x	$f(x)$
0	0
1	1
4	2
9	3



Domain: [0, ∞)   Range: [0, ∞)

x-intercept(s): (0, 0)   y-intercept: (0, 0)

Which does the function have: a maximum or a minimum? minimum

Max/Min Point: (0, 0)   Max/Min Value: 0

Positive: (0, ∞)   Negative: n/a

Increasing: (0, ∞)   Decreasing: n/a

Constant: n/a   Symmetry: none

Left End Behavior: $\lim_{x \to -\infty} f(x) =$ DNE

Right End Behavior: $\lim_{x \to \infty} f(x) =$ ∞

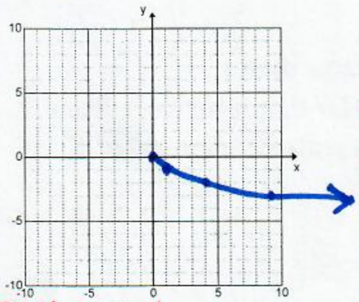
* Think about if the transformation is affecting the x-values or the y-values.

C. **Applying Transformations to the Function:**

Directions: For each of the following, create a table to show how the transformation changes the key points from the parent graph.

Vertical reflection: Use a table to create the following graph:

1. $f(x) = -\sqrt{x}$



The negative on the outside of the $\sqrt{\ }$ results in a vertical change to the graph.

Explain the difference between this graph and the graph of $f(x) = \sqrt{x}$.

The graph is reflected over the x-axis

What is the domain? $[0, \infty)$

What is the range? $(-\infty, 0]$

parent graph

$\cdot -1$

x	$y=\sqrt{x}$	$y=-\sqrt{x}$
0	0	0
1	1	-1
4	2	-2
9	3	-3

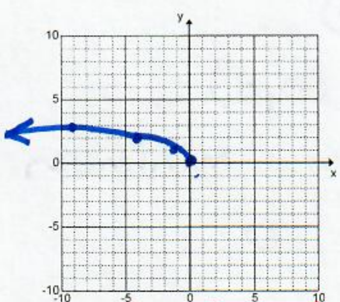
* y-coordinates are multiplied by -1.

* critical point : (0, 0)

It is important to know the location of the endpoint of a square root graph.

Horizontal reflection: Use a table to create the following graph:

2. $f(x) = \sqrt{-x}$



The negative on the inside of the square root results in a horizontal change to the graph

Explain the difference between this graph and the graph of $f(x) = \sqrt{x}$.

The graph is reflected over the y-axis.

What is the domain? $(-\infty, 0]$

What is the range? $[0, \infty)$

parent graph

$\cdot -1$	x	$y=\sqrt{x}$
0	0	0
-1	1	1
-4	4	2
-9	9	3

Endpoint: (0, 0)

The x-values get multiplied by -1

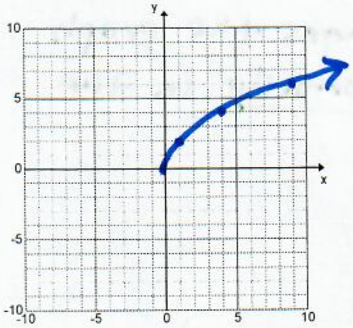
NOTE: Sometimes it is hard to see the effects of a horizontal reflection on a function. This is especially true if the graph is symmetric with respect to the y-axis.

* Think about if the transformation is affecting the x-values or the y-values.

Vertical stretch/compression (shrink): Create a table for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function.

↙ a number multiplied on the outside of the √ results in a vertical change to the graph.

3. $f(x)=2\sqrt{x}$



x	$y=\sqrt{x}$	$\cdot 2$
0	0	0
1	1	2
4	2	4
9	3	6

↑ multiply y-coordinates by 2.

Domain: $[0,\infty)$

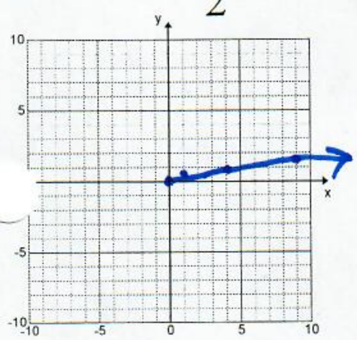
Range: $[0,\infty)$

Explain the difference between this graph and the graph of $f(x)=\sqrt{x}$.

This graph has been stretched vertically by a factor of 2.

Endpoint: (0,0)

4. $f(x)=\frac{1}{2}\sqrt{x}$



x	$y=\sqrt{x}$	$\cdot 1/2$
0	0	0
1	1	1/2
4	2	1
9	3	3/2

↑ multiply y-coordinates by 1/2.

Domain: $[0,\infty)$

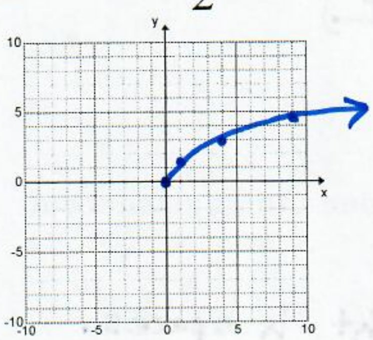
Range: $[0,\infty)$

Explain the difference between this graph and the graph of $f(x)=\sqrt{x}$.

This graph has been compressed vertically or there was a vertical shrink by a factor of 1/2.

Endpoint: (0,0)

5. $f(x)=\frac{3}{2}\sqrt{x}$



x	$y=\sqrt{x}$	$\cdot 3/2$
0	0	0
1	1	3/2
4	2	3
9	3	9/2

↑ multiply y-coordinates by 3/2.

Domain: $[0,\infty)$

Range: $[0,\infty)$

Explain the difference between this graph and the graph of $f(x)=\sqrt{x}$.

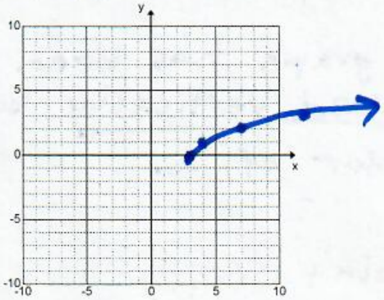
This graph has been stretched vertically by a factor of 3/2.

Endpoint: (0,0)

* Think about if the transformation is affecting the x-values or the y-values.

Horizontal Translation: Create a table for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function as well as the location of the endpoint.

6. $f(x)=\sqrt{x-3}$



* A number added or subtracted inside the $\sqrt{\ }$ changes the graph by shifting it horizontally in the opposite direction.

parent function

+3	x	$y=\sqrt{x}$
3	0	0
4	1	1
7	4	2
12	9	3

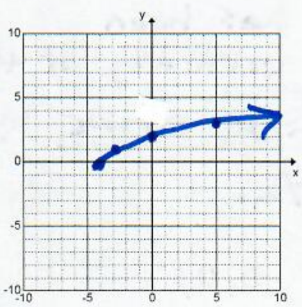
add 3 to the x-values (shift 3 spaces right.)

Domain: $[3,\infty)$

Range: $[0,\infty)$

Endpoint: $(3,0)$

7. $g(x)=\sqrt{x+4}$



parent function

-4	x	$y=\sqrt{x}$
-4	0	0
-3	1	1
0	4	2
5	9	3

subtract 4 from the x-values (shift 4 spaces left)

Domain: $[-4,\infty)$

Range: $[0,\infty)$

Endpoint: $(-4,0)$

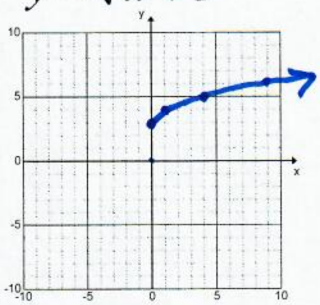
Compare the two graphs above with the graph of $f(x)=\sqrt{x}$. What conclusion can we make about functions that come in the form: $y=\sqrt{x-h}$?

For $y=\sqrt{x-h}$, shift the graph to the right h spaces.
For $y=\sqrt{x+h}$, shift the graph to the left h spaces.

*Think about if the transformation is affecting the x-values or the y-values.

Vertical Translation: Create a table for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function, as well as the location of the endpoint of the graph.

8. $y=\sqrt{x}+3$



x	$y=\sqrt{x}$	+3
0	0	3
1	1	4
4	2	5
9	3	6

add 3 to y-coordinates (shift up 3 spaces)

Explain the difference between this graph and the graph of $y=\sqrt{x}$.

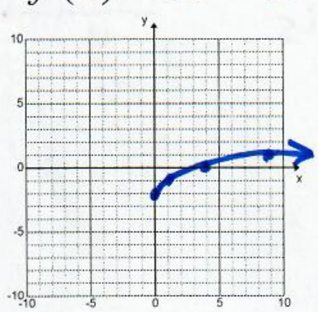
This graph was shifted vertically, up 3 units.

Domain: $[0,\infty)$

Range: $[3,\infty)$

Endpoint: $(0,3)$

9. $f(x)=\sqrt{x}-2$



x	$y=\sqrt{x}$	-2
0	0	-2
1	1	-1
4	2	0
9	3	1

Subtract 2 from the y-coordinates (shift down 2 spaces)

Domain: $[0,\infty)$

Range: $[-2,\infty)$

Endpoint: $(0,-2)$

Compare the two graphs above with the graph of $f(x)=\sqrt{x}$. What conjecture can we make about functions that come in the form: $y=\sqrt{x}+k$?

For $y=\sqrt{x}+k$, the graph is shifted up k spaces.

For $y=\sqrt{x}-k$, the graph is shifted down k spaces.

Putting It All Together:

Using the equation $y=a\sqrt{b(x-h)}+k$**, find a, b, h, and k.**

10. $y=2\sqrt{-(x+3)}-1$

a = 2

b = -1

h = -3

k = -1

11. $y=-\sqrt{\frac{1}{2}(x-4)}+3$

a = -1

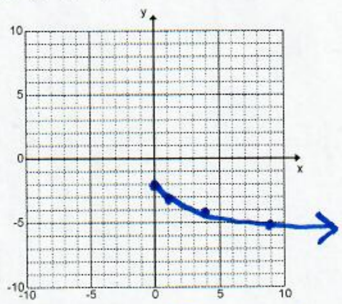
b = 1/2

h = 4

k = 3

For each of the following 1) List the transformations in the correct order. 2) Create a table to show the transformations on the key points. 3) State the domain and range. 4) State the endpoint.

12. $f(x)=-\sqrt{x}-2$



Domain: $[0,\infty)$

Range: $(-\infty,-2]$

Endpoint: $(0,-2)$

Transformations

1) reflect over the x-axis

2) shift down 2 units

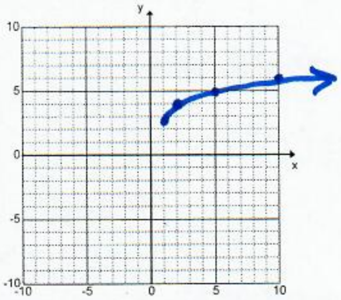
affects?

y

y

x	$y=\sqrt{x}$	$\cdot -1$	-2
0	0	0	-2
1	1	-1	-3
4	2	-2	-4
9	3	-3	-5

13. $f(x)=\sqrt{x-1}+3$



Domain: $[1, \infty)$

Range: $[3, \infty)$

Endpoint: $(1, 3)$

Transformations

shift right 1

shift up 3

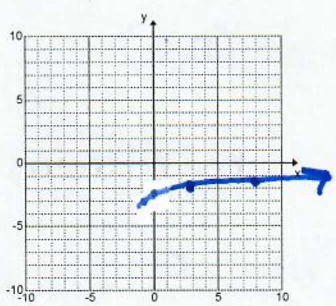
affects?

x

y

+1	x	$y=\sqrt{x}$	+3
1	0	0	3
2	1	1	4
5	4	2	5
10	9	3	6

14. $f(x)=\frac{1}{2}\sqrt{x+1}-3$



Domain: $[-1, \infty)$

Range: $[-3, \infty)$

Endpoint: $(-1, -3)$

Transformations

vertical shrink of $\frac{1}{2}$

shift left 1

shift down 3

affects?

y

x

y

-1	x	$y=\sqrt{x}$	$\cdot\frac{1}{2}$	-3
-1	0	0	0	-3
0	1	1	$\frac{1}{2}$	$-2\frac{1}{2}$
3	4	2	1	-2
8	9	3	$\frac{3}{2}$	$-1\frac{1}{2}$

1.5 Transformations –Absolute Value Functions and Quadratic Functions Notes

A. Types of transformations

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct "Order of Transformations."

1. **Reflection**:
   a. Vertical Reflection – graph is reflected over the x-axis
   b. Horizontal Reflection – graph is reflected over the y-axis
2. **Stretch/Compression (Shrink)**:
   a. Vertical Stretch – the y-coordinates are multiplied by a scalar that is greater than 1
   b. Vertical Compression/Shrink – the y-coordinates are multiplied by a scalar that is between 0 and 1
   c. Horizontal Stretch/Compression – the x-coordinates are multiplied by a scalar (reciprocal of b)
3. **Translation (or Shift)**:
   a. Horizontal Translation – graph is shifted to the left or right
   b. Vertical Translation – graph is shifted up or down

We will look at how transformations can be applied to the graphs of the Absolute Value Function and the Quadratic Function.

B. Parent Graph Analysis: Absolute Value Function

Fill in the given table to find some **key points** for the parent function. Use the table to generate ordered pairs for points on the graph and then sketch the graph.

$f(x)=|x|$

Where is the **vertex** of the graph of the function?

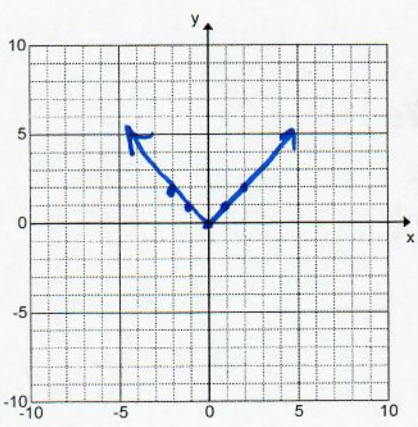
Fill in the following information about the parent function.

x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

x-intercept(s): $(0,0)$ y-intercept: $(0,0)$

Which does the function have: a maximum or a minimum? minimum



Max/Min Point: $(0,0)$ Max/Min Value: 0

Positive: $(-\infty,0)\cup(0,\infty)$ Negative: n/a

Increasing: $(0,\infty)$ Decreasing: $(-\infty, 0)$

Constant: n/a Symmetry: even

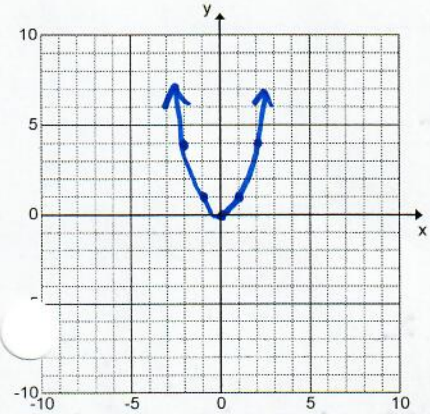
Left End Behavior: $\lim_{x\to-\infty} f(x) = \infty$ Right End Behavior: $\lim_{x\to\infty} f(x) = \infty$

C. Parent Graph Analysis: Quadratic Function (Parabola)

Fill in the table to find some **key points** for the parent function. Use the table to generate ordered pairs for points on the graph, then sketch the graph.

$f(x) = x^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



What is the **vertex** of the graph of the function? (0,0)

Fill in the following information about the parent function.

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

x-intercept(s): (0,0) y-intercept: (0,0)

Which does the function have: a maximum or a minimum? minimum

Max/Min Point: (0,0) Max/Min Value: 0

Positive: $(-\infty,0)\cup(0,\infty)$ Negative: n/a

Increasing: $(0,\infty)$ Decreasing: $(-\infty,0)$

Constant: n/a Symmetry: even

Left End Behavior: $\lim_{x\to-\infty} f(x) =$ ∞

Right End Behavior: $\lim_{x\to\infty} f(x) =$ ∞

Transformations of the parent graph:

	$f(x)=\|x\|$	$f(x)=x^2$	Effect on Parent Graph
$y=-f(x)$	$y=-\|x\|$	$y=-x^2$	reflect over the x-axis
$y=2f(x)$	$y=2\|x\|$	$y=2x^2$	vertical stretch by a factor of 2
$y=\frac{1}{2}f(x)$	$y=\frac{1}{2}\|x\|$	$y=\frac{1}{2}x^2$	vertical compression/shrink by a factor of 1/2
$y=f(x)+2$	$y=\|x\|+2$	$y=x^2+2$	shift up 2
$y=f(x)-2$	$y=\|x\|-2$	$y=x^2-2$	shift down 2
$y=f(x+2)$	$y=\|x+2\|$	$y=(x+2)^2$	shift left 2
$y=f(x-2)$	$y=\|x-2\|$	$y=(x-2)^2$	shift right 2
$y=f(-x)$	$y=\|-x\|$	$y=(-x)^2$	reflect over the y-axis
$y=f(2x)$	$y=\|2x\|$	$y=(2x)^2$	horizontal shrink by a factor of $\frac{1}{2}$

Examples: Describe the transformations of the parent graph needed to graph the following functions. Parent graph: $f(x)=|x|$. List the transformations in the order in which they should be applied.

a) $y=|x+5|-3$

shift left 5
shift down 3

b) $y=-\frac{1}{2}|x|+1$

reflect over x-axis
vertical shrink of $\frac{1}{2}$
shift up 1

c) $y=3|x-2|+4$

vertical stretch of 3
shift right 2
shift up 4

Parent graph: $f(x)=x^2$

a) $y=-x^2-3$

reflect over x-axis
shift down 3

b) $y=-4(x-3)^2$

reflect over x-axis
vertical stretch of 4
shift right 3

c) $y=\frac{1}{5}(x+2)^2-3$

vertical shrink of 1/5
shift left 2
shift down 3

To draw the graph of $y = a \cdot f\big(b(x-h)\big) + k$ using transformations.

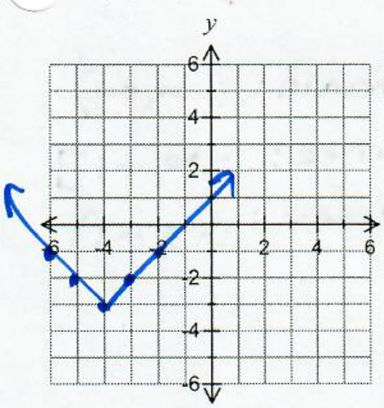
1. Draw the parent graph using the key points.
   - For $y = |x|$ or $y = x^2$, use (–2, 2), (–1, 1), (0, 0), (1, 1), (2, 2)
   - For $y = \sqrt{x}$, use (0, 0), (1, 1), (4, 2), (9, 3)

2. Reflections and/or stretches/compressions
   - Multiply the *x*-coordinates of the key points by the reciprocal of *b*.
   - Multiply the *y*-coordinates of the key points by *a*.

3. Translations
   - Move graph right if *h* is positive (equation has – sign)
   - Move graph left if *h* is negative (equation has + sign)
   - Move graph up if *k* is positive
   - Move graph down if *k* is negative

Examples: For each of the following:
- Identify *a, b, h,* and *k*.
- Graph the function using transformations. List each transformation.
- Give the coordinates of the key points for each step and the final graph.
- State the domain and range of the function.
- State the coordinates of the vertex or endpoint.

$f(x) = |x+4| - 3$



$a=1,\ b=1,\ h=-4,\ k=-3$

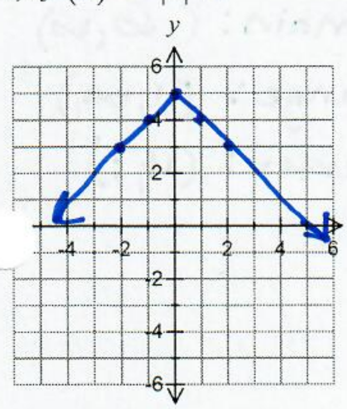
Shift left 4
Shift down 3

affects ?
x
y

-4	x	y=|x|	-3
-6	-2	2	-1
-5	-1	1	-2
-4	0	0	-3
-3	1	1	-2
-2	2	2	-1

Domain: $(-\infty, \infty)$
Range: $[-3, \infty)$
vertex: $(-4, -3)$

b) $f(x) = -|x| + 5$



$a=-1,\ b=1,\ h=0,\ k=5$

reflect over the x-axis
Shift up 5

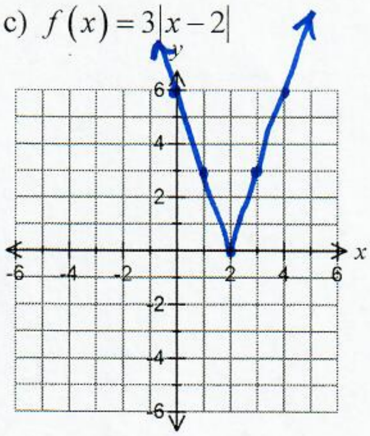
affects ?
y
y

x	y=|x|	·-1	+5
-2	2	-2	3
-1	1	-1	4
0	0	0	5
1	1	-1	4
2	2	-2	3

Domain: $(-\infty, \infty)$
range: $(-\infty, 5]$
vertex: $(0, 5)$

$a=3 \quad b=1 \quad h=2 \quad k=0$

c) $f(x)=3|x-2|$



vertical stretch of 3 — affects? y

Shift right 2 — x

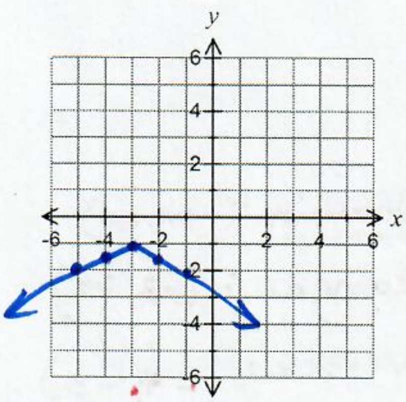
+2	x	y=|x|	·3
0	-2	2	6
1	-1	1	3
2	0	0	0
3	1	1	3
4	2	2	6

Domain: $(-\infty,\infty)$

range: $[0,\infty)$

vertex: $(2,0)$

d) $f(x)=-\frac{1}{2}|x+3|-1$



$a=-\frac{1}{2} \quad b=1 \quad h=-3 \quad k=-1$

affects?

reflect over x-axis — y

vertical shrink by a factor of ½ — y

Shift left 3 — x

shift down 1 — y

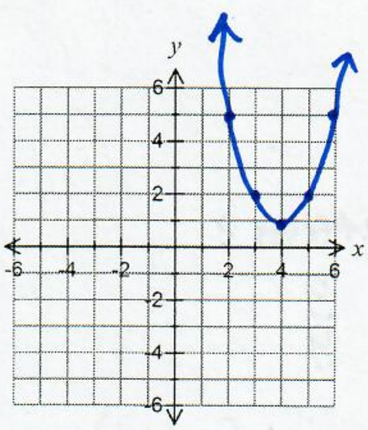
-3	x	y=|x|	·-½	-1
-5	-2	2	-1	-2
-4	-1	1	-½	-1½
-3	0	0	0	-1
-2	1	1	-½	-1½
-1	2	2	-1	-2

Domain: $(-\infty,\infty)$

Range: $(-\infty,-1]$

vertex: $(-3,-1)$

e) $f(x)=(x-4)^2+1$



$a=1 \quad b=1 \quad h=4 \quad k=1$

affects?

shift right 4 — x

shift up 1 — y

+4	x	$y=x^2$	+1
2	-2	4	5
3	-1	1	2
4	0	0	1
5	1	1	2
6	2	4	5

Domain: $(-\infty,\infty)$

Range: $[1,\infty)$

Vertex: $(4,1)$

f) $f(x) = -x^2 - 2$

$a=-1 \quad b=1 \quad h=0 \quad k=-2$

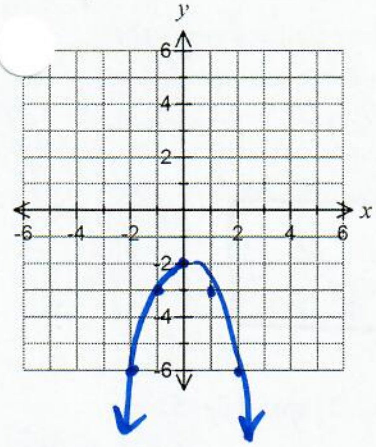
reflect over the x-axis

Shift down 2

affect?

y

y



x	$y=x^2$	$\cdot -1$	-2
-2	4	-4	-6
-1	1	-1	-3
0	0	0	-2
1	1	-1	-3
2	4	-4	-6

Domain: $(-\infty, \infty)$

Range: $(-\infty, -2]$

Vertex: $(0, -2)$

g) $f(x) = 2(-x+3)^2 - 4$

$= 2(-(x-3))^2 - 4$

$a=2 \quad b=-1 \quad h=3 \quad k=-4$

vertical stretch of 2

reflect over the y-axis

shift right 3

shift down 4

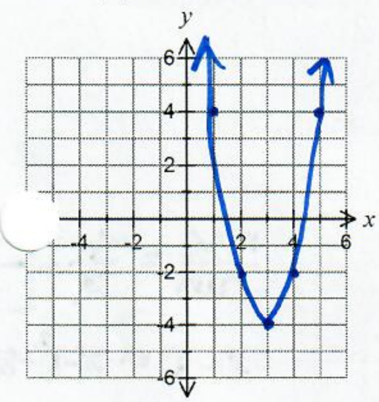
affect?

y

x

x

y



$+3$	$\cdot -1$	x	$y=x^2$	$\cdot 2$	-4
5	2	-2	4	8	4
4	1	-1	1	2	-2
3	0	0	0	0	-4
2	-1	1	1	2	-2
1	-2	2	4	8	4

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

Vertex: $(3, -4)$

h) $f(x) = -3(x-1)^2 + 5$

$a=-3 \quad b=1 \quad h=1 \quad k=5$

reflect over the x-axis

vertical stretch of 3

shift right 1

shift up 5

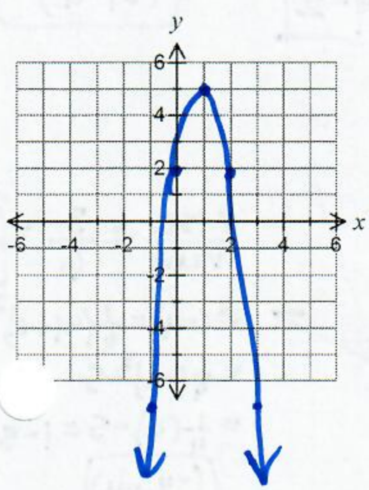
affect?

y

y

x

y



$+1$	x	$y=x^2$	$\cdot -3$	$+5$
-1	-2	4	-12	-7
0	-1	1	-3	2
1	0	0	0	5
2	1	1	-3	2
3	2	4	-12	-7

Domain: $(-\infty, \infty)$

Range: $(-\infty, 5]$

Vertex: $(1, 5)$

SM2H

SM2H – 1.6 Notes Average Rate of Change

The ***average rate of change*** between two points on a function is the slope of the line that connects the two points. For example, if the function $d(t)$ represents the distance in miles that a car has traveled after t hours, then finding the slope of the line connecting the points at $t = 1$ hour and $t = 4$ hours will give the average speed of the car (in miles per hour) during those three hours.

Slope Formula: The slope between the points (x_1, y_1) and (x_2, y_2) is $\boxed{m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$.

Examples: Find the slope between each pair of points.

a) $\overset{x_1,\ y_1}{(2,5)}$ and $\overset{x_2,\ y_2}{(4,9)}$

$m = \frac{9-5}{4-2} = \frac{4}{2} = \boxed{2}$

b) $(-1,4)$ and $(2,1)$

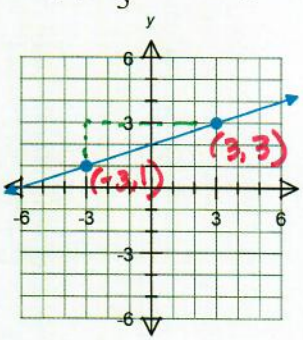
$m = \frac{1-4}{2--1} = \frac{-3}{3} = \boxed{-1}$

c) $(-4,-7)$ and $(2,-5)$

$m = \frac{-5--7}{2--4} = \frac{2}{6} = \boxed{\frac{1}{3}}$

Examples: For each of the following, draw the line that connects the two points. Write the coordinates of the two points, then calculate the average rate of change on the specified interval.

a) $f(x) = \frac{1}{3}x + 2$ on $[-3,3]$

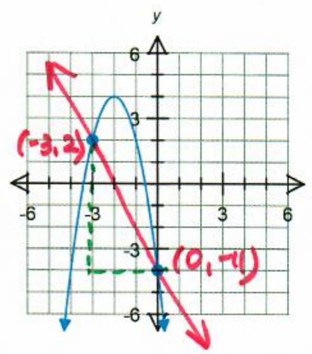


$\frac{\text{rise}}{\text{run}} = \frac{2}{6} = \frac{1}{3}$

(-3,1) and (3,3)

$m = \frac{3-1}{3--3} = \frac{2}{6} = \boxed{\frac{1}{3}}$

b) $f(x) = -2x^2 - 8x - 4$ on $[-3,0]$

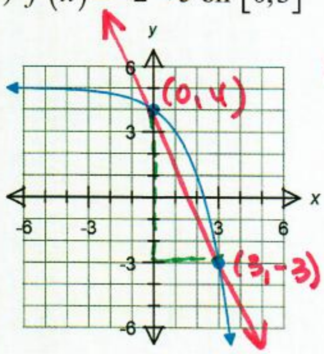


$\frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$

(-3,2) and (0,-4)

$m = \frac{-4-2}{0--3} = \frac{-6}{3} = \boxed{-2}$

c) $f(x) = -2^x + 5$ on $[0,3]$

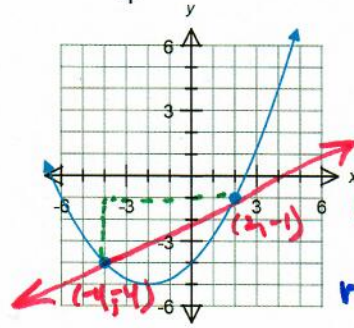


$\frac{\text{rise}}{\text{run}} = \frac{-7}{3}$

(0,4) and (3,-3)

$m = \frac{-3-4}{3-0} = \boxed{\frac{-7}{3}}$

d) $f(x) = \frac{1}{4}(x+2)^2 - 5$ on $[-4,2]$



$\frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$

(-4,-4) and (2,-1)

$m = \frac{-1--4}{2--4} = \frac{3}{6} = \boxed{\frac{1}{2}}$

Example: Find the average rate of change for $f(x) = 3x^2 - 5x + 4$ on the interval $[-1,3]$.

Step 1: Evaluate the value of the function at $x = -1$ and $x = 3$.

$f(-1) = 3(-1)^2 - 5(-1) + 4$
$= 3(1) + 5 + 4$
$= 12$

$f(3) = 3(3)^2 - 5(3) + 4$
$= 3(9) - 15 + 4$
$= 27 - 15 + 4$
$= 16$

Step 2: Write the two points as ordered pairs.

$(-1, \underline{12})$ $(3, \underline{16})$

Step 3: Use the slope formula to find the slope between the two points.

$m = \frac{16-12}{3--1} = \frac{4}{4} = \boxed{1}$

Examples: Find the average rate of change for each function on the specified interval.

a) $f(x) = -2x + 3$ on $[-5,7]$

$f(-5) = -2(-5) + 3$
$= 10 + 3$
$= 13$
$(-5, 13)$

$f(7) = -2(7) + 3$
$= -14 + 3$
$= -11$
$(7, -11)$

$m = \frac{-11-13}{7--5} = \frac{-24}{12} = \boxed{-2}$

b) $f(x) = -x^2 + 4$ on $[-4,2]$

$f(-4) = -(-4)^2 + 4$
$= -16 + 4$
$= -12$
$(-4, -12)$

$f(2) = -(2)^2 + 4$
$= -4 + 4$
$= 0$
$(2, 0)$

$m = \frac{0--12}{2--4} = \frac{12}{6} = \boxed{2}$

c) $f(x) = |x - 2|$ on $[-1,4]$

$f(-1) = |-1-2|$
$= |-3|$
$= 3$
$(-1, 3)$

$f(4) = |4-2|$
$= |2|$
$= 2$
$(4, 2)$

$m = \frac{2-3}{4--1} = \boxed{\frac{-1}{5}}$

d) $f(x) = 2(x-3)^2 - 5$ on $[0,10]$

$f(0) = 2(0-3)^2 - 5$
$= 2(-3)^2 - 5$
$= 2(9) - 5$
$= 13$
$(0, 13)$

$f(10) = 2(10-3)^2 - 5$
$= 2(7)^2 - 5$
$= 2(49) - 5$
$= 93$
$(10, 93)$

$m = \frac{93-13}{10-0} = \frac{80}{10} = \boxed{8}$

Examples:

a) Many of the elderly are placed in nursing care facilities. The cost of these has risen significantly since 1960. Use the table below to find the average rate of change from 2000 to 2010 and explain what your result means.

	Years Since 1960	Nursing Care Cost (billions of dollars)
1960	0	1
1970	10	4
1980	20	18
1990	30	53
2000	40	96
2010	50	157

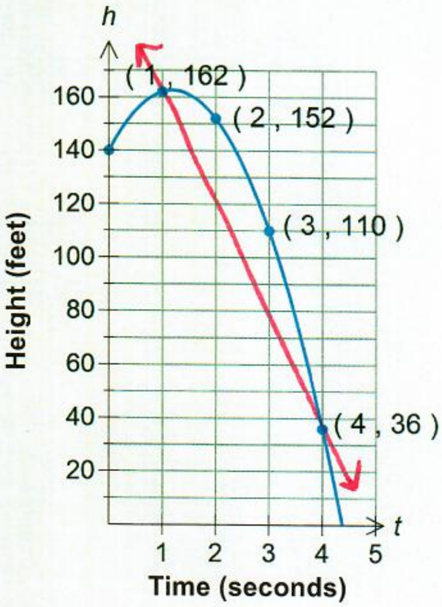
$(40, 96)$ $(50, 157)$

$m = \frac{157-96}{50-40} = \frac{\$61 \text{ billion}}{10 \text{ years}}$

$= \$6.1$ billion/year

Nursing care costs rose by an average of $6.1 billion per year from 2000 to 2010.

b) The graph below shows the height, in feet, of an object launched straight up from an initial height of 140 feet. Find the average rate of change from 1 to 4 seconds and explain what your answer means.



(1, 162) (4, 36)

$m = \frac{36-162}{4-1} = \frac{-126 \text{ feet}}{3 \text{ seconds}} = -42$ feet/sec

The height of the object decreased by an average of 42 feet per second from 1 second to 4 seconds after it was launched.

c) Suppose 25 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles in size every week. The equation $P(x) = 25 \cdot 2^x$ can be used to determine the number of beetles after x weeks. Complete the table below.

Week	Beetle Population
0	25
1	50
2	100
3	200
4	400
5	800

Calculate the average growth rate between weeks 1 and 3.

(1, 50) (3, 200)

$m = \frac{200-50}{3-1} = \frac{150 \text{ beetles}}{2 \text{ weeks}} = 75$ beetles per week

Calculate the average growth rate for the first five weeks [0,5].

(0, 25) (5, 800)

$m = \frac{800-25}{5-0} = \frac{775 \text{ beetles}}{5 \text{ weeks}} = 155$ beetles per week

Which average growth rate was higher? Why do you think it is higher?

The average growth rate was higher for weeks 0 to 5. This is because as the population increases, there are more beetles that can reproduce. So the longer you leave them, the faster the population will grow.