Worked Out Solutions

* you have proved the identity when both sides of equation are equal

① $\dfrac{\cot x}{\cos x} = \csc x$

$\dfrac{\frac{\cos x}{\sin x}}{\cos x} = \csc x$

$\dfrac{\cos x}{\sin x} \div \cos x = \csc x$

$\dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x} = \csc x$

$\dfrac{1}{\sin x}$

$\csc x = \csc x$

② $\cot x + \tan x = \csc x \sec x$

$\dfrac{\cos x}{\cos x} \cdot \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{\sin x} = \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

*add fractions (get common denominator)

$\dfrac{\cos^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} = \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

* add numerators

$\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

Pythag. Identity

$\dfrac{1}{\sin x \cos x} = \dfrac{1}{\sin x \cos x}$

* Remember Pythag. Identity
$\sin^2 x + \cos^2 x = 1$
so
$\underline{\sin^2 x = 1 - \cos^2 x}$

③ $\sin\theta + \dfrac{\cot\theta}{\sec\theta} = \csc\theta$

* Replace trig functions w/ $\sin\theta$ & $\cos\theta$

$\sin\theta + \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}} \cdot \dfrac{\cos\theta}{1} = \csc\theta$

Add Fractions get common den.

$\dfrac{\sin\theta \cdot \sin\theta}{\sin\theta} + \dfrac{\cos^2\theta}{\sin\theta} = \csc\theta$

Pythag. Identity

$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \csc\theta$

$\dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta}$

④ $\dfrac{1 - \cos^2 x}{\cos x} \cdot \csc x = \tan x$

$\dfrac{\sin^2 x}{\cos x} \cdot \dfrac{1}{\sin x} = \tan x$ *simplify

$\dfrac{\sin x}{\cos x} = \tan x$

$\tan x = \tan x$

⑤ $\sin^2\theta (1 + \cot^2\theta) = 1$

$\sin^2\theta (\csc^2\theta) = 1$

$\dfrac{\sin^2\theta}{1} \cdot \dfrac{1}{\sin^2\theta} = 1$

$1 = 1$

* Remember Pythg. Identities look in notes
$\underline{1 + \cot^2\theta = \csc^2\theta}$

* Replace trig functions w/ $\sin x$ & $\cos x$

⑥ $\sec x = \dfrac{\tan x}{\csc x} + \cos x$

$\dfrac{1}{\cos x} = \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\sin x}} + \cos x$

same as mult. by reciprocal

$\dfrac{1}{\cos x} = \dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{1} + \cos x$

$\dfrac{1}{\cos x} = \dfrac{\sin^2 x}{\cos x} + \dfrac{\cos x}{1} \cdot \dfrac{\cos x}{\cos x}$

* add fractions/get common den.

$\dfrac{1}{\cos x} = \dfrac{\sin^2 x}{\cos x} + \dfrac{\cos^2 x}{\cos x}$

$\dfrac{1}{\cos x} = \dfrac{\sin^2 x + \cos^2 x}{\cos x}$

$\dfrac{1}{\cos x} = \dfrac{1}{\cos x}$