SM2H 7.7 Quadrilaterals Day 2 Notes – Classifying Quadrilaterals in the Coordinate Plane

Graph and label the quadrilateral

Decide whether the quadrilateral is a PARALLELOGRAM using one of the following tests:

1. **slope formula:** Find the slope of all four sides to see if opposite sides are parallel (same slope).
2. **distance formula:** Find the length of all four sides to see if opposite sides are congruent.
3. **midpoint formula:** Find the midpoints of the diagonals to see if diagonals bisect each other. (If the midpoints are the same, the diagonals bisect each other.)

If the quadrilateral is a parallelogram:

1. **Decide whether the parallelogram is a RECTANGLE using one of the following tests:**
   a. **Slope formula:** See if consecutive sides are perpendicular (slopes are negative reciprocals—opposite signs and fraction flipped upside down).
   b. **Distance formula:** Find the length of both diagonals to see if they are congruent.
2. **Decide whether the parallelogram is a RHOMBUS using one of the following tests:**
   a. **Distance formula:** Find the length of all four sides to see if they are all congruent.
   b. **Slope formula:** Find the slope of the diagonals to see if they are perpendicular (slopes are negative reciprocals).
3. **Decide whether the parallelogram is a SQUARE:**
   a. If the parallelogram is both a rhombus and a rectangle, then it is a square.

If the quadrilateral is not a parallelogram:

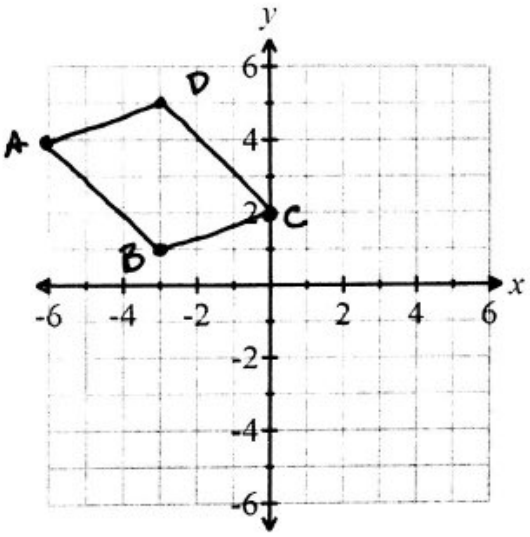
1. **Decide whether the quadrilateral is aTRAPEZOID:**
   a. **Slope Formula:** Find the slope of all four sides to see if one pair of opposite sides is parallel (same slope).
2. **If the quadrilateral is a trapezoid, check to see whether it is ISOSCELES using one of the following tests:**
   a. **Distance Formula:** Find the lengths of the non-parallel sides to see if they are congruent.
   b. **Distance Formula:** Find the lengths of the diagonals to see if they are congruent.

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ **Distance:** $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **Midpoint:** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ **Distance:** $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **Midpoint:** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Decide whether the quadrilateral with vertices at $A(-6,4)$, $B(-3,1)$, $C(0,2)$, and $D(-3,5)$ is a parallelogram.

Graph and label the quadrilateral



slope = rise ↕ / run ↔

Method 1: Slope Formula (easiest method)

Slope of $\overline{AB}$: $\frac{3}{-3} = -1$

Slope of $\overline{BC}$: $\frac{1}{3}$

Slope of $\overline{CD}$: $\frac{3}{3} = -1$

Slope of $\overline{DA}$: $\frac{1}{3}$

Is $\overline{AB} \parallel \overline{CD}$? yes AB & CD have same slope -1

Is $\overline{BC} \parallel \overline{DA}$? yes BC & DA have same slope $\frac{1}{3}$

Is $ABCD$ a parallelogram? Why or why not?

yes - Both pairs of opposite sides are parallel

Method 2: Distance Formula

Length of $\overline{AB}$: $\sqrt{(-4)^2 + (-3+6)^2} = \sqrt{18}$

Length of $\overline{BC}$: $\sqrt{(2-1)^2 + (0+3)^2} = \sqrt{10}$

Length of $\overline{CD}$: $\sqrt{(5-2)^2 + (-3-0)^2} = \sqrt{18}$

Length of $\overline{DA}$: $\sqrt{(5-4)^2 + (-3+6)^2} = \sqrt{10}$

Is $\overline{AB} \cong \overline{CD}$? yes AB & CD equal lengths $\sqrt{18}$

Is $\overline{BC} \cong \overline{DA}$? yes BC & DA equal lengths $\sqrt{10}$

Is $ABCD$ a parallelogram? Why or why not?

yes - opposite sides are $\cong$

Method 3: Midpoint Formula

Midpoint of $\overline{AC}$: $\left(\frac{-6+0}{2}, \frac{4+2}{2}\right)$

$(-3, 3)$

Midpoint of $\overline{BD}$: $\left(\frac{-3+-3}{2}, \frac{1+5}{2}\right)$

$(-3, 3)$

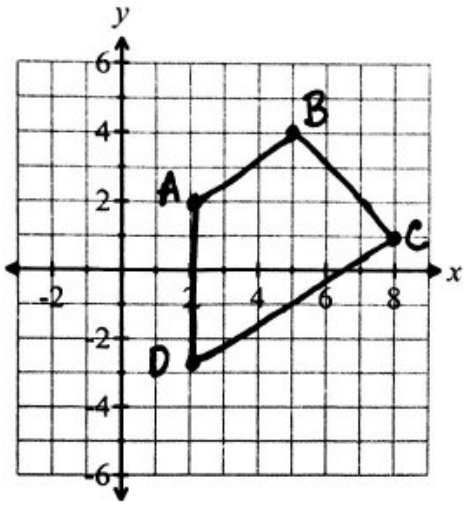
Are the midpoints of the diagonals the same? yes

Is $ABCD$ a parallelogram? Why or why not?

yes. If the midpoints are the same, the diagnols bisect each other

Decide whether the quadrilateral with vertices at $A(2,2)$, $B(5,4)$, $C(8,1)$, and $D(2,-3)$ is a parallelogram.

Graph and label the quadrilateral



Method 1: Slope Formula

Slope of $\overline{AB}$: $2/3$

Slope of $\overline{BC}$: -1

Slope of $\overline{CD}$: $\frac{4}{6} = 2/3$

Slope of $\overline{DA}$: $\frac{5}{0}$ undefined

Is $\overline{AB} \parallel \overline{CD}$? yes

Is $\overline{BC} \parallel \overline{DA}$? NO

Is $ABCD$ a parallelogram? Why or why not?

NO - Both pairs of opposite sides not parallel

Method 2: Distance Formula

Length of $\overline{AB}$: $\sqrt{(4-2)^2+(5-2)^2} = \sqrt{13}$

Length of $\overline{BC}$: $\sqrt{(1-4)^2+(8-5)^2} = \sqrt{18}$

Length of $\overline{CD}$: $\sqrt{(-3-1)^2+(2-8)^2} = \sqrt{52}$

Length of $\overline{DA}$: 5

Is $\overline{AB} \cong \overline{CD}$? NO

Is $\overline{BC} \cong \overline{DA}$? NO

Is $ABCD$ a parallelogram? Why or why not?

NO - opposite sides not congruent

Method 3: Midpoint Formula

Midpoint of $\overline{AC}$: $\left(\frac{2+8}{2}, \frac{2+1}{2}\right)$

$(5, 3/2)$

Midpoint of $\overline{BD}$: $\left(\frac{5+2}{2}, \frac{4+-3}{2}\right)$

$(7/2, 1/2)$

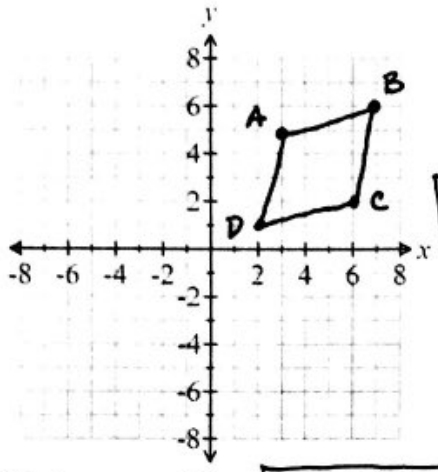
Are the midpoints of the diagonals the same? no

Is $ABCD$ a parallelogram? Why or why not?

no If midpoints are not the same, the diagonals bisect each other.

Graph and label each quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral. You may use any tests you want, but you must show all your work!

1. $A(3,5)$, $B(7,6)$, $C(6,2)$, $D(2,1)$



Rhombus

Distance AB $\sqrt{(6-5)^2+(7-3)^2} = \sqrt{17}$

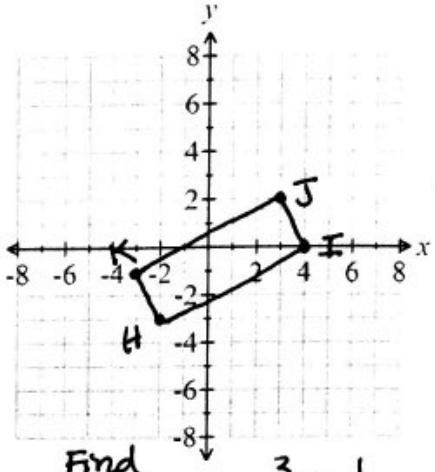
BC $\sqrt{(2-6)^2+(6-7)^2} = \sqrt{17}$

CD $\sqrt{(1-2)^2+(2-6)^2} = \sqrt{17}$

AD $\sqrt{(1-5)^2+(2-3)^2} = \sqrt{17}$

all 4 sides $\cong$

2. $H(-2,-3)$, $I(4,0)$, $J(3,2)$, $K(-3,-1)$



Rectangle

Find Slopes $KJ = \frac{3}{6} = \frac{1}{2}$

$KH = \frac{2}{-1} = -2$

slopes are opposite reciprocals

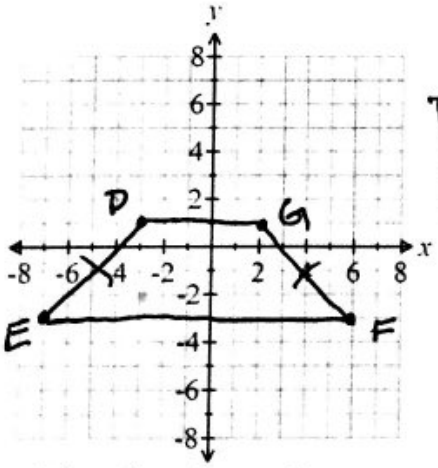
$HI = \frac{3}{6} = \frac{1}{2}$

$IJ = \frac{2}{-1} = -2$

slopes are opposite reciprocals

* consecutive sides are perperdicular

3. $D(-3,1)$, $E(-7,-3)$, $F(6,-3)$, $G(2,1)$



Isosceles Trapezoid

Slopes $DG = 0$

$EF = 0$

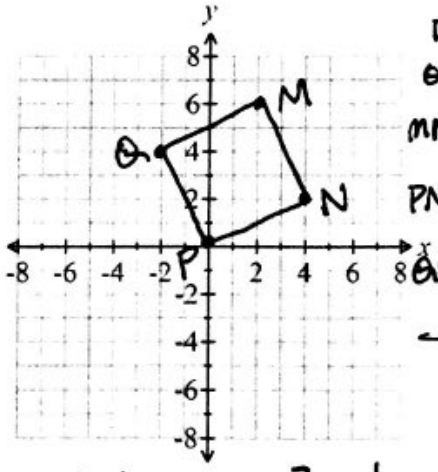
$DG \parallel EF$ same slope

Distance $DE \sqrt{(-3-1)^2+(-7+3)^2} = \sqrt{32}$

$GF \sqrt{(1+3)^2+(2-6)^2} = \sqrt{32}$

legs are $\cong$

4. $M(2,6)$, $N(4,2)$, $P(0,0)$, $Q(-2,4)$



Distance

QM $\sqrt{(4-6)^2+(-2-2)^2} = \sqrt{}$

MN $\sqrt{(2-6)^2+(4-2)^2} = \sqrt{}$

PN $\sqrt{(0-2)^2+(0-4)^2} = \sqrt{}$

QP $\sqrt{(4-0)^2+(-2-0)^2} = \sqrt{}$

Square

Find slopes of sides $QM = \frac{2}{4} = \frac{1}{2}$

$MN = \frac{4}{-2} = -2$

slopes opposite reciprocals

$PN = \frac{2}{4} = \frac{1}{2}$

$QP = \frac{4}{-2} = -2$

slopes opposite reciprocals

* consecutive sides perpendicular

* Equal sides