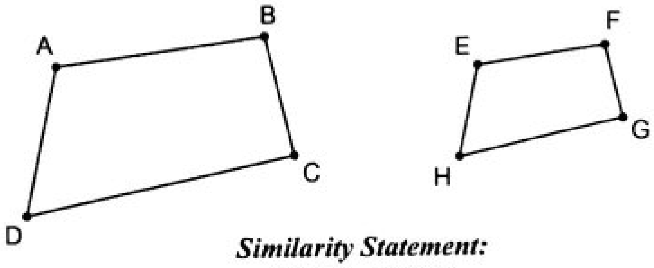
SM2H 7.5 Similarity Notes

Congruent Figures: Same shape and same size.
Similar Figures: Same shape.

If two polygons are similar, then:

- Their **corresponding angles are congruent.**
- The lengths of their **corresponding sides are proportional.**

Examples:



Similarity Statement:
ABCD ~ EFGH

1. List all pairs of congruent angles.

$\angle A \cong \angle E$ $\quad \angle D \cong \angle H$
$\angle B \cong \angle F$ $\quad \angle C \cong \angle G$

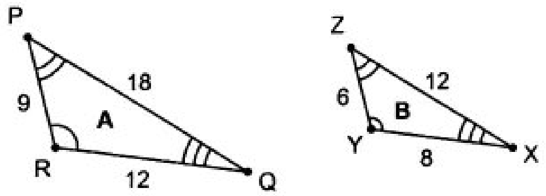
2. Write a ***statement of proportionality*** for the sides.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

Scale Factor: The ratio of the lengths of two corresponding sides in similar polygons.

Examples: Decide whether each set of figures are similar. If they are similar, write a similarity statement and find the scale factor.

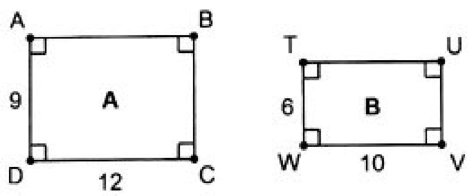
1.



$\frac{6}{9} = \frac{2}{3}$ $\quad \frac{8}{12} = \frac{2}{3}$ $\quad \frac{12}{18} = \frac{2}{3}$

$\triangle PQR \sim \triangle ZXY$ $\quad$ scale factor $= 2/3$ or $3/2$

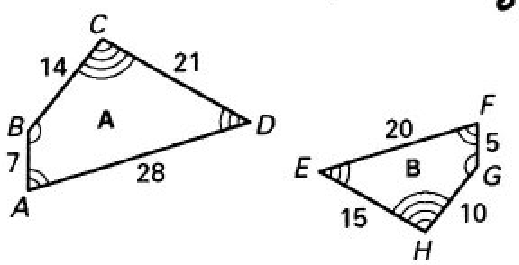
2.



$\frac{6}{9} = \frac{2}{3}$ $\quad \frac{10}{12} = \frac{5}{6}$

NOT SIMILAR

3.



$\frac{5}{7}$

$\frac{10}{14} = \frac{5}{7}$

$\frac{15}{21} = \frac{5}{7}$

$\frac{20}{28} = \frac{5}{7}$

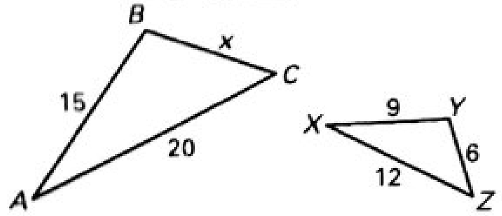
Similar

$ABCD \sim FGHE$

Scale factor $= 5/7$ or $7/5$

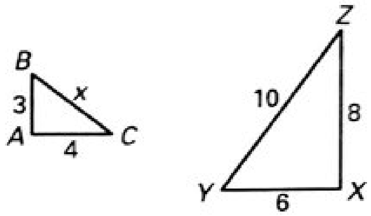
Examples: $\Delta ABC \sim \Delta XYZ$. Find the value of x.

1.



$\frac{x}{6} = \frac{15}{9}$ $9x = 90$ $\boxed{x = 10}$

2.

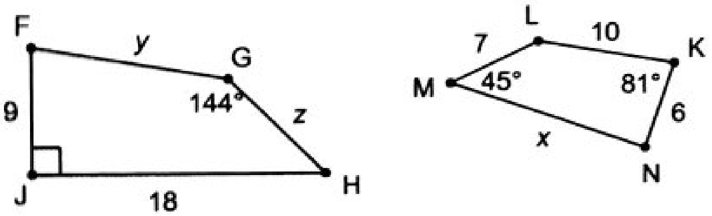


$\frac{x}{10} = \frac{3}{6}$

$6x = 30$

$\boxed{x = 5}$

Examples: In the diagram below, $FGHJ \sim KLMN$.



1. List all pairs of congruent angles.

$\angle F \cong \angle K$ $\angle H \cong \angle M$

$\angle G \cong \angle L$ $\angle J \cong \angle N$

2. Write a statement of proportionality.

3. Find $m\angle F$.

81°

4. Find $m\angle H$.

45°

5. Find $m\angle L$.

144°

6. Find $m\angle N$.

90°

7. Find the value of x.

$\frac{x}{18} = \frac{6}{9}$

$9x = 108$

$\boxed{x = 12}$

8. Find the value of y.

$\frac{y}{10} = \frac{9}{6}$

$6y = 90$

$\boxed{y = 15}$

9. Find the value of z.

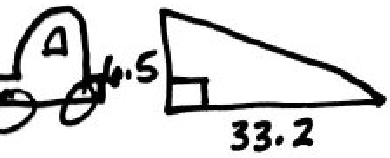
$\frac{z}{7} = \frac{9}{6}$

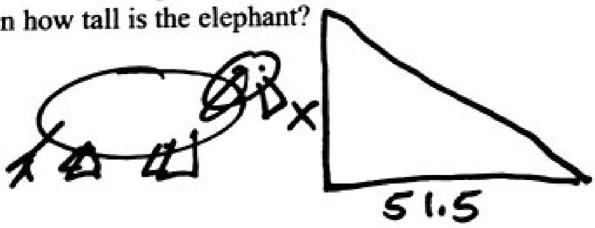
$6z = 63$

$z = 63/6 = \boxed{21/2 = 10.5}$

Examples:

1. A 6.5 ft. tall car standing next to an adult elephant casts a 33.2 ft. shadow. If the adult elephant casts a shadow that is 51.5 ft. long, then how tall is the elephant?



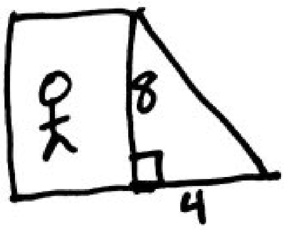


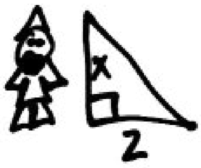
$\frac{x}{6.5} = \frac{51.5}{33.2}$

$\frac{33.2x}{33.2} = \frac{334.75}{33.2}$

$\boxed{x = 10.1 ft}$

2. A telephone booth that is 8 ft. tall casts a shadow that is 4 ft. long. Find the height of a nearby lawn ornament that casts a 2 ft. shadow.





$\frac{x}{8} = \frac{2}{4}$

$4x = 16$

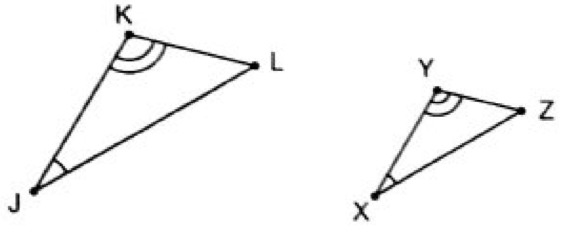
$\boxed{x = 4 ft}$

Triangle Similarity Theorems

So far, if we wanted to show that two figures are similar, we've had to show that *all* of the corresponding angles are congruent and *all* of the corresponding sides are proportional. Luckily, there are some shortcuts for triangles.

Angle-Angle Similarity Postulate (AA Similarity):

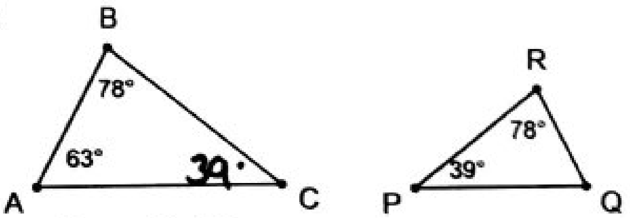
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



If $\angle J \cong \angle X$ and $\angle K \cong \angle Y$, then $\Delta JKL \sim \Delta XYZ$.

Examples: Determine whether the triangles are similar. Explain your reasoning. If they are similar, write a similarity statement.

a)



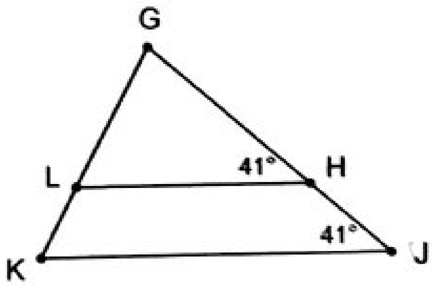
$180-78-63=39$

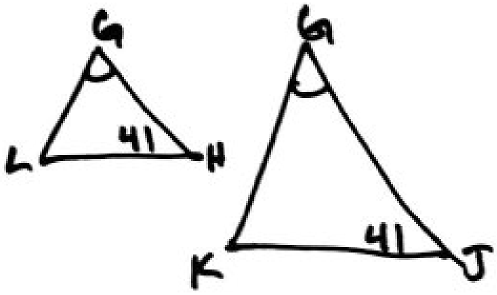
A $\angle C \cong \angle P$

A $\angle B \cong \angle R$

$\Delta ABC \sim \Delta RQP$ by AA Similarity

b)



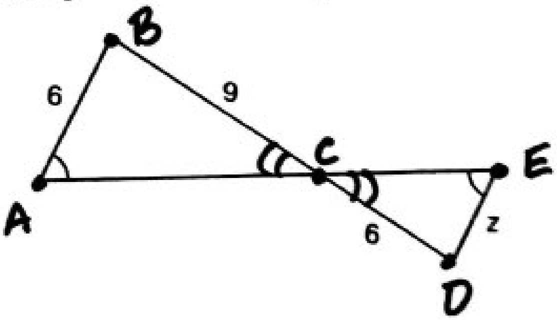


A $\angle G \cong \angle G$

A $\angle H \cong \angle J$

$\Delta GLH \sim \Delta GKJ$ by AA similarity

Example: Write a similarity statement for the triangles. Then find the value of z.



A $\angle BCA \cong \angle DCE$

A $\angle A \cong \angle E$

$\Delta ABC \sim \Delta EDC$ by AA Similarity

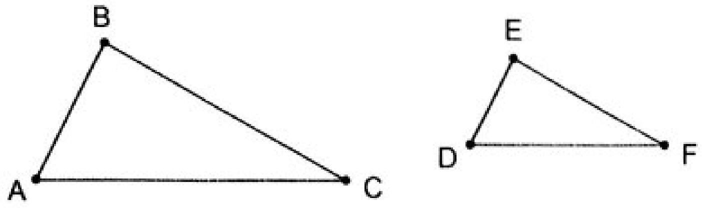
$\frac{z}{6} = \frac{6}{9}$

$9z = 36$

$\boxed{z=4}$

Side-Side-Side Similarity Theorem (SSS Similarity)

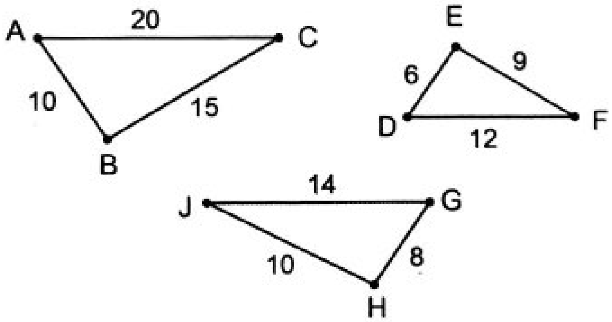
If the corresponding sides of two triangles are proportional, then the triangles are similar.



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then $\Delta ABC \sim \Delta DEF$.

★ **TIP:** When testing for SSS similarity, compare the shortest sides, longest sides, and medium sides.

Example: Is either ΔDEF or ΔGHJ similar to ΔABC?



SIDE SIDE SIDE

$\frac{10}{6} = \frac{5}{3}$ $\frac{15}{9} = \frac{5}{3}$ $\frac{20}{12} = \frac{5}{3}$

$\Delta ABC \sim \Delta DEF$ by SSS

SIDE SIDE SIDE

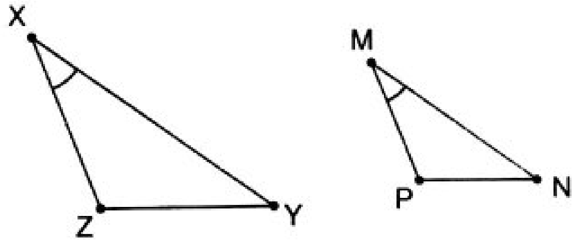
$\frac{10}{8} = \frac{5}{4}$ $\frac{15}{10} = \frac{3}{2}$ $\frac{20}{14} = \frac{10}{7}$

ΔABC not similar ΔGHJ

Fractions not same

Side-Angle-Side Similarity Theorem (SAS Similarity)

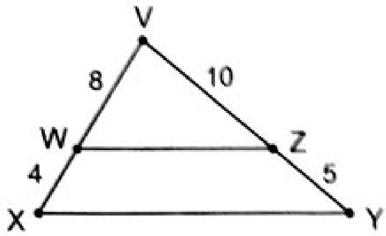
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar.

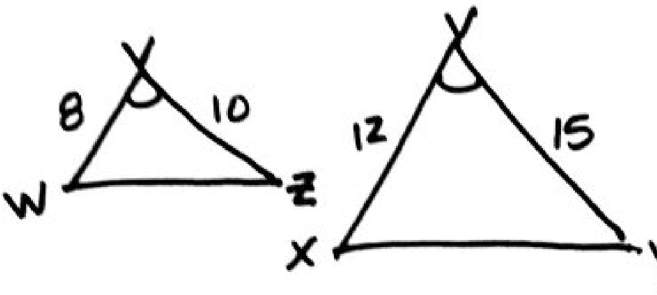


If $\angle X \cong \angle M$ and $\frac{PM}{ZX} = \frac{MN}{XY}$, then $\Delta XYZ \sim \Delta MNP$.

Examples: Determine whether the triangles are similar. If they are similar, write a similarity statement and determine the scale factor.

a)





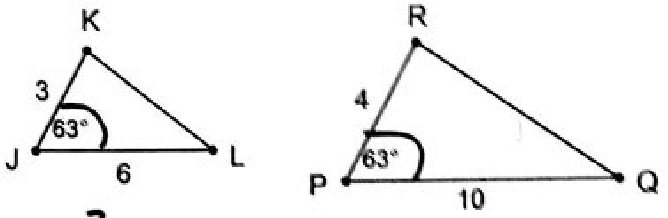
S $\frac{8}{12} = \frac{2}{3}$

A $\angle V \cong \angle V$

S $\frac{10}{15} = \frac{2}{3}$

$\Delta WVZ \sim \Delta XVY$

b)



S $\frac{3}{4}$

A $\angle J \cong \angle P$

S $\frac{6}{10} = \frac{3}{5}$

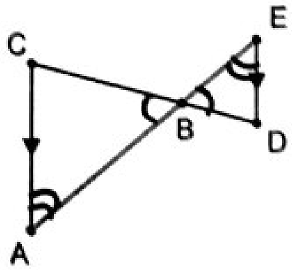
NOT SIMILAR

Fractions not same

Complete the following proof:

Given: $\overline{AC} \parallel \overline{DE}$

Prove: $\Delta ABC \sim \Delta EBD$



Statements	Reasons
1. $\overline{AC} \parallel \overline{DE}$	1. Given
2. $\angle A \cong \angle E$	2. Alternate Interior $\angle$'s $\cong$
3. $\angle CBA \cong \angle DBE$	3. Vertical Angles Theorem
4. $\Delta ABC \sim \Delta EBD$	4. AA Similarity