Review Factions

$\frac{8}{8}=1$ $\quad \frac{3\pi}{3}=\pi$ $\quad \frac{4\pi}{4}=\pi$

SM2H 9.1 Notes – Degrees and Radians

Fraction Review:

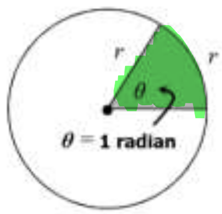
common denominator

1. $\frac{2\pi}{5}\cdot\frac{1}{4}=\frac{\cancel{2}\pi}{\cancel{20}_{10}}=\boxed{\frac{\pi}{10}}$

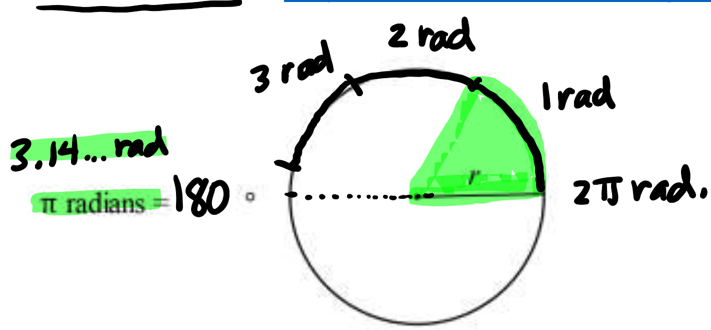
2. $\frac{3\pi}{8}\cdot\frac{4}{\pi}=\frac{\cancel{12}^{3}\cancel{\pi}}{\cancel{8}_{2}\cancel{\pi}}=\boxed{\frac{3}{2}}$ $\quad \frac{3\cancel{\pi}}{\cancel{8}_{2}}\cdot\frac{\cancel{4}^{1}}{\cancel{\pi}_{1}}=\boxed{\frac{3}{2}}$

3. $\frac{4\cdot 5x}{4\cdot 3}-\frac{x\cdot 3}{4\cdot 3}$ $\quad \frac{20x}{12}-\frac{3x}{12}=\boxed{\frac{17x}{12}}$

4. $\frac{\pi}{6}+\frac{\pi\cdot 2}{3\cdot 2}$ $\quad \frac{\pi}{6}+\frac{2\pi}{6}=\frac{\cancel{3}^{1}\pi}{\cancel{6}_{2}}=\boxed{\frac{\pi}{2}}$

Radians vs. Degrees	A **radian** is a unit of angle measure based on arc length. One radian is defined as the measure of the angle formed when the radius is equivalent to the length of the intercepted arc. Recall that the circumference of a circle is $2r\pi$, therefore: $360^\circ =$ 2π ; $180^\circ =$ π	
	**Converting Degrees → Radians**	**Converting Radians → Degrees**
	mult. by $\frac{\pi}{180}$ Radians = Degrees $\cdot\left(\frac{\pi \text{ radians}}{180}\right)$	mult. by $\frac{180}{\pi}$ Degrees = Radians $\cdot\left(\frac{180}{\pi \text{ radians}}\right)$

What is a radian? https://commons.wikimedia.org/wiki/File:Circle_radians.gif#/media/File:Circle_radians.gif



$\frac{\pi}{180}=1$

$\frac{180}{\pi}=1$

Convert each degrees to radians and radians to degrees:

1. $\frac{45^\circ}{1}\cdot\frac{\pi}{180}=\frac{\cancel{45}\pi}{_4\cancel{180}}=\boxed{\frac{\pi}{4}}$

Graphing calc.
$45\div180=.25=\frac{1}{4}$
math
#1

2. $\frac{2\cancel{\pi}}{3}\cdot\frac{180}{\cancel{\pi}}=\frac{360}{3}=\boxed{120^\circ}$

3. $-\frac{11\cancel{\pi}}{6}\cdot\frac{180}{\cancel{\pi}}=\frac{-1980}{6}=\boxed{-330^\circ}$

4. $\frac{-935^\circ}{1}\cdot\frac{\pi}{180}=\frac{-935\pi}{180}=\boxed{\frac{-187\pi}{36}}$

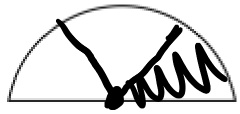
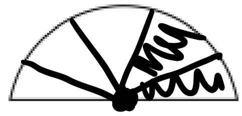
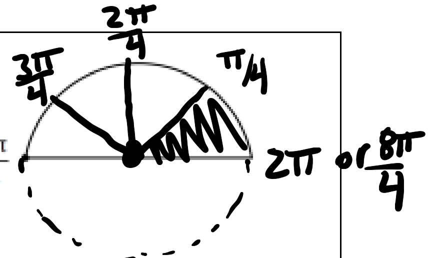
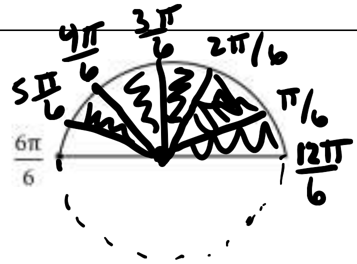
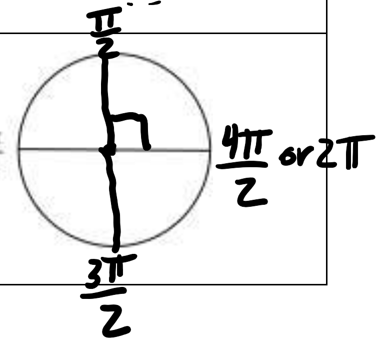
5. $\frac{7\cancel{\pi}}{4}\cdot\frac{180}{\cancel{\pi}}=\frac{1260}{4}$ $\boxed{315^\circ}$

6. $\frac{80^\circ}{1}\cdot\frac{\pi}{180}=\frac{80\pi}{180}=\boxed{\frac{4\pi}{9}}$

90° same as $\frac{\pi}{2}$ rad.

Coterminal and Reference Angles

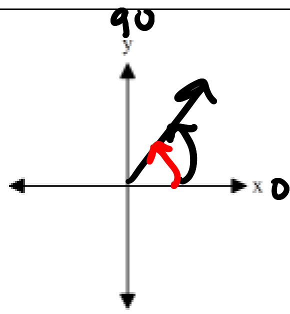
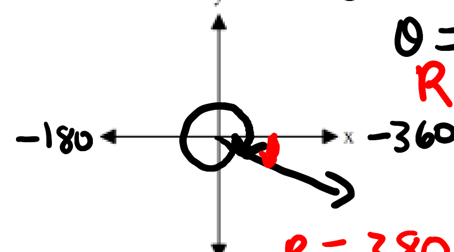
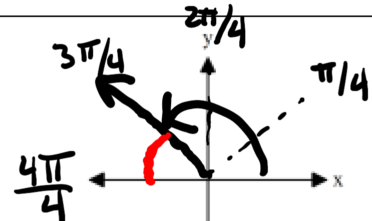
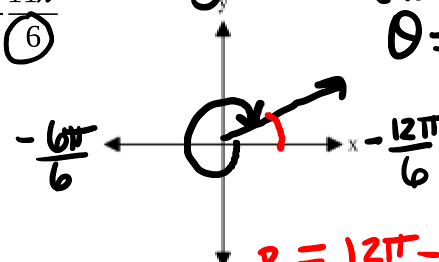
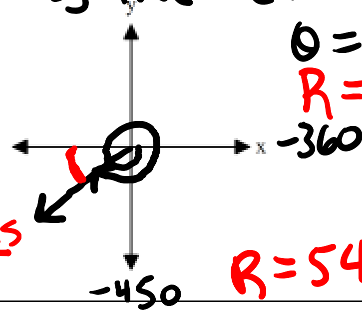
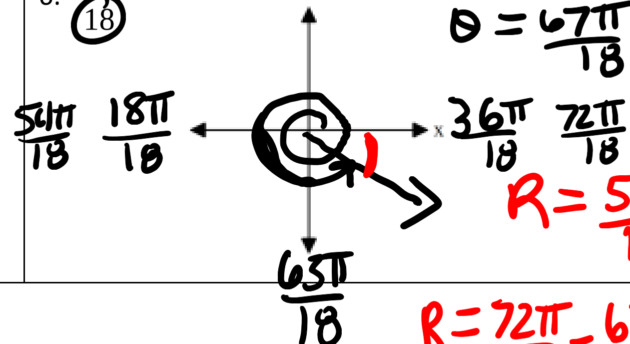
A. Shade the appropriate portion of the semi-circle.

1. $\frac{1}{3}$ 	2. $\frac{2}{5}$ 	3. $\frac{\pi}{4}$ 
4. $\frac{5\pi}{6}$ 	5. $\frac{9\pi}{8}$ 	6. $\frac{3\pi}{2}$ 

B. Drawing Angles

- Draw and Label the Given angle.
- Draw and Label the *reference angle*. (Remember- A **reference angle** is formed by the *terminal side* of the angle and the *x-axis*. This means the angle will always be less than 90°.) positive always

negatives angles clockwise

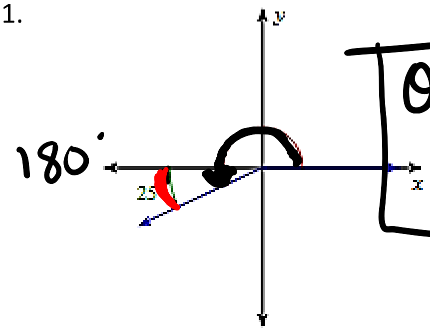
1. 60°  $\theta = 60°$ $R = 60°$	2. −380°  $\theta = -380°$ $R = 20°$ $R = 380 - 360 = 20°$
3. $\frac{3\pi}{4}$  $\theta = \frac{3\pi}{4}$ $R = \frac{\pi}{4}$ $R = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$	4. $-\frac{11\pi}{6}$ negative clockwise  $\theta = -\frac{11\pi}{6}$ $R = \frac{\pi}{6}$ $R = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$
5. −525° negative clockwise  $\theta = -525°$ $R = 15°$ $R = 540 - 525$	6. $\frac{67\pi}{18}$ counterclockwise  $\theta = \frac{67\pi}{18}$ $R = \frac{5\pi}{18}$

$R = \frac{72\pi}{18} - \frac{67\pi}{18} = \frac{5\pi}{18}$

C. Measures of Angles

Find the measure of each angle and then find the reference angle.

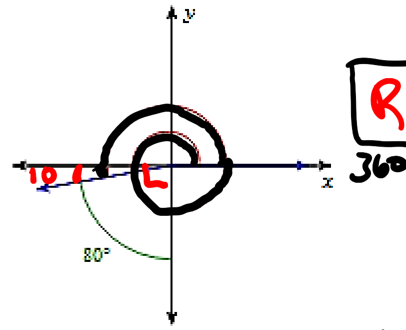
1.



$\boxed{\theta = 205^\circ \quad R = 25^\circ}$

$180 + 25 = 205^\circ$

2.

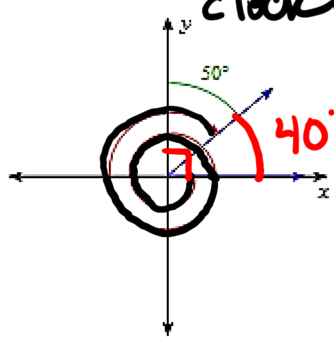


$\boxed{R = 10^\circ}$

360
180
10

$\boxed{\theta = 550^\circ}$

3. clockwise <u>negative</u>

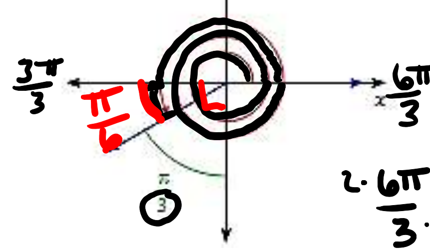


−360
−180
− 90
− 50

$\boxed{\theta = -680^\circ}$

$\boxed{R = 40^\circ}$

4. counterclockwise – positive



$\frac{3 \cdot \pi}{3 \cdot 2} - \frac{2\pi}{2 \cdot 3} = \frac{\pi}{6}$

$\frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$

$\boxed{R = \frac{\pi}{6}}$

$\frac{2 \cdot 6\pi}{3 \cdot 2}$ $\frac{12\pi}{6}$

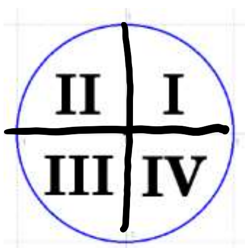
$\frac{6\pi}{3}$ $\frac{12\pi}{6}$

$\frac{2 \cdot 3\pi}{3 \cdot 2}$ $\frac{6\pi}{6}$

$+ \ \frac{\pi}{6}$ $\frac{1\pi}{6}$

$\boxed{\frac{31\pi}{6}}$

D. Determine the quadrant of each angle.



540	1. 480° 450, 180, 360 $\boxed{\text{II}}$	2. $\frac{9\pi}{4}$ $\frac{4\pi}{4}$, $\frac{8\pi}{4}$ $\boxed{\text{I}}$
	clockwise 3. $-\frac{7\pi}{3}$ $-\frac{3\pi}{3}$, $-\frac{6\pi}{3}$ $\boxed{\text{IV}}$	clockwise 4. -256° -270, -180, -90 $\boxed{\text{II}}$
counterclockwise 5. $\frac{13\pi}{6}$ $\frac{6\pi}{6}$, $\frac{12\pi}{6}$ $\boxed{\text{I}}$	negative / clockwise 6. $-\frac{19\pi}{4}$ $-\frac{12\pi}{4}$, $-\frac{20\pi}{4}$, $-\frac{4\pi}{4}$, $-\frac{8\pi}{4}$, $-\frac{16\pi}{4}$ $\boxed{\text{III}}$	counterclockwise pos. 7. $\frac{24\pi}{6} = 4\pi$ $\frac{18\pi}{6}$, $\frac{6\pi}{6}$, $\frac{12\pi}{6}$, $\frac{24\pi}{6}$ $\boxed{\text{none}}$

E. Finding *coterminal angles* between 0° and 360° **or** 0 and 2π.

Watch youtube video : https://www.youtube.com/watch?v=A8NoBcYQJ1U

Coterminal angles are the same angles with different measures. Coterminal angles share the terminal side of the angle.

What happens when you go around a circle more than once? add or subtract 360° or 2π

Fill in the blank to create an angle equal to 2π. $$2\pi = \frac{16\pi}{8} = \frac{6\pi}{3} = \frac{10\pi}{5} = \frac{32\pi}{16}$$

In fractions that are equal to 2π, **the numerator must be double the denominator.**

You can tell if the angle is bigger than 2π if the numerator is ***more*** than double the denominator.

Find a coterminal angle between $\boxed{0^\circ \text{ and } 360^\circ \textbf{ or } 0 \text{ and } 2\pi.}$

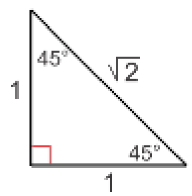
Many coterminal angles.

1. 580° $580-360=220^\circ$	2. -92° $-92+360=268^\circ$	3. $\frac{9\pi}{4}$ $\frac{9\pi}{4}-2\pi$ $\frac{9\pi}{4}-\frac{8\pi}{4}=\frac{\pi}{4}$
4. $-\frac{2\pi}{3}$ $-\frac{2\pi}{3}+2\pi$ $-\frac{2\pi}{3}+\frac{6\pi}{3}=\frac{4\pi}{3}$	5. -225° $-225+360=135^\circ$	6. $\frac{116\pi}{45}$ $\frac{116\pi}{45}-2\pi$ $\frac{116\pi}{45}-\frac{90\pi}{45}=\frac{26\pi}{45}$

SM2H 9.2 Notes – The Unit Circle

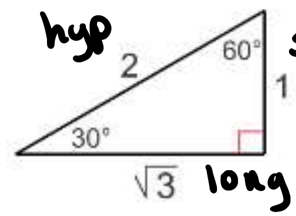
Review Special Right Triangles

In a $45° - 45° - 90°$ triangle,
the sides are in the ratio $1:1:\sqrt{2}$.



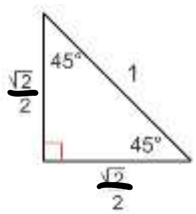
hyp = leg $\cdot \sqrt{2}$

In a $30° - 60° - 90°$ triangle,
the sides are in the ratio $1:\sqrt{3}:2$.

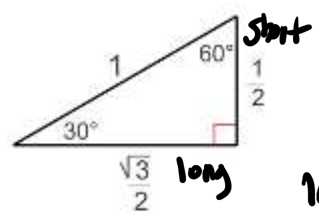


short
long = short $\cdot \sqrt{3}$
hyp = short $\cdot 2$

Now lets make the hypotenuse 1 so we'll be able to use the special right triangles in the Unit Circle. The Unit Circle is a circle where the radius is equal to 1 unit. Using the rules of special right triangles, the new side lengths are:



$$\text{leg} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



Short leg = $1 \div 2$

short leg = $\frac{1}{2}$

long = $\frac{1}{2} \cdot \frac{\sqrt{3}}{1}$

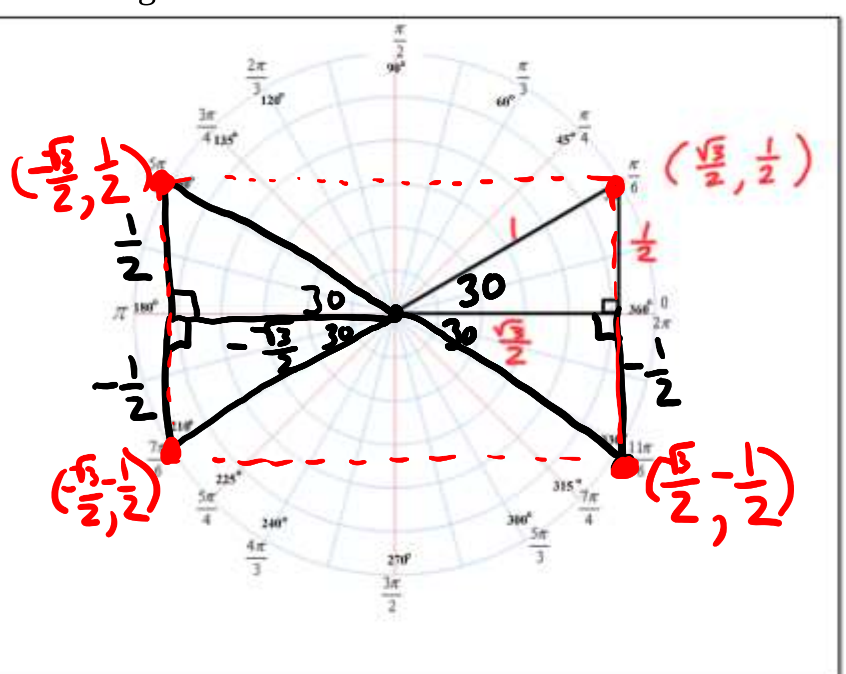
long = $\frac{\sqrt{3}}{2}$

<u>Please watch this video on understanding the Unit Circle **</u>**

https://www.youtube.com/watch?v=n6L7VkdMv2g

The 30° triangle in the Unit Circle:

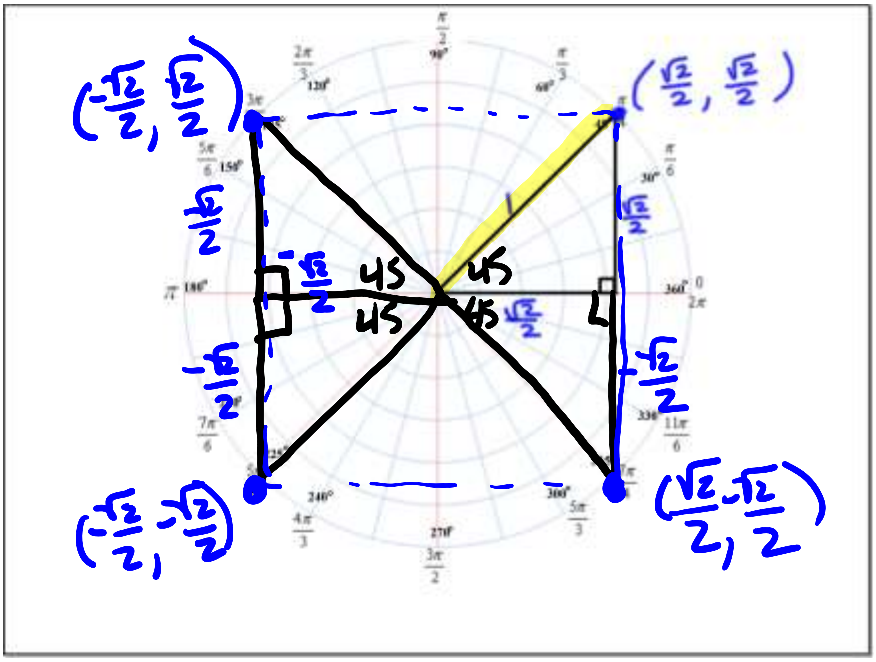
The above ratios of the $30° - 60° - 90°$ triangle were used to fill in the side lengths and coordinates of the **30°** triangles in the Unit Circle.



Sketch in the other three **30°** triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.

The 45° triangle in the Unit Circle:

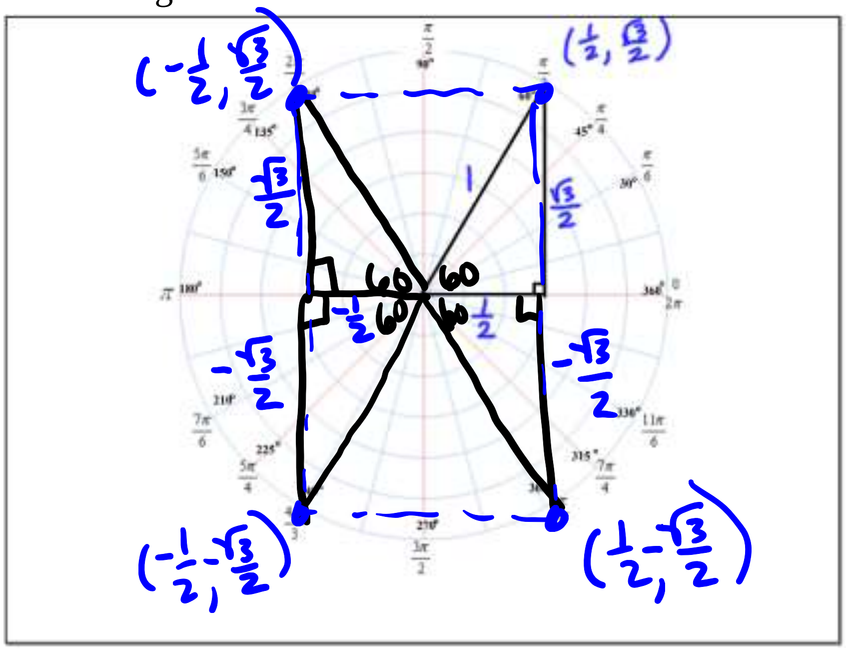
The above ratios of the $45° - 45° - 90°$ triangle were used to fill in the side lengths and coordinates of the **45°** triangle in the Unit Circle.



Sketch in the other three **45°** triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.

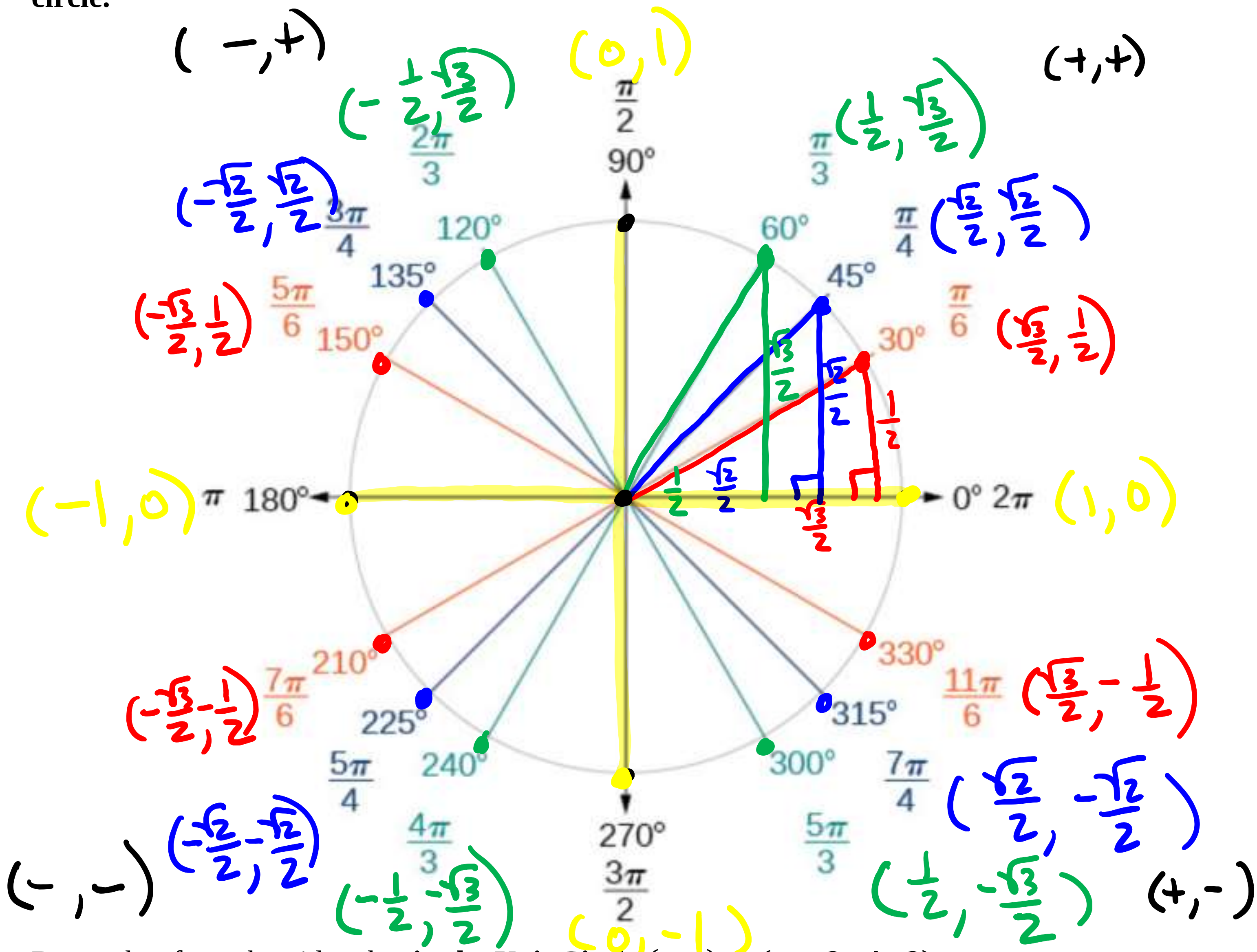
The 60° triangle in the Unit Circle:

The above ratios of the $30° - 60° - 90°$ triangle were used to fill in the side lengths and coordinates of the **60°** triangle in the Unit Circle.



Sketch in the other three **60°** triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.

Fill in the coordinates of the first quadrant of the Unit Circle and then reflect those coordinates over the x and y axes to find the coordinates of all the other angles on the unit circle.



Remember from the video that **in the Unit Circle** $(x, y) = (\boldsymbol{cos\theta}, \boldsymbol{sin\theta})$

$\boldsymbol{x = cos\theta}$ $\boldsymbol{y = sin\theta}$ $\boldsymbol{\frac{y}{x} = tan\theta} = \frac{\sin\theta}{\cos\theta}$ no decimals/no calc.

Using the Unit Circle find the exact value of:

a) $\cos 60° = \frac{1}{2}$

d) $sin\ 270° = -1$

g) $tan\frac{2\pi}{3} =$ $\frac{\sqrt{3}}{2} \div -\frac{1}{2} = \frac{\sqrt{3}}{\cancel{2}} \cdot -\frac{\cancel{2}}{1} = -\sqrt{3}$

b) $sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

e) $cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

h) $\tan 90° = \frac{1}{0} =$ undefind

c) $\tan 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

f) $\tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \div$

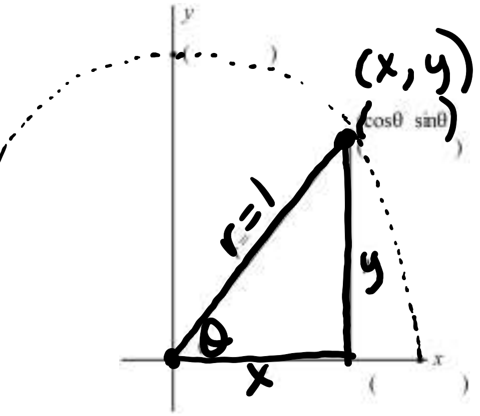
$\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

i) $tan\frac{7\pi}{6} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$-\frac{1}{2} \div -\frac{\sqrt{3}}{2} = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{-\sqrt{3}} = \frac{-1\sqrt{3}}{-\sqrt{3}\sqrt{3}}$ $\boxed{\frac{\sqrt{3}}{3}}$

SM2H 9.3 Notes Using the Unit Circle

SOH CAH TOA

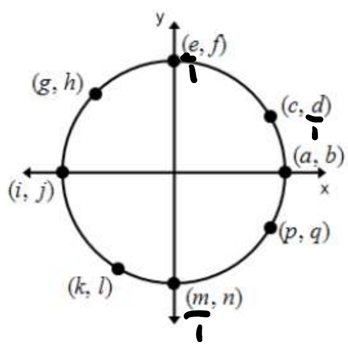


$\cos\theta = \frac{x}{1} = x$ $\sec\theta = \frac{1}{x}$ or $\frac{1}{\cos\theta}$ reciprocal cosθ

$\sin\theta = \frac{y}{1} = y$ $\csc\theta = \frac{1}{y}$ or $\frac{1}{\sin\theta}$ reciprocal of sinθ

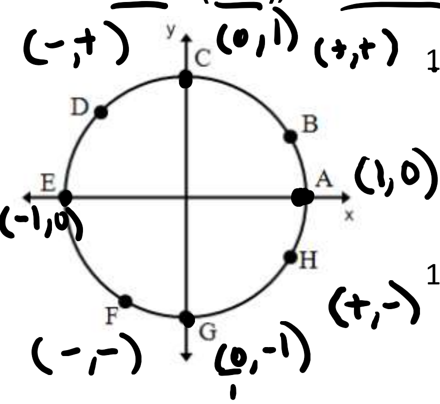
$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$ or $\frac{\cos\theta}{\sin\theta}$ reciprocal of tanθ

Refer to the diagram below. Give the letter or letters that could stand for the function value.



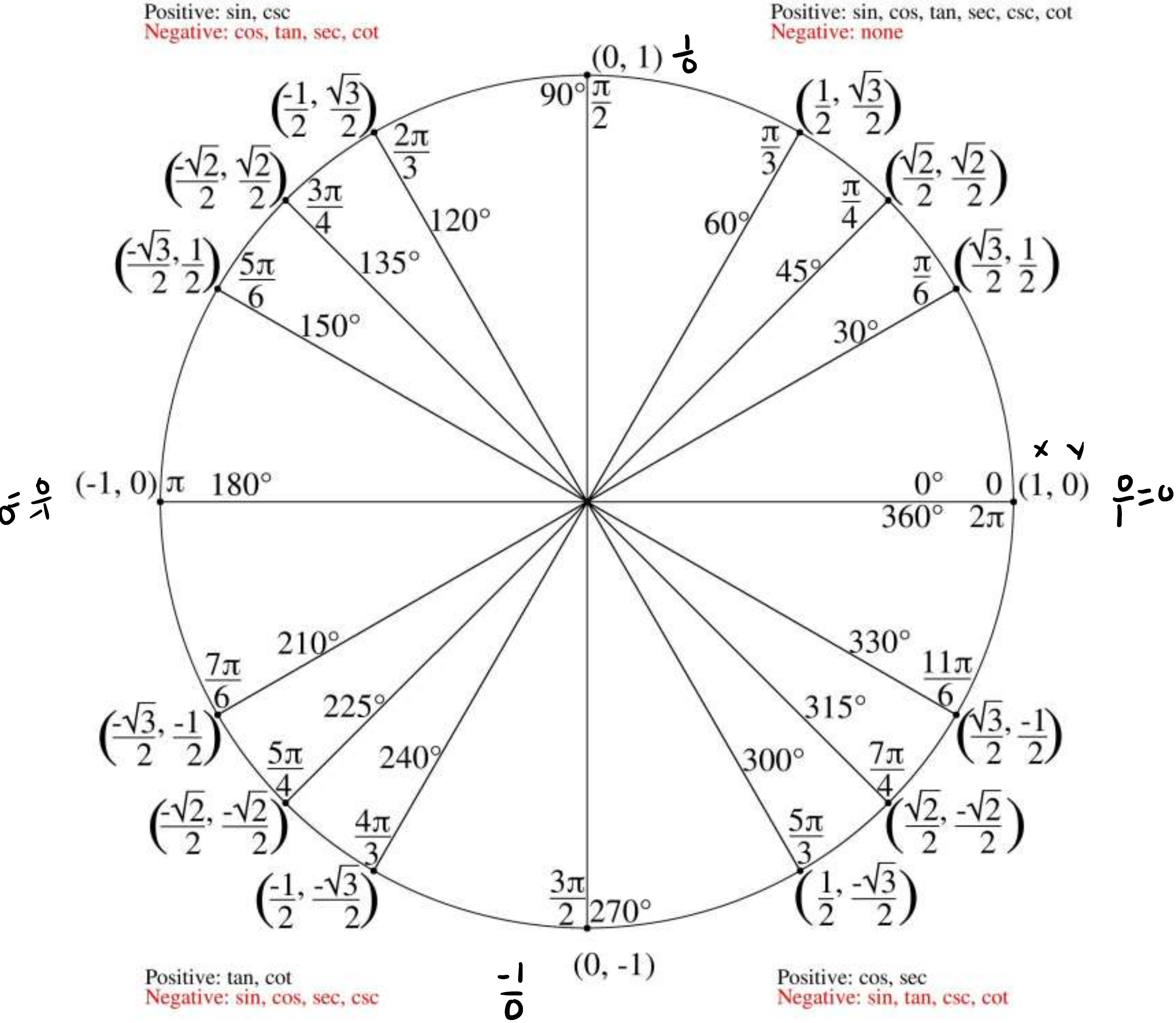
1. $\cos 180°$ i
2. $\tan 270°$ $\frac{n}{m}$
3. $\sin\frac{11\pi}{6}$ q
4. $\sec 270°$ $\frac{1}{m}$
5. $\csc 30°$ $\frac{1}{d}$
6. $\cos 135°$ g
7. $\cot 330°$ $\frac{p}{q}$
8. $\sec\frac{\pi}{2}$ $= \frac{1}{e}$
9. $\tan\frac{4\pi}{3}$ $\frac{l}{k}$
10. $\cos -\frac{11\pi}{6}$ c
11. $\sin -\frac{2\pi}{3}$ l
12. $\cot -\frac{5\pi}{4}$ $\frac{g}{h}$

Refer to the diagram below. For the indicated point, tell if the value for $\sin\theta$ *or* $\cos\theta$ is positive, negative, neither (zero), or undefined. $\frac{1}{0}$



13. $\sin D$ positive
14. $\sec B$ positive
15. $\cos G$ neither
16. $\cot A$ $\frac{1}{0}$ undefined
17. $\cos F$ negative
18. $\cos A$ Positive
19. $\sin H$ negative
20. $\csc H$ negative
21. $\cot F$ $\frac{-}{-}$ positive
22. $\sec G$ $\frac{1}{0}$ undefined

The Unit Circle



no dec. / no calc.

Use the unit circle to find the exact value of each ~~trigo~~nometric function.

1. $\sin 30°$ $\boxed{\frac{1}{2}}$

2. $\cos 30°$ $\boxed{\frac{\sqrt{3}}{2}}$

3. $\tan 30°$ $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\div$

$\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1\sqrt{3}}{\sqrt{3}\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

4. $\csc 30°$ $\boxed{2}$

5. $\sec 30°$ $\frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

6. $\cot 30°$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\div$

$\frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \boxed{\sqrt{3}}$

7. $\cos\frac{5\pi}{6}$ $\boxed{-\frac{\sqrt{3}}{2}}$

8. $\csc\frac{3\pi}{2}$ $\boxed{-1}$

9. $\cot\frac{11\pi}{6}$

$\frac{\sqrt{3}}{2} \div \frac{-1}{2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{-\cancel{2}}{1} = \boxed{-\sqrt{3}}$

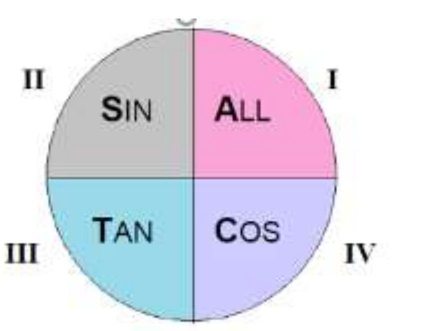
10. $\tan 90°$ $\boxed{\text{undefined}}$ $\frac{1}{0}$

11. $\sec\frac{5\pi}{3}$ $\boxed{2}$

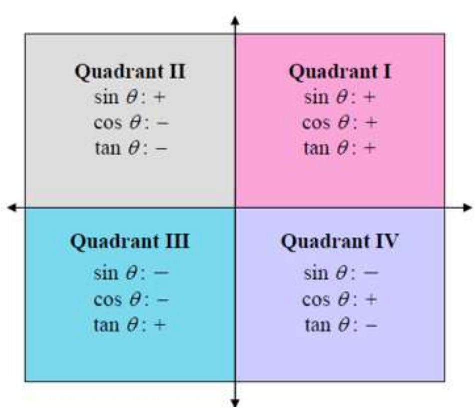
12. $\cot\frac{\pi}{2}$ $\frac{0}{1} = \boxed{0}$

Signs of the Trig Functions: Where are the trig functions Positive?

S | A
T | C



All Students Take Calculus



Name the Quadrant in which the angle lies.

pos neg

1) $\tan \theta > 0$, $\sin \theta < 0$

A) I B) II C) III D) IV

neg neg

2) $\cos \theta < 0$, $\sin \theta < 0$

A) I B) II C) III D) IV

pos neg

3) $\sin \theta > 0$, $\cos \theta < 0$

A) I B) II C) III D) IV

neg pos

4) $\tan \theta < 0$, $\cos \theta > 0$

A) I B) II C) III D) IV

pos. pos.

5) $\sin \theta > 0$, $\cos \theta > 0$

A) I B) II C) III D) IV

neg neg

6) $\cos \theta < 0$, $\tan \theta < 0$

A) I B) II C) III D) IV

neg neg

7) $\tan \theta < 0$, $\sin \theta < 0$

A) I B) II C) III D) IV

pos neg

8) $\cos \theta > 0$, $\sin \theta < 0$

A) I B) II C) III D) IV

Find the exact measures of the angles $[0°, 360°)$ using the unit circle.

where is sin pos.

1. $\sin \theta = \frac{\sqrt{3}}{2}$ S A T C

wher y value $\frac{\sqrt{3}}{2}$

60° and 120°

$\frac{y}{x}$

2. $\tan \theta = 0°$

0 and 180°

3. $\cos \theta = -\frac{\sqrt{2}}{2}$ S A T C

where is x value $-\frac{\sqrt{2}}{2}$

135°, 225°

4. $\sin \theta = -1$

where is y value = -1

270°

SM2H Notes 9.4 Trigonometric Identities

Trigonometry Function Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

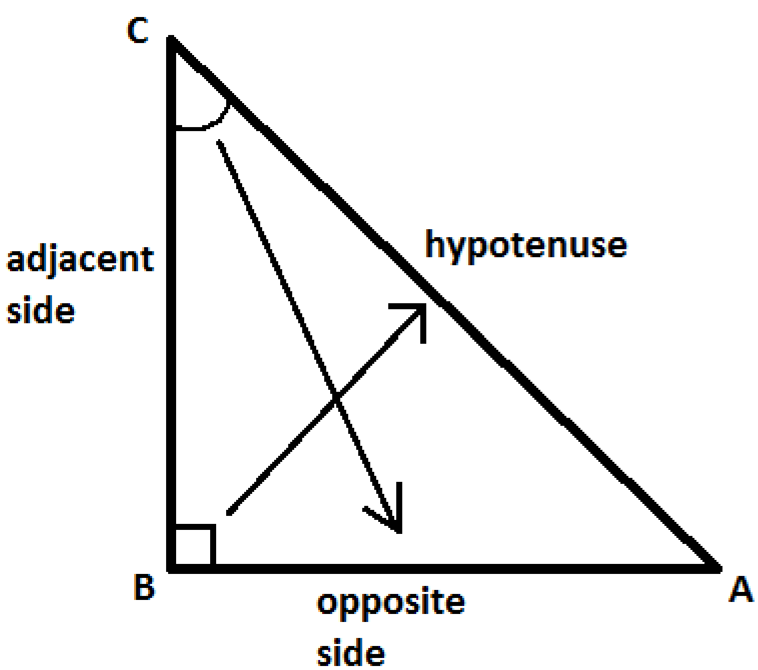
Reciprocal Indentities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \boxed{\csc\theta = \frac{1}{\sin\theta}}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \boxed{\sec\theta = \frac{1}{\cos\theta}}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \boxed{\cot\theta = \frac{1}{\tan\theta}}$$

SOH CAH TOA



$$\sin\ c = \frac{\text{opposite}}{\text{hypotenuse}} =$$

$$\cos\ c = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\ c = \frac{\text{opposite}}{\text{adjacent}} =$$

$$\cot\ c = \frac{\text{adjacent}}{\text{opposite}}$$ reciprocal tanc

$$\sec\ c = \frac{\text{hypotenuse}}{\text{adjacent}}$$ reciprocal cosc

$$\csc\ c = \frac{\text{hypotenuse}}{\text{opposite}}$$ reciprocal sinC

Review

1. Multiply $\frac{\cancel{15}^{3}}{\cancel{16}_{4}} \cdot \frac{\cancel{4}^{1}}{\cancel{25}_{5}} = \frac{3}{20}$

2. Add $\frac{3 \cdot 3}{3 \cdot 8} + \frac{5 \cdot 4}{6 \cdot 4}$

$$\frac{9}{24} + \frac{20}{24} = \frac{29}{24}$$

* 3. Divide $\frac{\frac{8}{25}}{\frac{2}{5}} \div$

$$\frac{8}{25} \div \frac{2}{5} = \frac{\cancel{8}^{4}}{\cancel{25}_{5}} \cdot \frac{\cancel{5}^{1}}{\cancel{2}_{1}} = \frac{4}{5}$$

SOH CAH TOA

Prove the identities, using opposite, adjacent, and hypotenuse.

4. $\sin\theta = \frac{1}{\csc\theta}$

$\frac{opp}{hyp} = \frac{1}{\frac{hyp}{opp}} \div$

$1 \div \frac{hyp}{opp}$

$1 \cdot \frac{opp}{hyp}$

✓ $\frac{opp}{hyp} = \frac{opp}{hyp}$

5. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{opp}{adj.} = \frac{\frac{opp}{hyp}}{\frac{adj.}{hyp}} \div$

$\frac{opp}{hyp} \div \frac{adj.}{hyp}$

$\frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj.} \rightarrow \frac{opp}{adj.} = \frac{opp}{adj.}$ ✓

Write each expression in terms of sine and/or cosine, then simplify.

6. $\sec x \cdot \sin x$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$

7. $\frac{\tan x}{\sin x}$

$\frac{\frac{\sin x}{\cos x}}{\sin x} \div$

$\frac{\sin x}{\cos x} \div \frac{\sin x}{1}$

$\frac{\cancel{\sin x}^{1}}{\cos x} \cdot \frac{1}{\cancel{\sin x}_{1}} = \frac{1}{\cos x} = \sec x$

Prove the trigonometric identities.

8. $\cos x \sec x = 1$

$\frac{\cancel{\cos x}^{1}}{1} \cdot \frac{1}{\cancel{\cos x}_{1}} = 1$

$1 = 1$ ✓

9. $\cos\theta \csc\theta = \cot\theta$

$\frac{\cos\theta}{1} \cdot \frac{1}{\sin\theta} = \cot\theta$

$\frac{\cos\theta}{\sin\theta}$

$\cot\theta = \cot\theta$ ✓

SM2H Notes 9.5 Trigonometric Identities Day 2

Review Concepts

1. $\dfrac{\frac{1}{3}}{5}$ ÷

$\frac{1}{3} \div \frac{5}{1}$

$\frac{1}{3} \cdot \frac{1}{5} = \frac{1}{15}$

2. $(x+6)(x-6)$

$x^2 - \cancel{6x} + \cancel{6x} - 36$

$x^2 - 36$

3. $\dfrac{y\cdot 3}{y\cdot x} + \dfrac{5\cdot x}{y\cdot x}$

$\frac{3y}{xy} + \frac{5x}{xy} = \frac{3y+5x}{xy}$

$x + x = 2x$

4. $\sin x + \sin x = 2\sin x$

$x \cdot x = x^2$

5. $\sin x \cdot \sin x = \sin^2 x$

6. $(1-\cos x)(1+\cos x) =$

$1 + \cancel{\cos x} - \cancel{\cos x} - \cos^2 x$

$1 - \cos^2 x$

7. $\boldsymbol{tan^2 x} = \mathbf{tan}\,\boldsymbol{x} \cdot \mathbf{tan}\,\boldsymbol{x}$ so $\mathbf{tan}\,\boldsymbol{x} \cdot \mathbf{tan}\,\boldsymbol{x} = \dfrac{\sin x \cdot \sin x}{\cos x \cdot \cos x}$ or $\boldsymbol{tan^2 x} = \dfrac{sin^2 x}{cos^2 x}$

Factor out the GCF

$x^2 + x$

8. $5x^3 - 25x^2 + 5x$

$5x(x^2 - 5x + 1)$

9. $\sin^2 x + \sin x$

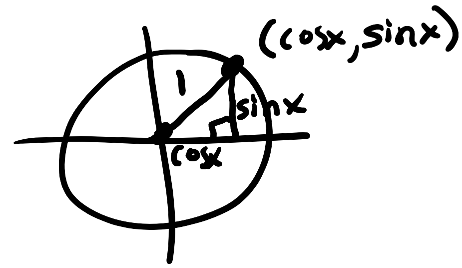
$\sin x(\sin x + 1)$

The Pythagoren Identity:

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$



If we divide each term of the fundamental identity by $\sin^2 x$ or $\cos^2 x$, we can derive two more identities. These are called Pythagorean Identities because they are related to the Pythagorean Theorem:

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

You can rewrite the Pythagorean Identities by using the addition property of addition and subtraction.

$\sin^2 x + \cos^2 x = 1$

✓ $\sin^2 x = 1 - \cos^2 x$

✓ $\cos^2 x = 1 - \sin^2 x$

$1 + \cot^2 x = \csc^2 x$

✓ $1 = \csc^2 x - \cot^2 x$

✓ $\cot^2 x = \csc^2 x - 1$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

A General Strategy for Proving Identities

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.
4. Write a single rational expression as a sum of two rational expressions.
5. Combine a sum of two rational expressions into a single rational expression.
6. If both sides simplify to a third expression, then the equation is an identity.
7. NO solving (NO whatever you do to one side do to the other)

Prove each identity.

1. $\dfrac{\tan x}{\sin x} = \sec x$

$\dfrac{\frac{\sin x}{\cos x}}{\sin x}$ ÷

$\dfrac{\sin x}{\cos x} \div \dfrac{\sin x}{1}$

$\dfrac{\cancel{\sin x}}{\cos x} \cdot \dfrac{1}{\cancel{\sin x}}$

$\dfrac{1}{\cos x}$

✓ $\sec x = \sec x$

Factoring GCF

2. $\sin x + \sin x \cdot \cot^2 x = \csc x$

$\sin x\,(1 + \cot^2 x)$

$\sin x \cdot \csc^2 x$

$\dfrac{\cancel{\sin x}}{1} \cdot \dfrac{1}{\sin^{\cancel{2}} x}$

$\dfrac{1}{\sin x}$

$\csc x = \csc x$

3. $\sin\theta \cdot \cos\theta \cdot \tan\theta = 1 - \cos^2\theta$

$\dfrac{\sin\theta}{1} \cdot \dfrac{\cancel{\cos\theta}}{1} \cdot \dfrac{\sin\theta}{\cancel{\cos\theta}}$

check identity $\sin^2\theta = 1 - \cos^2\theta$

✓ $1 - \cos^2\theta = 1 - \cos^2\theta$

4. $\cos x + \sin x \cdot \tan x = \sec x$

$\cos x + \dfrac{\sin x}{1} \cdot \dfrac{\sin x}{\cos x}$

combine fractions

$\dfrac{\cos x \cos x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x}$

$\dfrac{1}{\cos x}$

$\sec x = \sec x$

common denominator

5. $\dfrac{\sin x \,\sin x}{\sin x\,(1+\cos x)} + \dfrac{(1+\cos x)(1+\cos x)}{\sin x\,(1+\cos x)} = 2\csc x$

$\dfrac{\sin^2 x}{\sin x\,(1+\cos x)} + \dfrac{1 + \cancel{\cos x + \cos x} + \cos^2 x}{\sin x\,(1+\cos x)}$ $2\cos x$

1

$\dfrac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{\sin x\,(1+\cos x)}$

GCF $\dfrac{2 + 2\cos x}{\sin x\,(1+\cos x)}$

$\dfrac{2\,\cancel{(1+\cos x)}}{\sin\,\cancel{(1+\cos x)}} = \dfrac{2}{1} \cdot \dfrac{1}{\sin x}$

$2\csc x = 2\csc x$

6. $\dfrac{\sin^2 x}{\cos x} = \sec x - \cos x$

$\dfrac{1 - \cos^2 x}{\cos x}$

$\dfrac{1}{\cos x} - \dfrac{\cos^{\cancel{2}} x}{\cancel{\cos x}}$

$\sec x - \cos x = \sec x - \cos x$