

9.4 Trigonometric Identities 2018-19

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Prove the reciprocal identities, using opposite, adjacent and hypotenuse.

$$1. \csc x = \frac{1}{\sin x}$$

$$\frac{\text{hyp}}{\text{opp}} = \frac{1}{\frac{\text{opp}}{\text{hyp}}}$$

$$2. \cot x = \frac{1}{\tan x}$$

$$\frac{\text{adj}}{\text{opp}} = \frac{1}{\frac{\text{opp}}{\text{adj}}}$$

$$3. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{\text{adj}}{\text{opp}} = \frac{\frac{\text{adj}}{\text{hyp}}}{\frac{\text{opp}}{\text{hyp}}}$$

$$\frac{\text{hyp}}{\text{opp}} = 1 \div \frac{\text{opp}}{\text{hyp}}$$

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$$\frac{\text{adj}}{\text{opp}} = \frac{\text{adj}}{\text{hyp}} \div \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\text{hyp}}{\text{opp}} = 1 \cdot \frac{\text{hyp}}{\text{opp}} \Rightarrow \frac{\text{hyp} = \text{hyp}}{\text{opp} = \text{opp}}$$

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$$\frac{\text{adj}}{\text{opp}} = \frac{\text{adj}}{\text{hyp}} \cdot \frac{\text{hyp}}{\text{opp}} \Rightarrow \frac{\text{adj} = \text{adj}}{\text{opp} = \text{opp}}$$

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Write each expression in terms of sine, cosine, or secant, and then simplify.

4. $\sin x \cdot \sec x$

$$\frac{\sin x}{1} \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x} = \tan x$$

5. $\frac{\csc x}{\cot x}$

$$\frac{\csc x}{\cot x} = \frac{1}{\sin x} \div \frac{\cos x}{\sin x}$$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$= \sec x$$

6. $\tan x \cdot \csc x \cdot \cos x$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\cos x}{1} = 1$$

Prove the trigonometric identities.

7. $\sec x \cot x = \csc x$

$$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

or

$$\csc x = \csc x$$

8. $\sin x \sec x = \tan x$

$$\sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x} = \frac{\sin x}{\cos x}$$

$$\tan x = \tan x$$

9. $-\tan x \cos x = -\sin x$

$$-\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = -\sin x$$

$$-\sin x = -\sin x$$

10. $\csc x \sin x = \cot x \tan x$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{1} = \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}$$

$$1 = 1$$