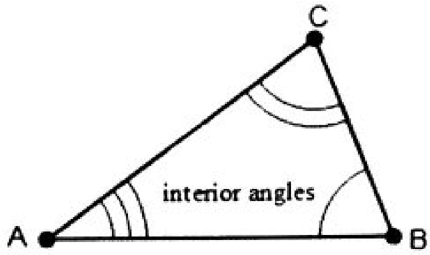
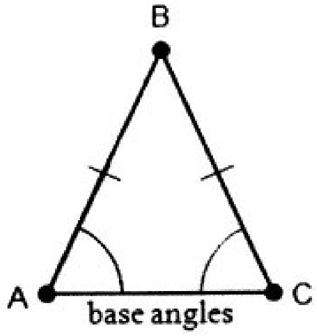
SM2H 7.4 Triangle Notes

VOCABULARY

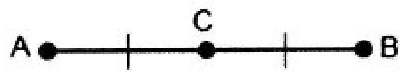
The angles inside of a triangle are called **interior angles**.



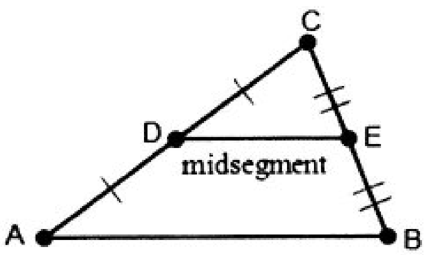
A triangle with at least two congruent sides is called an **isosceles triangle**. In an isosceles triangle, the angles that are opposite the congruent sides are called **base angles**.



A point that is halfway between the endpoints of a segment is called the **midpoint**. Point C is the midpoint of $\overline{AB}$.

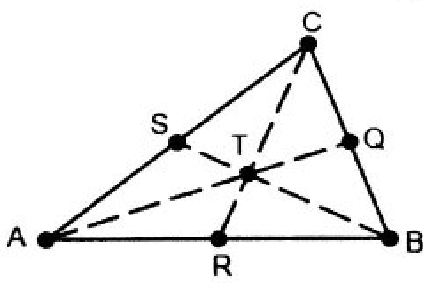


A segment whose endpoints are the midpoints of two sides of a triangle is called the **midsegement of a tiangle**.



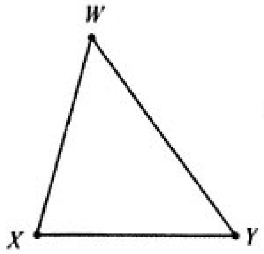
The line connecting midpoints to the opposite vertex of a triangle is called the **median**. Point S is the midpoint of $\overline{AC}$. Point Q is the midpoint of $\overline{BC}$. Point R is the midpoint of $\overline{AB}$.

The point where all three medians of a triangle intersect is called a **centroid**. Point T is the centroid of $\triangle ABC$.



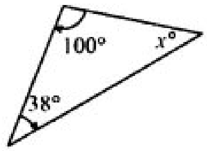
<u>Other Triangle Theorems:</u>

Angle Sum Theorem: The sum of the measures of the angles of a triangle is 180°.



$m\angle W + m\angle X + m\angle Y = 180°$

Example: Find x.

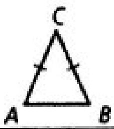


$x+38+100=180$

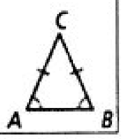
$x=42°$

Isosceles Triangle Theorem	*Converse of the Isosceles Triangle Theorem*
If two sides of a triangle are congruent, then the angles opposite those sides are congruent	If two angles of a triangle are congruent , then the sides opposite those angles are congruent

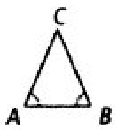
If . . . $\overline{AC} \cong \overline{BC}$



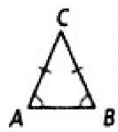
Then . . . $\angle A \cong \angle B$



If . . . $\angle A \cong \angle B$

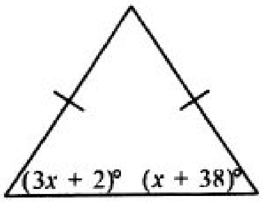


Then . . . $\overline{AC} \cong \overline{BC}$



Examples: Find x.

a)

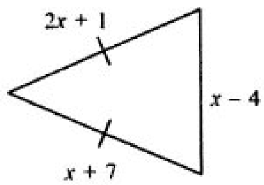


$3x+2=x+38$

$2x=36$

$x=18$

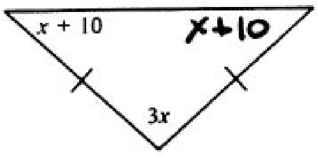
b)



$2x+1=x+7$

$x=6$

c)

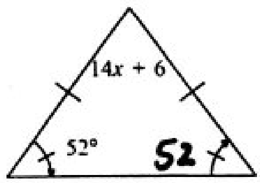


$x+10+x+10+3x=180$

$5x=160$

$x=32$

d)



$14x+6+52+52=180$

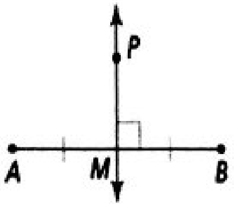
$14x=70$

$x=5$

Perpendicular Bisector Theorem

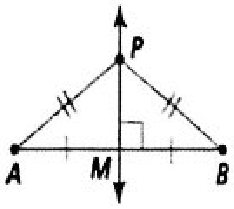
If . . .

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



Then . . .

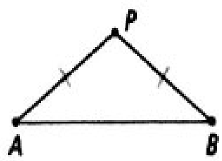
$PA = PB$



Converse of Perpendicular Bisector Theorem

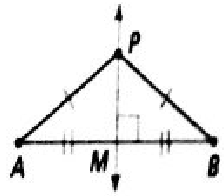
If . . .

$PA = PB$

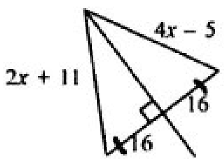


Then . . .

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



a) Find x.

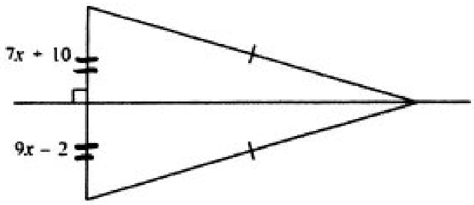


$2x+11 = 4x-5$

$16 = 2x$

$8 = x$

b) Find x.

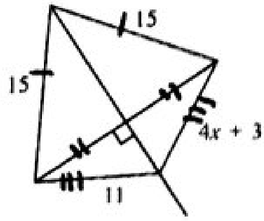


$7x+10 = 9x-2$

$12 = 2x$

$6 = x$

c) Find x.

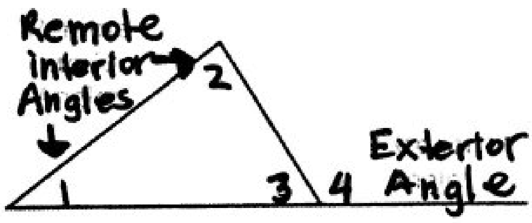


$4x+3 = 11$

$4x = 8$

$x = 2$

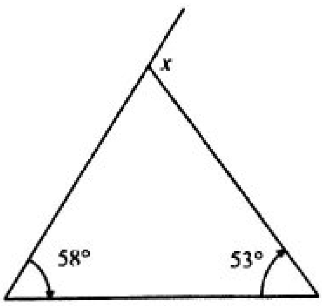
Exterior Angle Theorem: The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.



Find x.

$m\angle 1 + m\angle 2 = m\angle 4$

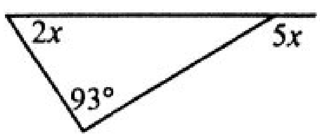
d)



$x = 58+53$

$x = 111$

e)

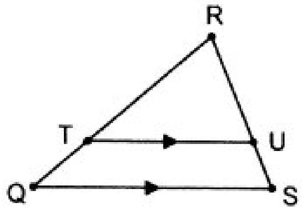


$5x = 2x + 93$

$3x = 93$

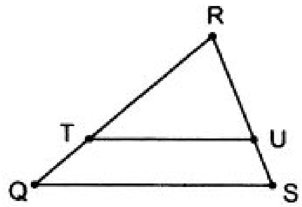
$x = 31$

Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the sides proportionally.



In $\triangle QRS$, if $\overline{TU} \parallel \overline{QS}$,
then $\dfrac{RT}{TQ} = \dfrac{RU}{US}$.

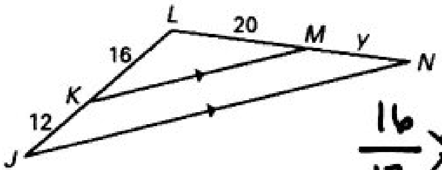
Converse of the Triangle Proportionality Theorem: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



In $\triangle QRS$, if $\dfrac{RT}{TQ} = \dfrac{RU}{US}$,
then $\overline{TU} \parallel \overline{QS}$.

Examples: Find the value of the variable.

a)



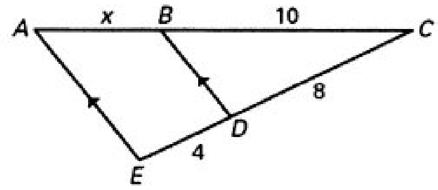
$\dfrac{16}{12} \times \dfrac{20}{y}$

$16y = 20 \cdot 12$

$16y = 240$

$\boxed{y = 15}$

b)

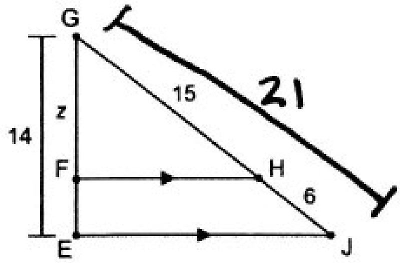


$\dfrac{x}{10} \times \dfrac{4}{8}$

$8x = 40$

$\boxed{x = 5}$

c)



$\dfrac{z}{14} \times \dfrac{15}{21}$

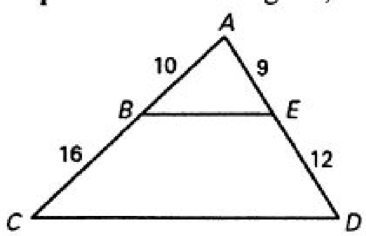
$21z = 15 \cdot 14$

$21z = 210$

$\boxed{z = 10}$

Examples: Given the diagram, determine whether $\overline{BE} \parallel \overline{CD}$. Show work to support your answer.

a)

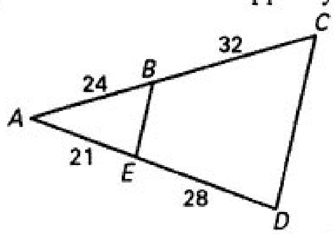


$\dfrac{10}{16} \stackrel{?}{=} \dfrac{9}{12}$

$\dfrac{5}{8} \neq \dfrac{3}{4}$

$\overline{BE} \nparallel \overline{CD}$ Ratios $\neq$

b)



$\dfrac{21}{28} \stackrel{?}{=} \dfrac{24}{32}$

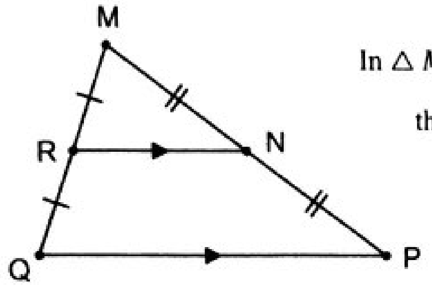
$\dfrac{3}{4} = \dfrac{3}{4}$

$\overline{BE} \parallel \overline{CE}$ Ratios =

Scanned with CamScanner

Midsegment of a Triangle: A segment that connects the midpoints of two sides of a triangle.

Midsegment Theorem*:* The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

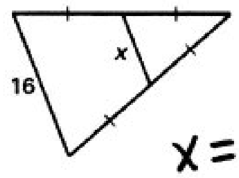


In $\triangle MPQ$, if $MR = RQ$ and $MN = NP$,

then $\overline{RN} \parallel \overline{QP}$ and $RN = \frac{1}{2}QP$.

Examples: Find the value of the variable.

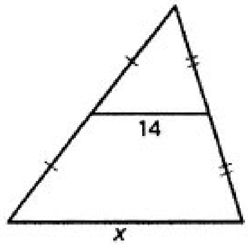
a)



$x = 16 \div 2$

$x = 8$

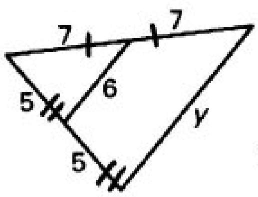
b)



$x = 14 \cdot 2$

$x = 28$

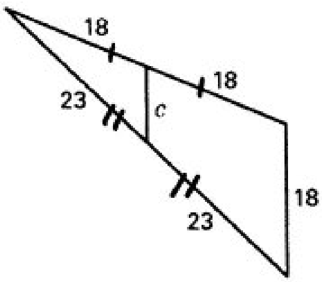
c)



$y = 6 \cdot 2$

$y = 12$

d)

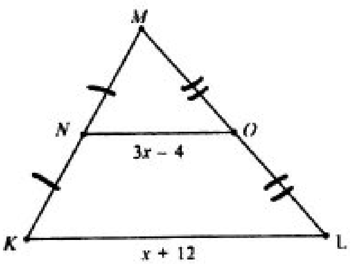


$c = 18 \div 2$

$c = 9$

Given that $\overline{NO}$ is a midsegment of the triangle, find x.

e)



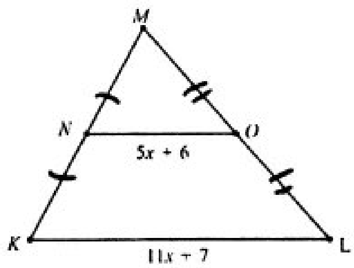
$2(3x-4) = x+12$

$6x - 8 = x + 12$

$5x = 20$

$x = 4$

f)



$2(5x+6) = 11x + 7$

$10x + 12 = 11x + 7$

$5 = x$