SM2H Unit 1 Analyzing Functions Notes

1.1 Functions

Function: When each domain element is paired with a unique range element.

- Only 1 x-value for every y-value
- For every input there is one and only one output.

Domain: The set of all inputs (the x-values) of a relation.

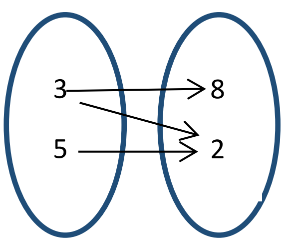
- If a relation is represented by a graph or ordered pairs, the domain is the set of all x-coordinates of points on the graph.
- When modeling a real-life situation, the domain is the set of x's that make sense in the problem.
- When given an equation, the domain is the set of x-values for which the equation is defined.

Range: The set of all outputs (the y-values) of a relation.

- If a relation is represented by a graph or ordered pairs, the range is the set of all y-coordinates of points on the graph.
- When modeling a real-life situation, the range is the set of y's that make sense in the problem.

<u>Examples:</u> Decide whether each relation is a function. Then write the relation as a set of ordered pairs. Then find the domain and range.

1.



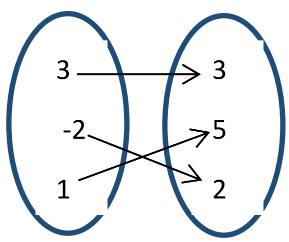
Function?

Ordered Pairs:

Domain:

Range:

2.



Function?

Ordered Pairs:

Domain:

Range:

3.

x	y
-2	5
-1	7
0	9
1	-2
2	-2

Function?

Domain:

Range:

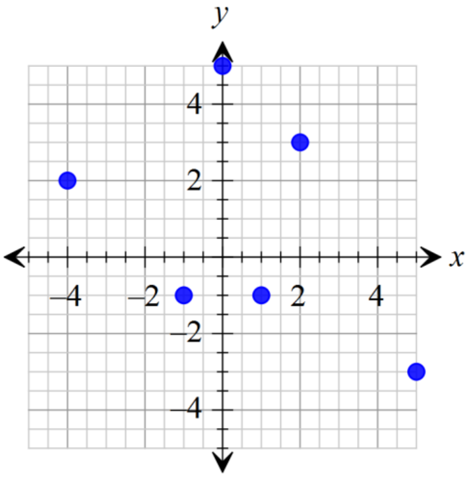
4. $\{(4,-7),(-1,5),(8,2),(4,5)\}$

Function?

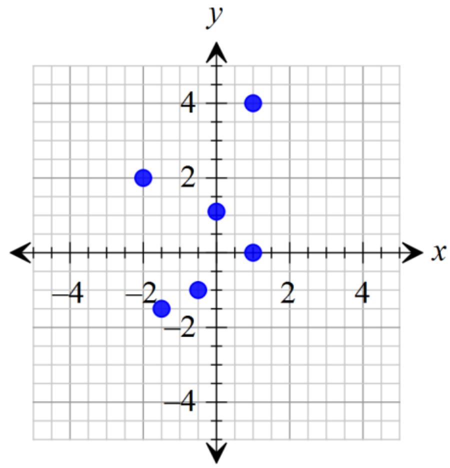
Domain:

Range:

5.



6.



Function?

Ordered Pairs:

Domain:

Range:

Function?

Ordered Pairs:

Domain:

Range:

Evaluating a function or finding a value <u>means substituting the given value for x in the equation</u>. Evaluate the expression.

Examples: Find each value if $f(x) = x^2 - 2x + 3$, $g(x) = 3x - 5$, and $h(x) = \frac{x}{4-2x}$. Leave your answers as simplified fractions, if necessary. Show all your work.

a) $f(2)$

b) $g(-1)$

c) $h(4)$

d) $g\left(\frac{2}{3}\right)$

e) $f(-5)$

f) $h(-3)$

Function Notation:

Examples: The graph of $y = f(x)$ is shown below. Use it to answer the following questions.

a) Find $f(-4)$.

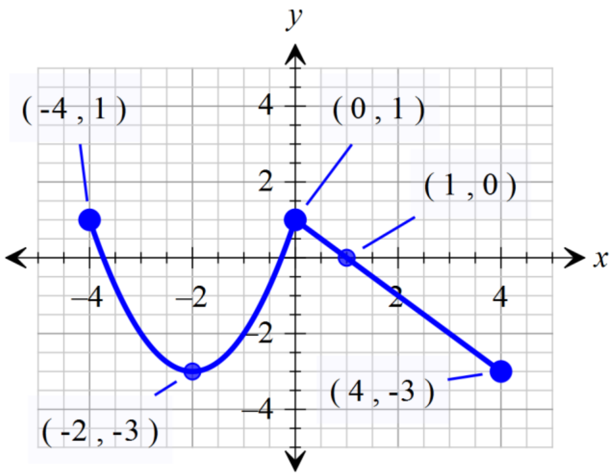
b) Find $f(0)$.

c) For what values of x is $f(x) = 0$?

d) For what values of x is $f(x) = -3$?

e) What is the domain?

f) What is the range?



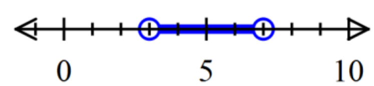
Interval Notation

Domain and range are often written in ***interval notation*.**

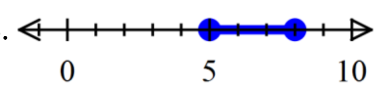
- The numbers are where the interval starts and stops.
- If an endpoint is *included*, put it in a bracket [or]
- If an endpoint is *not included*, put it in parentheses (or)
- If the interval goes on forever, use $-\infty$ or ∞. These always get put in parentheses (or).

Examples:

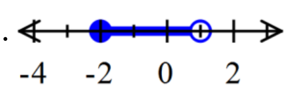
- ______ means everything between 3 and 7, not including either 3 or 7.



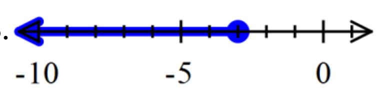
- ______ means everything between 5 and 8, including both 5 and 8.



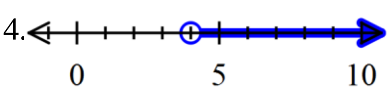
- _______ means everything between –2 and 1, including –2, but not including 1.



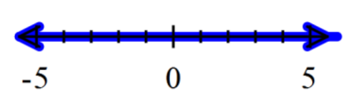
- _________ means everything less than or equal to –3.



- _______ means everything greater than 4.



- _______ means all real numbers (goes on forever in both directions).



1.2 Analyzing Functions

Domain - given the equation of a function. When given an equation, think about what x-values will "work" as input into the equation. Are there any values of x for which the equation is not defined?

Examples: Find the domain.

1. $f(x) = 2x + 6$

2. $p(x) = 2x^2 - 3x + 6$

3. $g(x) = \sqrt{x-5}$

4. $h(x) = |x| - 2$

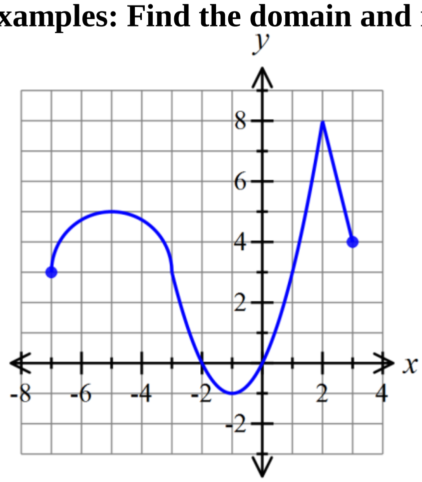
Tips for finding Domain and Range

Remember: Domain is the set of all inputs (**the *x*-values**) of a relation
Range is the set of all outputs (**the *y*-values**) of a relation.

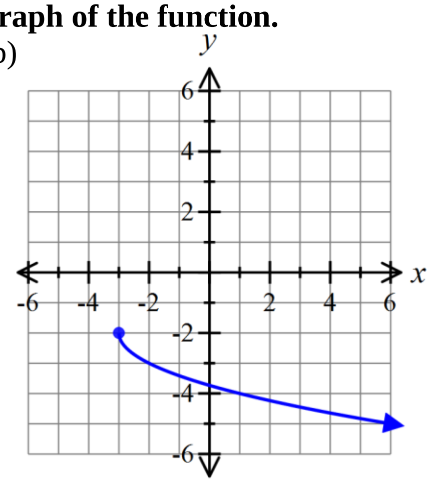
- Use parentheses (or) around numbers that *are not included* in the domain or range.
  - This happens when there is an ***open circle*** at a point or an ***asymptote*** (a line that the graph gets really close to, but never actually touches).
- Use brackets [or] around endpoints that *are included* in the domain or range.
  - It there is a point on the graph with the given *x*- or *y*-coordinate, use a bracket.
- Always use parentheses around $-\infty$ and ∞.
- ***Read the domain from left to right and the range from down to up.***
  - Write the lower value or $-\infty$ first and the higher value or ∞ last.

Examples: Find the domain and range given the graph of the function.

a)

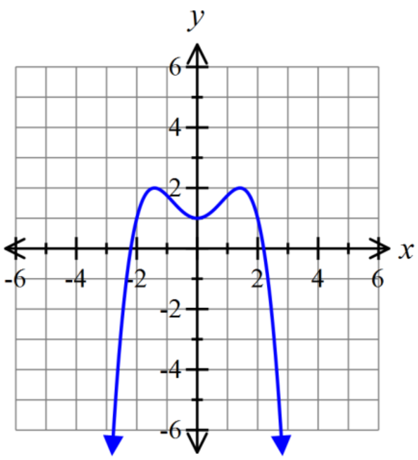


b)

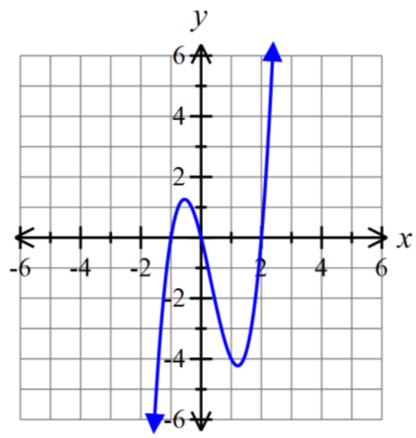


Find the domain and range given the graph of the function.

c)



d)



Domain and Range in Real Life Situations: In real life situations, it's important to think through what values make sense in the problem. You also need to think about what *x* stands for and what *y* stands for.

Examples: Find the real world domain and range for each situation.

1. You are getting ready for the Homecoming dance. Your dad is going to let you borrow his new car, but you need to wash and fill it. The car wash costs $5 and the gas costs $3.89 per gallon. The car can hold 15 gallons of gas. What are the domain and range if the total cost is a function of the number of gallons and the car is completely empty when you pull into the gas station?

   Circle which unit represents the domain: **Students or Chaperones**

   Domain:

   Range:

2. Your cell phone plan charges a flat fee of $10 for up to 1000 texts and $0.10 per text over 1000. What are the domain and range?

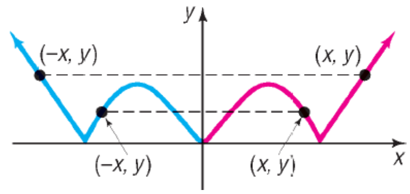
   Circle which unit represents the domain: **Students or Chaperones**

   Domain:

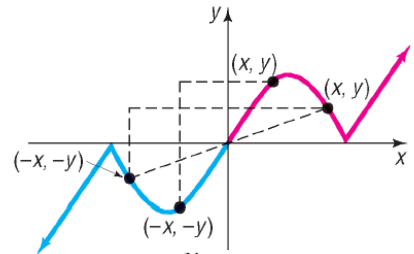
   Range:

Symmetry

- A function has ***even symmetry*** or ***y-axis symmetry*** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. If you folded the graph along the *y*-axis, the two sides would overlap.

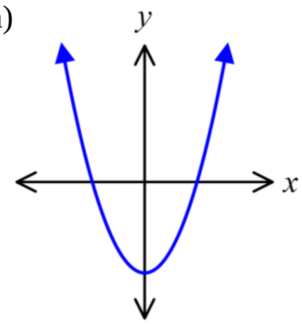


- A function has ***odd symmetry*** or ***origin symmetry*** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. If you rotated the graph 180°, it would end up in the same place it started.

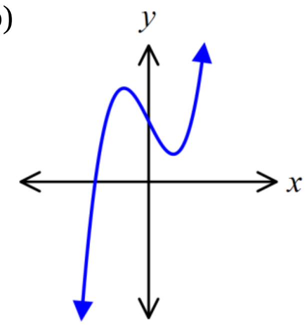


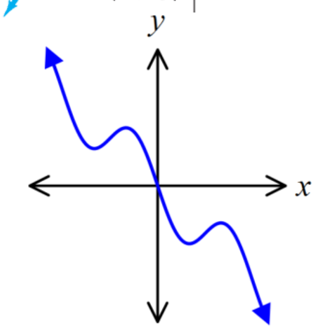
Examples: Determine what type of symmetry each function has, if any.

a)



b)





Determine the Symmetry of a Function Algebraically

- A function is **symmetric with respect to the *y*-axis** or is an **even function**, if $f(-x) = f(x)$. In other words, if you substitute $-x$ in for every x, you end up with the original function. When looking at the graph, you could "fold" the graph along the *y-axis* and both sides are the same.
- A function is **symmetric with respect to the origin** or is an **odd function**, if $f(-x) = -f(x)$. In other words, if you substitute $-x$ in for every x, you end up with the opposite of the original function. When looking at the graph, there is a mirror image in Quadrants 1 & 3 or in Quadrants 2 & 4. The graph has rotational symmetry about the origin.
- A function has **no symmetry** if it is neither **even** nor **odd**. If you substitute $-x$ in for every x and you end up with something that is not the original function and is not its opposite, then there is no symmetry for this function.

Examples: Determine algebraically the type of symmetry for each of the following functions.

1. $f(x) = |x| + 5$
2. $g(x) = 8x^3$
3. $h(x) = x^2 + 2x$
4. $d(x) = |2x - 1|$
5. $p(x) = -2x^2$
6. $q(x) = -x^3$

SM2H

SM2H Unit 1 Analyzing Functions Notes

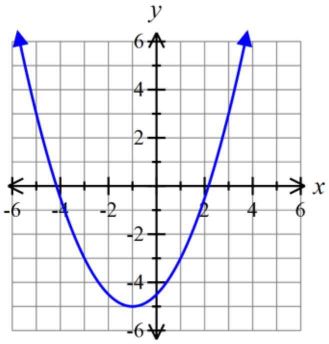
End behavior describes what is happening to the *y*-values of a graph when *x* goes to the far right (∞) or when *x* goes to the far left ($-\infty$).

Right End Behavior: $\lim_{x\to\infty} f(x) = c$ **Left End Behavior:** $\lim_{x\to-\infty} f(x) = c$

- If *y* is getting larger and larger (the graph is pointing up) with no upper limit, then the limit would be equal to ∞.
- If *y* is getting smaller and smaller (the graph is pointing down) with no lower limit, then the limit would be equal to $-\infty$.
- If the graph has an asymptote (gets closer and closer to a number, but never touches it), then the limit is equal to wherever the asymptote is.
- If there's an endpoint, the limit does not exist.

Examples: Describe the end behavior of each graph using limits.

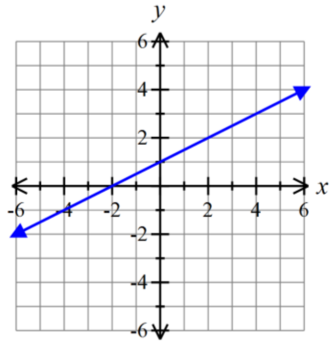
a)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

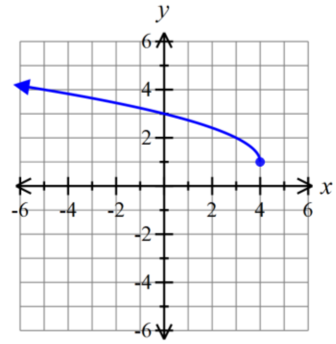
b)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

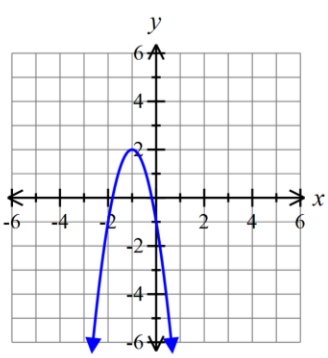
c)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

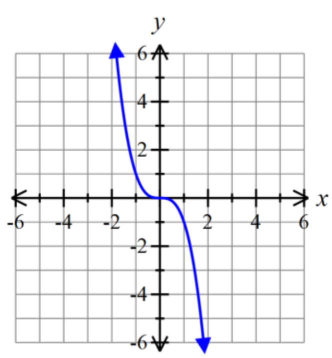
d)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

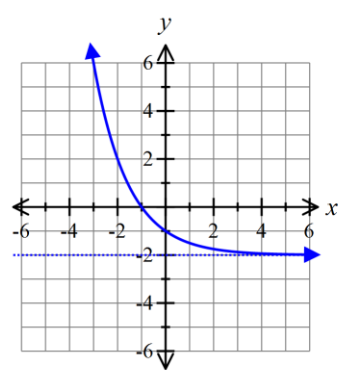
e)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

f)



Left: $\lim_{x\to-\infty} f(x) =$ ______

Right: $\lim_{x\to\infty} f(x) =$ ______

End Behavior

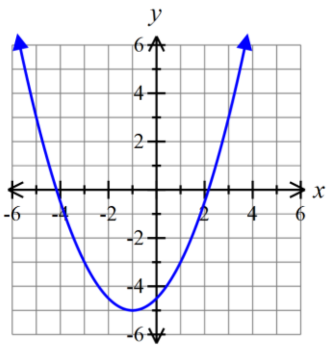
End behavior describes what is happening to the y-values of a graph when x goes to the far right (∞) or when x goes to the far left ($-\infty$).

Right End Behavior: $\lim_{x \to \infty} f(x) = c$ **Left End Behavior:** $\lim_{x \to -\infty} f(x) = c$

- If y is getting larger and larger (the graph is pointing up) with no upper limit, then the limit would be equal to ∞.
- If y is getting smaller and smaller (the graph is pointing down) with no lower limit, then the limit would be equal to $-\infty$.
- If the graph has an asymptote (gets closer and closer to a number, but never touches it), then the limit is equal to wherever the asymptote is.
- If there's an endpoint, the limit does not exist.

Examples: Describe the end behavior of each graph using limits.

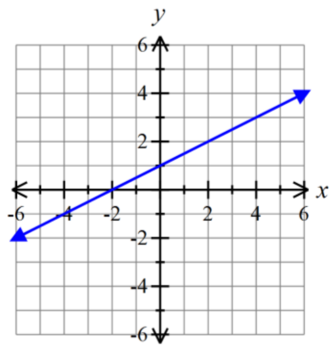
a)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

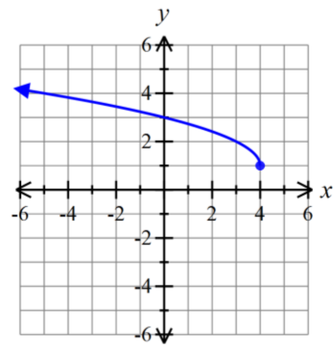
b)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

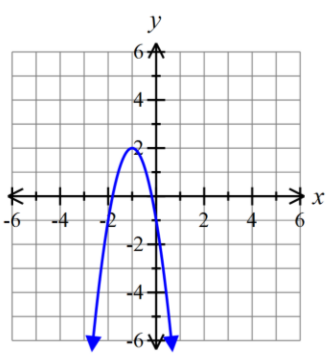
c)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

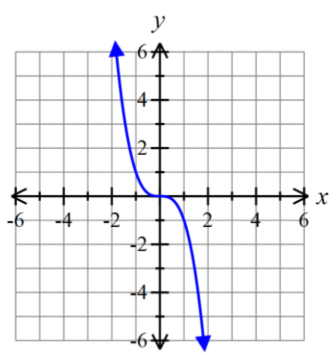
d)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

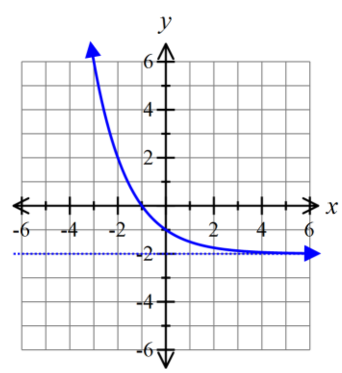
e)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

f)



Left: $\lim_{x \to -\infty} f(x) =$ _____

Right: $\lim_{x \to \infty} f(x) =$ _____

1.3 Analyzing Functions Day 2

x-Intercepts: The points where a graph crosses the *x*-axis. They have the form $(x, 0)$.

- To find the *x*-intercept(s), set $y = 0$ and solve for x.

y-Intercepts: The points where a graph crosses the *y*-axis. They have the form $(0, y)$.

- To find the *y*-intercept(s), set $x = 0$ and solve for y.

Examples: Find the intercepts of each graph. Write the intercepts as ordered pairs.

a) $f(x) = 2x + 6$

b) $f(x) = -3x + 2$

c) $3x + 2y = 12$

d) $x - 2y = 5$

Relative Maxima and Minima

When a point is higher that all the points near it, it is called a ***relative maximum.*** When a point is lower than all the points near it, it is called a ***relative minimum.***

Relative Maximum

Relative Minimum

- If you are asked for a ***maximum point or a minimum point,*** write the answer as an ordered pair.
- If you are asked for a ***maximum value or a minimum value,*** the answer is the ***y-coordinate.***

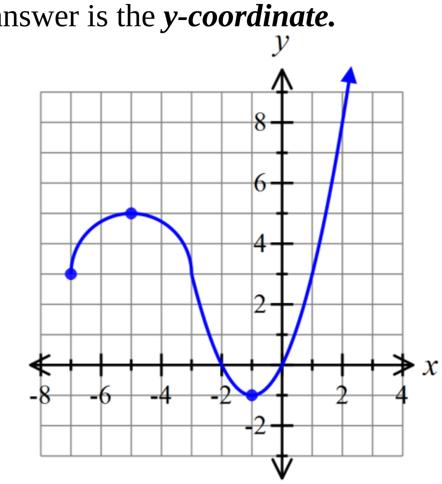
Example:

a) Find the relative maximum point and the relative maximum value.

b) Find the relative minimum points and the relative minimum values.

c) Find the absolute maximum point and the absolute maximum value.

d) Find the absolute minimum points and the absolute minimum values.



Increasing, Decreasing, and Constant

If you look from left to right along the graph of the function, you will notice parts are *rising*, parts are *falling* and parts are *horizontal*. In such cases, the function is described as *increasing*, *decreasing*, or *constant*, respectively.

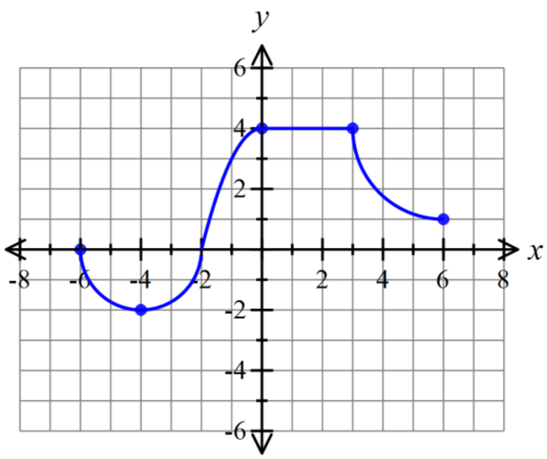
Increasing **Decreasing** **Constant**

★ When you are asked to state where the graph is increasing, decreasing, and constant, write the intervals of ***x*-coordinates** from **left to right.**

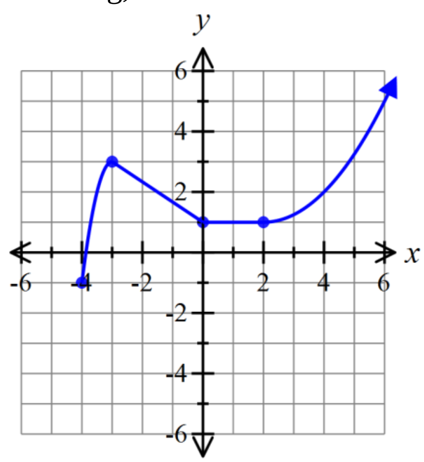
★ Always use ( ) for increasing, decreasing, and constant. Never use [ ].

Example: Determine where each graph is increasing, decreasing, and constant.

a)



b)



Positive and Negative

- A function is ***positive*** where the *y*-coordinates are positive. The graph is ***above the x-axis.***
- A function is ***negative*** where the *y*-coordinates are negative. The graph is ***below the x-axis.***

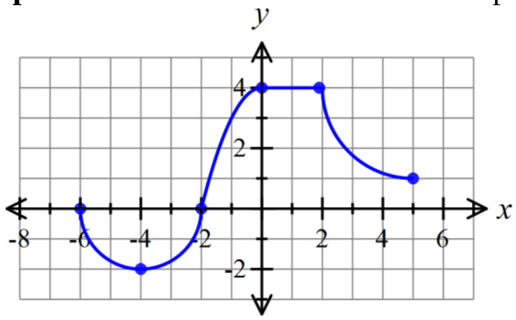
★ When you are asked to state where the graph is positive and negative, write the intervals of the ***x*-coordinates** from **left to right.**

★ Use (or) at the *x*-intercepts, where the graph crosses over from positive to negative. The *y*-coordinate is zero at the intercepts, so the graph is neither positive or negative there. That means those points are not included in the interval.

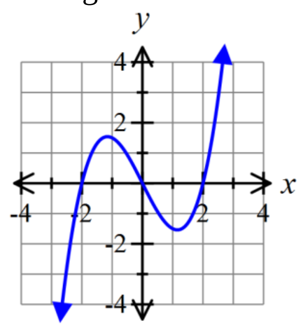
★ Use [or] if the graph has an *endpoint* somewhere above or below the *x*-axis.

Example: Determine where the function is positive and where it is negative.

a)



b)



1.4 Transformations

A. <u>Types of transformations</u>

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct **"Order of Transformations."**

1. **<u>Vertical Reflection (the negative in front of the equation)</u>**:
   a. Vertical Reflection – graph is reflected over the x-axis
      ***The y values are multiplied by -1**
2. **<u>Vertical Stretch/Compression (the number in front of the equation or a)</u>:**
   We will restrict our attention to Vertical stretches/compressions.
   a. Vertical Stretch – the **y-coordinates are multiplied** by a scalar that is greater than 1
   b. Vertical Compression – the **y-coordinates are multiplied** by a scalar that is between 0 & 1.
3. **<u>Horizontal Reflection (the negative under the √ in front of the b):</u>**
   a. Horizontal Reflection – graph is reflected over the y-axis
      ***The x values are multiplied by -1**
4. **<u>Horizontal Stretch/Compression (b)</u>:**
   a. Horizontal Stretch – the x-coordinates are multiplied by a scalar that is greater than 1
   b. Horizontal Compression – the x-coordinates are multiplied by a scalar that is between 0 and 1.
5. **<u>Translation or Shift (h then k)</u>**:
   a. Horizontal Translation – graph is shifted to the left or right (the opposite of the number with x, or in the parentheses with x, or h)
   b. Vertical Translation – graph is shifted up or down (the number that is added or subtracted to the equation or k)

B. <u>Finding the number that tells you the transformation</u>

Answer the following questions using the equations: $\boldsymbol{y = a\sqrt{b(x-h)} + k}$,
Given the following equations find $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{h}$, and $\boldsymbol{k}$.

A. $y = 3\sqrt{2(x-6)} + 8$

a=______________________

b=______________________

h=______________________

k=______________________

B. $y = -\sqrt{(x-6)} - 3$

a=_________________________

b =_________________________

h=_________________________

k=_________________________

C. $y = 7\sqrt{-4x}$

a=______________________

b=______________________

h=______________________

k=______________________

C. Order of Transformations should be written in order from left to right reflection, a, b, h, and then k.

List the transformations in the correct order.

$y=-2\sqrt{x}-1$

Transformations:

1.

2.

3.

$y=3\sqrt{-(x+4)}-6$

Transformations:

1.

2.

3.

4.

$y=-7\sqrt{-2x}+3$

Transformations:

1.

2.

3.

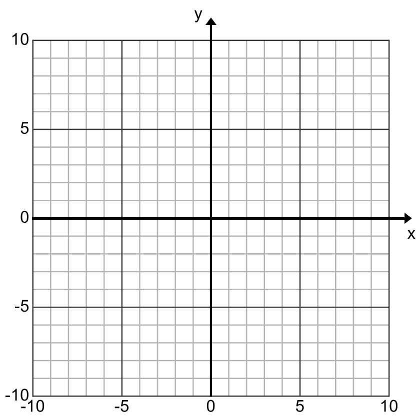
4.

B. <u>Parent Graph Analysis</u>: Square Root Function

Fill in the table to find some **key points** for the parent functions. Use the table to generate ordered pairs for points on the graph, then sketch the graph.

$$f(x)=\sqrt{x}$$

x	$f(x)$
0	
1	
4	
9	



Fill in the following information about the parent function.

Domain:____________________ Range:____________________

x-intercept(s):______________ y-intercept:______________

Which does the function have: a maximum or a minimum? _____________

Max/Min Point:________________ Max/Min Value:________________

Positive:________________ Negative:________________

Increasing:________________ Decreasing:________________

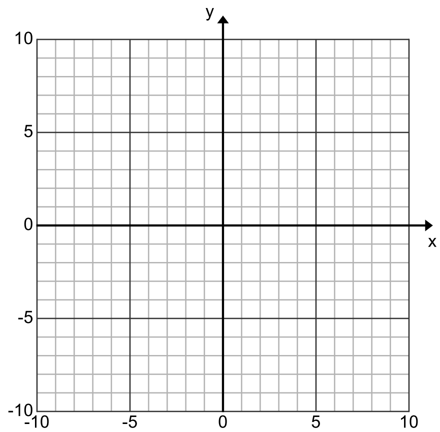
Constant:________________ Symmetry: ________________

Left End Behavior $\lim_{x\to-\infty} f(x)=$ _____ Right End Behavior: $\lim_{x\to\infty} f(x)=$ _____

Applying Transformations to the Function:
Directions: For each of the following, create a table to show how the transformation changes the key points from the parent graph.

Vertical reflection: Use a table to create the following graph:

1. $f(x) = -\sqrt{x}$



Explain the difference between this graph and the graph of $f(x) = \sqrt{x}$.

What is the domain?

What is the range?

Parent Graph

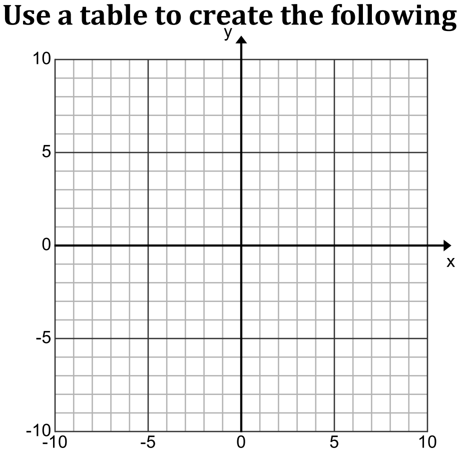
$f(x) = \sqrt{x}$

x	$f(x)$
0	
1	
4	
9	

x	$f(x)$

Horizontal reflection: Use a table to create the following graph:

2. $f(x) = -\sqrt{x}$



Explain the difference between this graph and the graph of $f(x) = \sqrt{x}$.

What is the domain?

What is the range?

Parent Graph

$f(x) = \sqrt{-x}$

x	$f(x)$
0	
1	
4	
9	

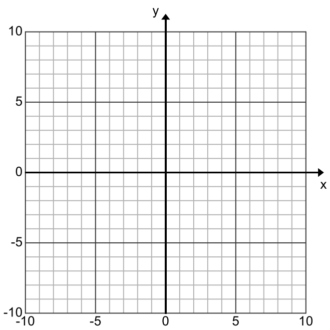
x	$f(x)$

NOTE: Sometimes it is hard to see the effects of a horizontal reflection on a function. This is especially true if the graph is symmetric with respect to the y-axis.

<u>Vertical stretch/compression</u>: Create tables for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function.

Parent Graph **Transformation:**

3. $f(x)=2\sqrt{x}$



Parent Graph: $f(x)=\sqrt{x}$

x	$f(x)$
0	
1	
4	
9	

Transformation:

x	$f(x)$

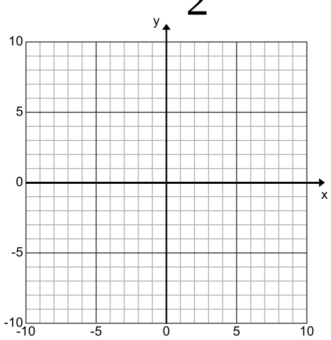
Domain:

Range:

Endpoint:

Explain the difference between this graph and the graph of $f(x)=\sqrt{x}$.

4. $f(x)=\dfrac{1}{2}\sqrt{x}$



Parent Graph

$f(x)=\sqrt{x}$

x	$f(x)$
0	
1	
4	
9	

x	$f(x)$

Domain:

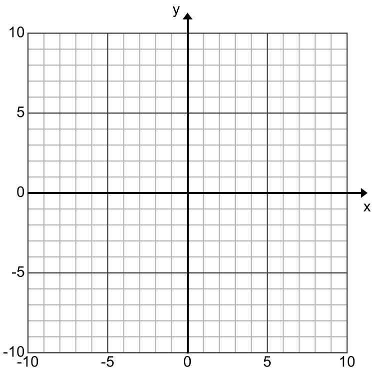
Explain the difference between this graph and the graph of $f(x)=\sqrt{x}$.

Range:

Endpoint:

<u>Horizontal Translation</u>: Create <u>your own tables</u> for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function as well as the location of the endpoint.

5. $f(x)=\sqrt{x-3}$

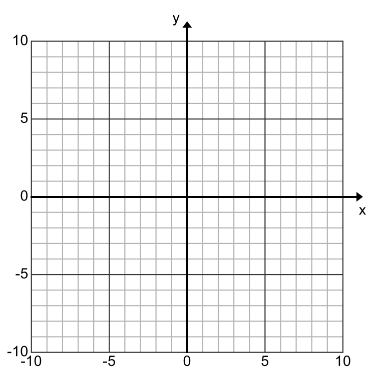


Domain:

Range:

Endpoint:

6. $g(x)=\sqrt{x+4}$



Domain:

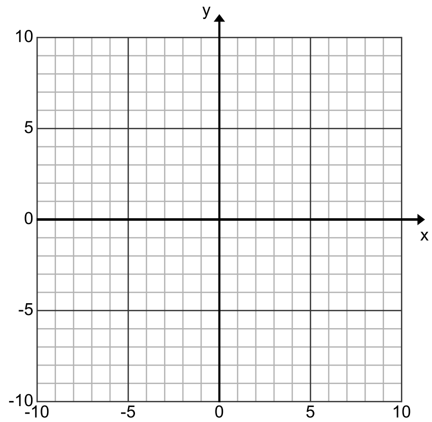
Endpoint:

Range:

Compare the two graphs above with the graph of $f(x)=\sqrt{x}$ **. What conclusion can we make about functions that come in the form:** $y=\sqrt{x-h}$ **?**

<u>Vertical Translation</u>: Create your own tables for each of the following to show the effects of the transformation on the key points from the Parent Function. State the domain and the range for each transformed function, as well as the location of the endpoint of the graph.

7. $y = \sqrt{x} + 3$



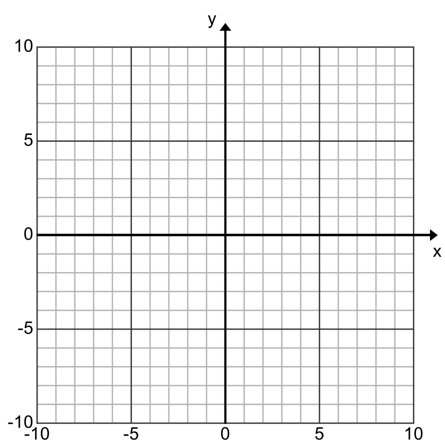
Explain the difference between this graph and the graph of $y = \sqrt{x}$.

Domain:

Range:

Endpoint:

8. $f(x) = \sqrt{x} - 2$



Domain:

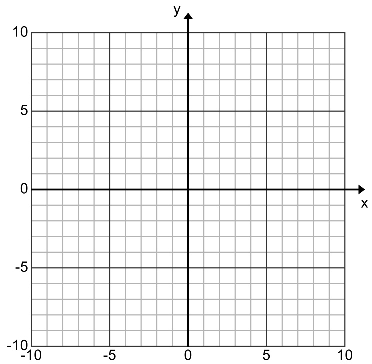
Range:

Endpoint:

Compare the two graphs above with the graph of $f(x) = \sqrt{x}$. What conjecture can we make about functions that come in the form: $y = \sqrt{x} + k$?

<u>Putting It All Together</u>: For each of the following 1) List the transformations in the correct order. 2) Create tables to show the transformations on the key points. 3) State the domain and range. 4) State the endpoint.

9. $f(x) = 2\sqrt{x-3} + 2$

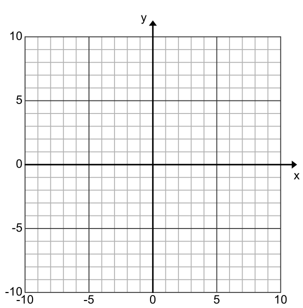


Domain:

Range:

Endpoint:

10. $f(x) = \frac{1}{2}\sqrt{x+1} - 3$



Domain:

Range:

Endpoint:

1.5 Transforming Absolute Value and Quadratic Functions

A. <u>Parent Graph Analysis</u>: Absolute Value Function

Fill in the given table to find some **key points** for the parent function. Use the table to generate ordered pairs for points on the graph and then sketch the graph.

$$f(x)=|x|$$

Where is the **vertex** of the graph of the function?

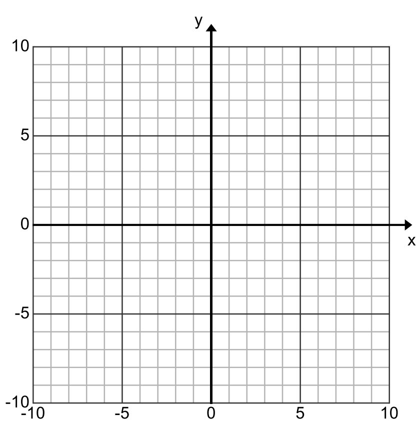
x	$f(x)$
-2	
-1	
0	
1	
2	

Fill in the following information about the parent function.

Domain:________________ Range:________________

x-intercept(s):________________ *y*-intercept:________________

Which does the function have: a maximum or a minimum? ______________



Max/Min Point:________________ Max/Min Value:________________

Positive:________________ Negative:________________

Increasing:________________ Decreasing:________________

Constant:________________ Symmetry: ________________

Left End Behavior: $\lim\limits_{x\to-\infty} f(x) =$ _______

Right End Behavior: $\lim\limits_{x\to\infty} f(x) =$ _______

B. <u>Parent Graph Analysis</u>: Quadratic Function (Parabola)

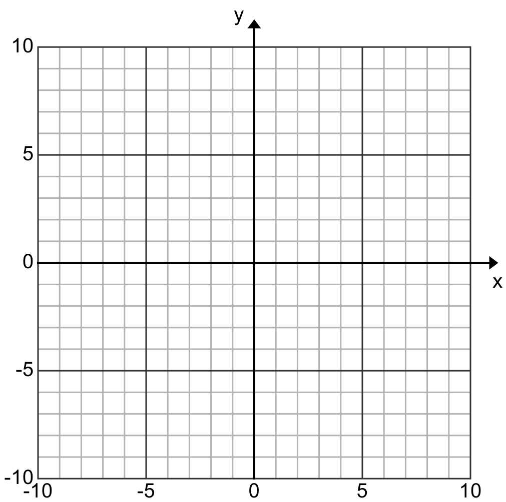
Fill in the table to find some **key points** for the parent function. Use the table to generate ordered pairs for points on the graph, then sketch the graph.

$$f(x) = x^2$$

What is the **vertex** of the graph of the function?

Fill in the following information about the parent function.

x	$f(x)$
-2	
-1	
0	
1	
2	



Domain:________________ Range:________________

x-intercept(s):________________ y-intercept:________________

Which does the function have: a maximum or a minimum? ______________

Max/Min Point:______________ Max/Min Value:________________

Positive:________________ Negative:________________

Increasing:________________ Decreasing:________________

Constant:________________ Symmetry: ________________

Left End Behavior: $\lim_{x \to -\infty} f(x) =$ _______

Right End Behavior: $\lim_{x \to \infty} f(x) =$ _______

C. **Transformations** of the parent graph:

	$f(x)=\|x\|$	$f(x)=x^2$	Effect on Parent Graph
$y=-f(x)$			
$y=2f(x)$			
$y=\frac{1}{2}f(x)$			
$y=f(x)+2$			
$y=f(x)-2$			
$y=f(x+2)$			
$y=f(x-2)$			

Examples: Describe the transformations of the parent graph needed to graph the following functions. Parent graph: $f(x)=|x|$. List the transformations in the order in which they should be applied.

a) $y=|x+5|-3$ b) $y=-\frac{1}{2}|x|+1$ c) $y=3|x-2|+4$

Parent graph: $f(x)=x^2$

a) $y=-x^2-3$ b) $y=-4(x-3)^2$ c) $y=\frac{1}{5}(x+2)^2-3$

D. To draw the graph of $y = a \cdot f(x-h)+k$ using transformations.

1. Draw the parent graph using the key points.
    - For $y=|x|$ or $y=x^2$, use (–2, 2), (–1, 1), (0, 0), (1, 1), (2, 2)
    - For $y=\sqrt{x}$, use (0, 0), (1, 1), (4, 2)

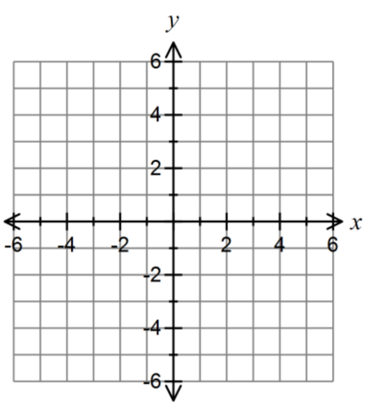
2. Reflections and/or stretches/compressions
    - Multiply the *y*-coordinates of the key points by *a*.

3. Translations
    - Move graph right if *h* is positive (equation has – sign)
    - Move graph left if *h* is negative (equation has + sign)
    - Move graph up if *k* is positive
    - Move graph down if *k* is negative

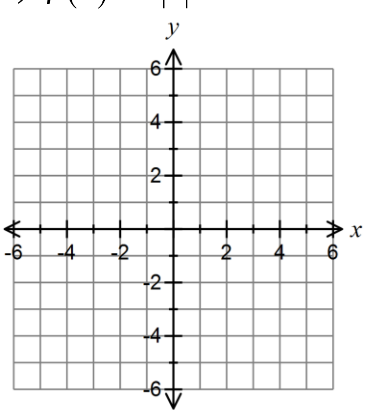
Examples: For each of the following:

- Identify *a*, *h*, and *k*.
- Graph the function using transformations. List each transformation.
- Give the coordinates of the key points for each step and the final graph.
- State the domain and range of the function.
- State the coordinates of the vertex or endpoint.

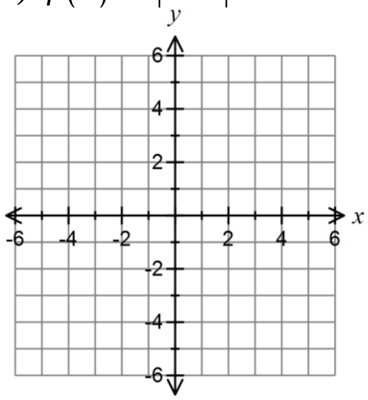
a) $f(x) = |x+4| - 3$



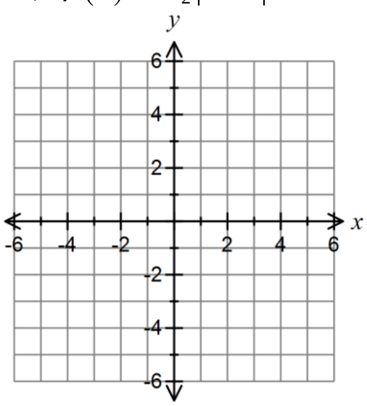
b) $f(x) = -|x| + 5$



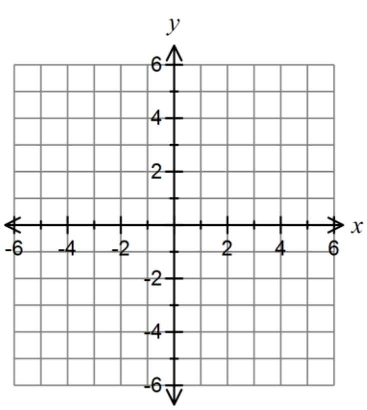
c) $f(x) = 3|x-2|$



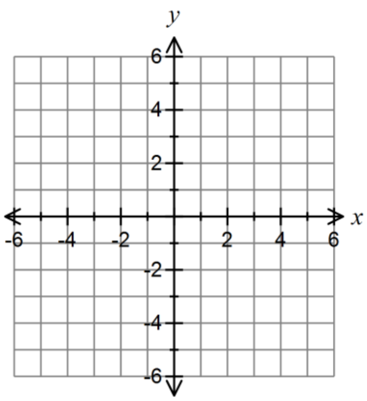
d) $f(x) = -\frac{1}{2}|x+3| - 1$



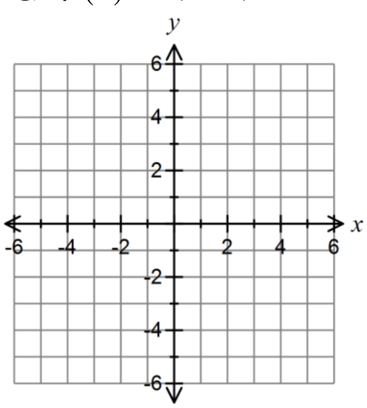
e) $f(x) = (x-4)^2 + 1$



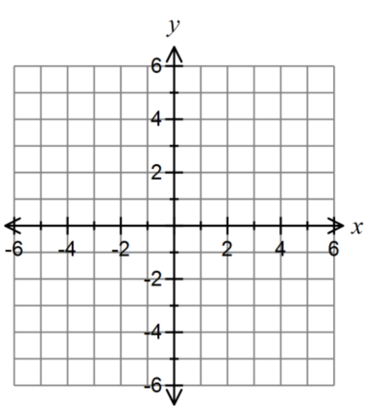
f) $f(x) = -x^2 - 2$



g) $f(x) = 2(x+3)^2 - 4$



h) $f(x) = -3(x-1)^2 + 5$



<u>1.6 Average Rate of Change</u>

The ***average rate of change*** between two points on a function is the slope of the line that connects the two points. For example, if the function $d(t)$ represents the distance in miles that a car has traveled after *t* hours, then finding the slope of the line connecting the points at $t=1$ hour and $t=4$ hours will give the average speed of the car (in miles per hour) during those three hours.

Slope Formula: The slope between the points (x_1, y_1) and (x_2, y_2) is $m=\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}$.

Examples: Find the slope between each pair of points.

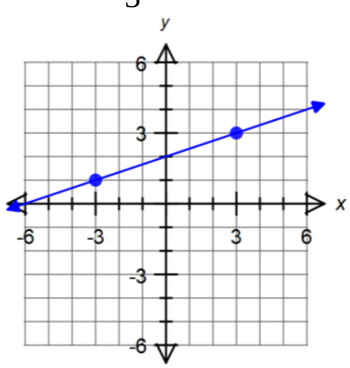
a) $(2,5)$ and $(4,9)$

b) $(-1,4)$ and $(2,1)$

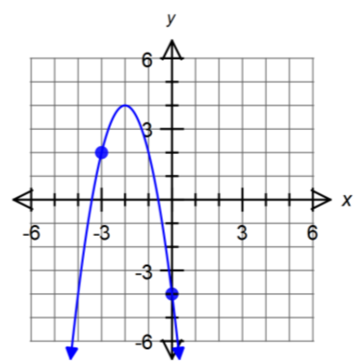
c) $(-4,-7)$ and $(2,-5)$

Examples: For each of the following, draw the line that connects the two points. Write the coordinates of the two points, then calculate the average rate of change on the specified interval.

a) $f(x)=\dfrac{1}{3}x+2$ on $[-3,3]$

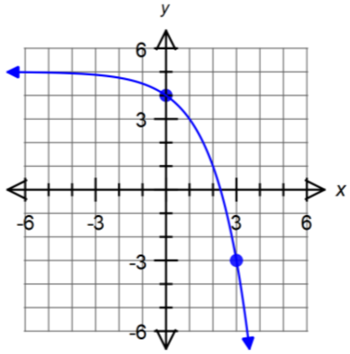


b) $f(x)=-2x^2-8x-4$ on $[-3,0]$

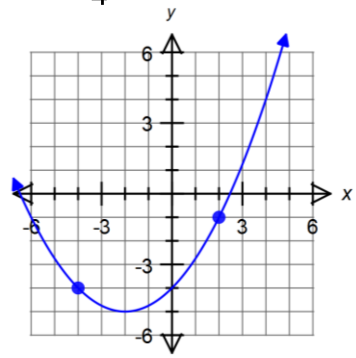


Continued: For each of the following, draw the line that connects the two points. Write the coordinates of the two points, then calculate the average rate of change on the specified interval.

c) $f(x) = -2^x + 5$ on $[0,3]$



d) $f(x) = \frac{1}{4}(x+2)^2 - 5$ on $[-4,2]$



Example: Find the average rate of change for $f(x) = 3x^2 - 5x + 4$ on the interval $[-1,3]$.

Step 1: Evaluate the value of the function at $x = -1$ and $x = 3$.

$f(-1) =$

$f(3) =$

Step 2: Write the two points as ordered pairs.

$(-1, \_\_\_)$

$(3, \_\_\_)$

Step 3: Use the slope formula to find the slope between the two points.

Examples: Find the average rate of change for each function on the specified interval.

a) $f(x) = -2x + 3$ on $[-5, 7]$

b) $f(x) = -x^2 + 4$ on $[-4, 2]$

c) $f(x) = |x - 2|$ on $[-1, 4]$

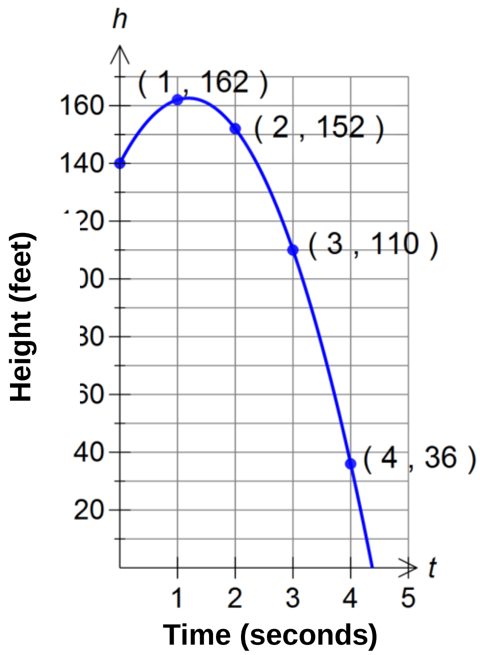
d) $f(x) = 2(x-3)^2 - 5$ on $[0, 10]$

Examples:

a) Many of the elderly are placed in nursing care facilities. The cost of these has risen significantly since 1960. Use the table below to find the average rate of change from 2000 to 2010 and explain what your result means.

Years Since 1960	**Nursing Care Cost (billions of dollars)**
0	1
10	4
20	18
30	53
40	96
50	157

b) The graph below shows the height, in feet, of an object launched straight up from an initial height of 140 feet. Find the average rate of change from 1 to 4 seconds and explain what your answer means.



c) Suppose 25 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles in size every week. The equation $P(x)=25\cdot 2^x$ can be used to determine the number of beetles after *x* weeks. Complete the table below.

Week	Beetle Population
0	
1	
2	
3	
4	
5	

Calculate the average growth rate between weeks 1 and 3.

Calculate the average growth rate for the first five weeks [0,5].

Which average growth rate was higher? Why do you think it is higher?