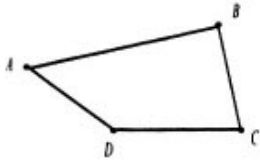


SM2H 7.6 Quadrilaterals Notes

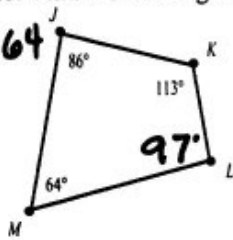
Quadrilateral Interior Angles Theorem: The measures of the interior angles of a quadrilateral add up to 360° .



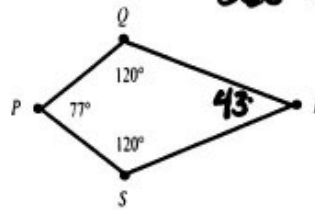
$$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$$

Examples: Find the missing angle measures in each quadrilateral.

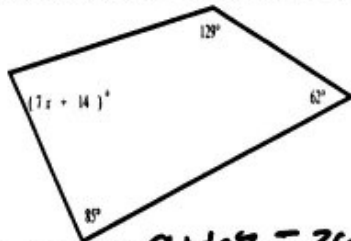
$$360 - 86 - 113 - 64 = 97$$



$$360 - 120 - 120 - 77 = 43$$



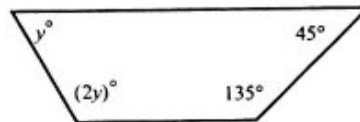
Examples: Find the value of the variable in each quadrilateral.



$$7x + 14 + 85 + 129 + 62 = 360$$

$$7x + 290 = 360$$

$$7x = 70 \quad \boxed{x = 10}$$



$$2y + y + 45 + 135 = 360$$

$$3y + 180 = 360$$

$$3y = 180$$

$$\boxed{y = 60}$$

Parallelogram: A quadrilateral with two pairs of parallel sides.

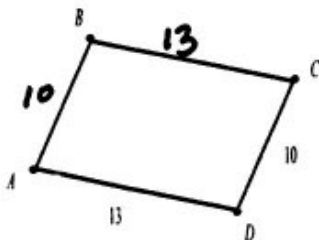
Properties:

- Opposite sides are parallel. (Definition)
- Opposite sides are congruent. (Theorem)
- Opposite angles are congruent. (Theorem)
- Consecutive angles are supplementary. (Theorem)
- Diagonals bisect each other. (Theorem)



Examples:

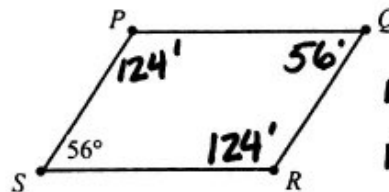
Find AB and BC in $\Delta ABCD$.



$$AB = 10$$

$$BC = 13$$

Find the missing angle measures in $\Delta PQRS$.



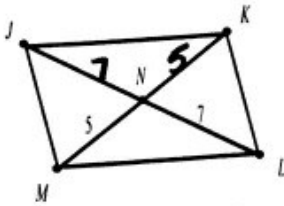
$$m\angle Q = 56'$$

$$m\angle P = 124'$$

$$m\angle R = 124'$$

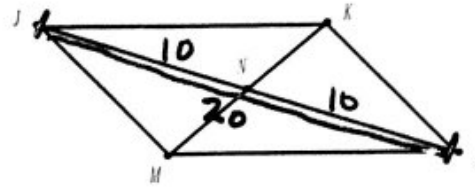
*Diagonals bisect each other.

JKLM is a parallelogram. Find the requested measures.
Find JN and KN.



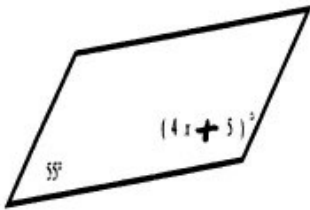
JN = 7
NK = 5

Find JN and NL if JL = 20.

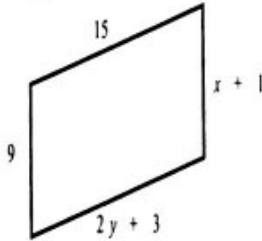


JN = 10

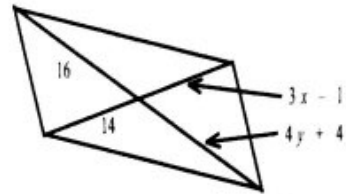
Find the value of the variables in each parallelogram. *Consecutive \angle 's supplementary



$4x + 5 + 55 = 180$
 $4x + 60 = 180$
 $4x = 120$
 $x = 30$

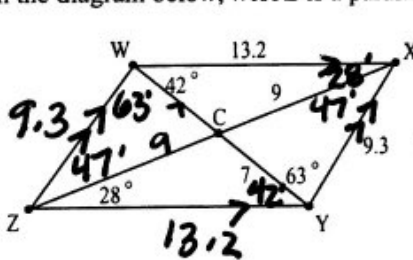


$x + 1 = 9$
 $x = 8$
 $2y + 3 = 15$
 $2y = 12$
 $y = 6$



$3x - 1 = 14$
 $3x = 15$
 $x = 5$
 $4y + 4 = 16$
 $4y = 12$
 $y = 3$

In the diagram below, WXYZ is a parallelogram. Find the requested measures.



$m\angle XWZ = 105^\circ$
 $m\angle WXY = 75^\circ$
 $m\angle WZC = 47^\circ$
 $m\angle WCX = 110^\circ$

YZ = 13.2
CZ = 9
WY = 14
WZ = 9.3

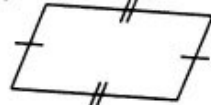
$360 - 63 - 42 - 63 - 42 - 28 - 28 = 94$
 $94 \div 2 = 47$

To Prove that a Quadrilateral is a Parallelogram:

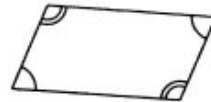
Show that both pairs of opposite sides are parallel. (Definition of parallelogram)



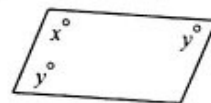
Show that both pairs of opposite sides are congruent. (Theorem)



Show that both pairs of opposite angles are congruent. (Theorem)

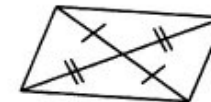


Show that one angle is supplementary to both of its consecutive angles. (Theorem)

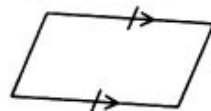


$$x^\circ + y^\circ = 180^\circ$$

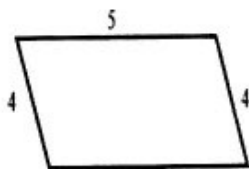
Show that the diagonals bisect each other. (Theorem)



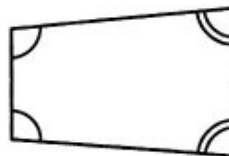
Show that one pair of sides are both parallel and congruent. (Theorem)



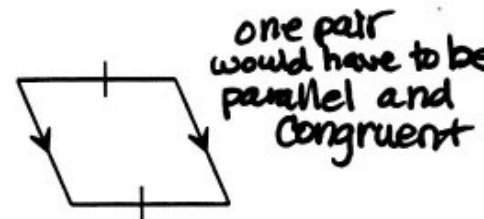
Examples: Decide whether each quadrilateral is a parallelogram. Explain your reasoning. Hint: On each problem, list everything that the diagram tells you. Then think about whether you can use that information to say anything else about the diagram. Finally, decide whether you have enough information to use one of the theorems above.



Parallelogram
Opposite sides \cong

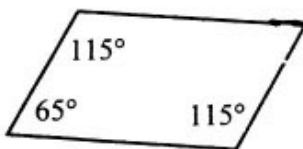


NOT Parallelogram
Opposite angles not \cong

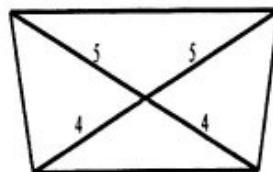


one pair would have to be parallel and congruent

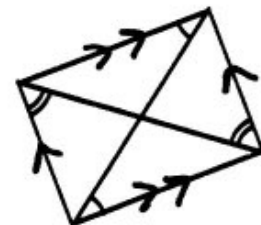
NOT Parallelogram



Parallelogram
One angle is supplementary to both consecutive angles.



NOT Parallelogram
Diagonals not bisected



Parallelogram
Both pairs opposite sides parallel because of alternate interior angles

Rectangle: A parallelogram with four right angles.

Properties:

All properties of parallelograms apply (Rectangles are parallelograms)

Four right angles (Definition)

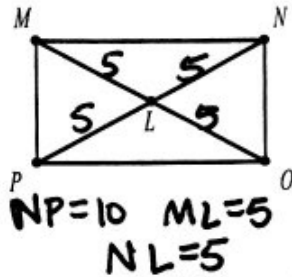
Congruent diagonals (Theorem)

Rectangle Corollary: If a quadrilateral has four right angles, then it is a rectangle. This means you don't have to know that it is a parallelogram to show it is a rectangle.

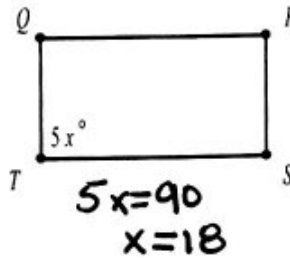


Examples: Each of the quadrilaterals below is a rectangle. Find the requested values.

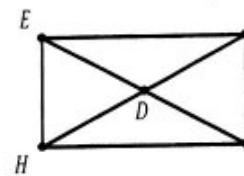
If $MO = 10$, find NP , ML , and NL .



Find the value of x .



Find the value of y , EG and DG .



$EG = 3y + 9$
 $FH = 6y$

$3y + 9 = 6y$
 $9 = 3y$
 $3 = y$

Diagonals are \cong

Rhombus: A parallelogram with four congruent sides.

Properties:

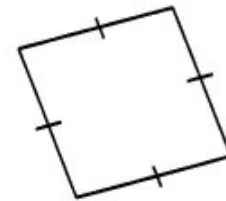
All properties of parallelograms apply (Rhombi are parallelograms)

All four sides are congruent (Definition)

Diagonals are perpendicular bisectors (Theorem)

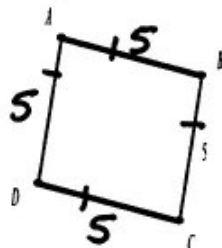
Each diagonal bisects a pair of opposite angles

Rhombus Corollary: If a quadrilateral has four congruent sides, then it is a rhombus. This means you don't have to know that it is a parallelogram to show it is a rhombus.



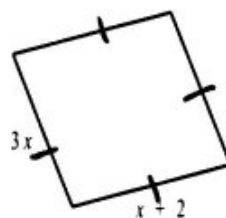
Examples: Each of the quadrilaterals below is a rhombus. Find the requested values.

Find AB , CD , and AD .



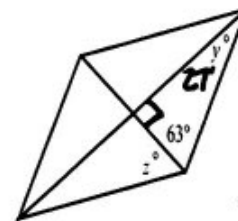
$AB = 5$
 $CD = 5$
 $AD = 5$

Find the value of x .



$3x = x + 2$
 $2x = 2$
 $x = 1$

Find the values of y and z .



$180 - 90 - 63 = 27$

$y = 27$

$z = 63$

Diagonals bisected

Square: A parallelogram with four congruent sides and four right angles.

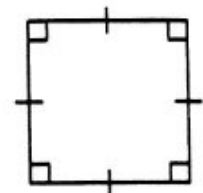
Properties:

All properties of parallelograms apply (All squares are parallelograms)

All properties of rectangles apply (All squares are rectangles)

All properties of rhombi apply (All squares are rhombi)

Square Corollary: If a quadrilateral has four congruent sides and four right angles, then it is a square. This means you don't have to know that it is a parallelogram to show it is a square.



Trapezoid: A quadrilateral with *exactly one* pair of parallel sides.

Bases of a trapezoid: The parallel sides of a trapezoid.

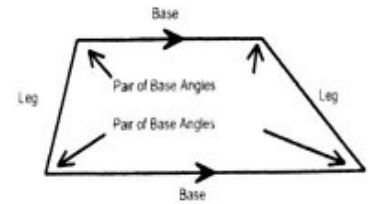
Base angles: Two angles that share a base.

A trapezoid has two pairs of base angles.

Legs of a trapezoid: The non-parallel sides of a trapezoid.

Properties:

The angles on either side of each leg are supplementary (Same-side interior angles).



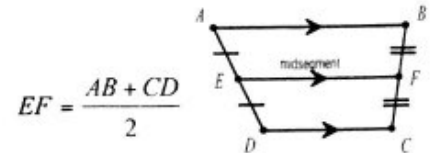
Midsegment of a trapezoid: The segment that joins the midpoints of the legs.

Properties of the midsegment of a trapezoid:

Bisects the legs (definition)

Parallel to the two bases

Length is the average of the lengths of the bases (add the lengths of the bases and divide by two).

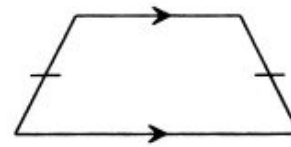


Isosceles Trapezoid: A trapezoid with congruent legs.

Properties:

Diagonals are congruent.

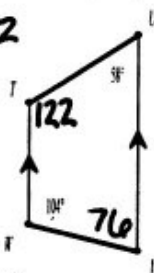
Base angles are congruent.



Examples:

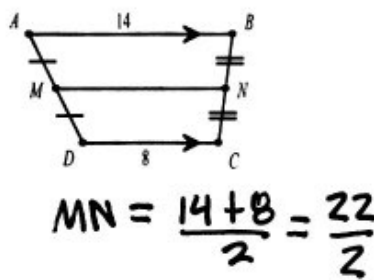
Find the missing angle measures.

$$180 - 58 = 122$$



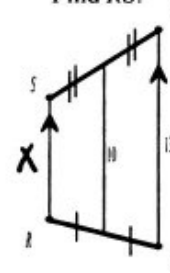
$$180 - 104 = 76$$

Find the length of midsegment \overline{MN} .



$$MN = 11$$

Find RS.

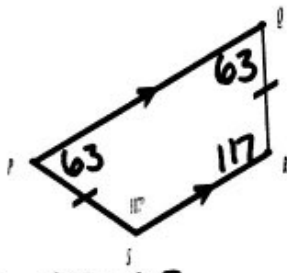


$$\frac{X + 13}{2} = 10$$

$$X + 13 = 20$$

$$X = 7$$

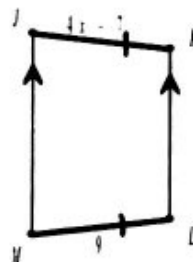
$PQRS$ is an isosceles trapezoid. Find the missing angle measures.



$$180 - 117 = 63$$

base \angle 's \cong

$JKLM$ is an isosceles trapezoid. Find the value of x .



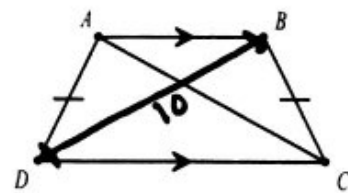
$$4x - 7 = 9$$

$$4x = 16$$

$$x = 4$$

legs \cong

$ABCD$ is an isosceles trapezoid. Find AC if $BD = 10$.



$$AC = 10$$

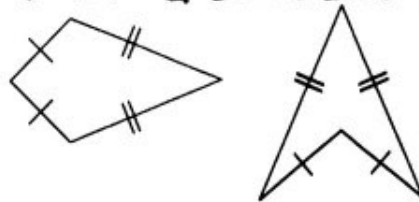
Diagonals \cong

Kite: A quadrilateral with two pairs of congruent consecutive sides and no congruent opposite sides.

Properties:

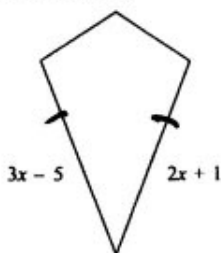
No parallel sides

Diagonals are perpendicular.



Examples:

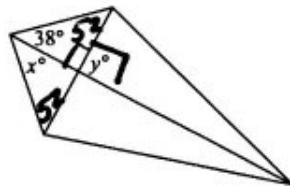
Solve for x.



$$3x - 5 = 2x + 1$$

$$x = 6$$

Find the missing angle measures.



$$y = 90^\circ$$

$$x = 38^\circ$$

Directions: Put an "x" in the box if the statement is always true for each type of quadrilateral.

	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid
Both pairs of opposite sides are parallel	X	X	X	X		
Diagonals are congruent		X		X		X
Both pairs of opposite angles are congruent	X	X	X	X		
Diagonals bisect each other	X	X	X	X		
All pairs of consecutive angles are supplementary	X	X	X	X		
Diagonals are perpendicular			X	X		
Exactly one pair of parallel sides					X	X
Both pairs of opposite sides are congruent	X	X	X	X		
All four sides are congruent			X	X		
Diagonals are angle bisectors			X	X		
Has four right angles		X		X		

