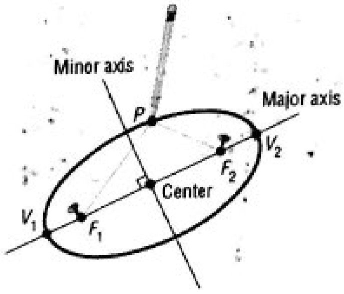
The Ellipse

Objectives:

- **Given the general equation of an ellipse, identify the foci and vertices.**
- **Given the foci and the vertices of an ellipse, write an equation for the ellipse.**
- **Sketch the graph of a circle, given the equation.**



Ellipse: The collection of all points in the plane, the sum of whose distances from two fixed points, called the **foci**, F_1 and F_2, is a constant.

Major Axis: The line containing the foci. (longer axis w/vertices and foci)
a

Center: The midpoint of the line segment joining the two foci.

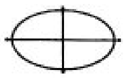
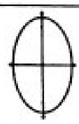
Minor Axis: The line through the center and perpendicular to the major axis.
b

Vertices: The points of intersection of the ellipse and the major axis.
a

Covertices: The points of intersection of the ellipse and the minor axis.
b

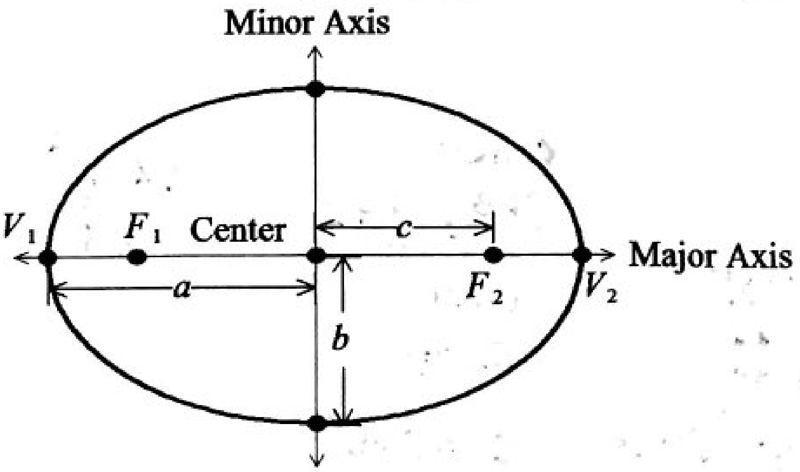
a^2 is always biggest #

Standard Form of the Equation of an Ellipse with Center at (h,k)

Equation	Description	Picture
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to x-axis	
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to y-axis	

$c^2=a^2-b^2$

a = Distance from center to vertices
b = Distance from center to covertices
c = Distance from center to foci



Examples: Find the center, foci, and vertices of each ellipse. Graph each equation.

bigger # ellipse

a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

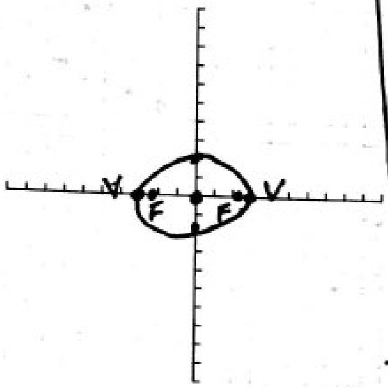
$a^2 = 9 \quad b^2 = 4$

$a = 3 \quad b = 2$

center $(0,0)$

vertices $(3,0)$ $(-3,0)$

Foci $(\sqrt{5}, 0)$ $(-\sqrt{5}, 0)$



$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$\sqrt{c^2} = \pm\sqrt{5}$

Distance from center to foci $c = \pm\sqrt{5}$ 2.2

b) $\frac{x^2}{16} + \frac{y^2}{36} = 1$

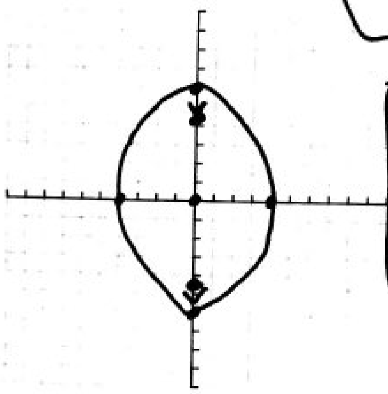
$b^2 \quad a^2$

$b = 4 \quad a = 6$

center $(0,0)$

vertices $(0,6)$ $(0,-6)$

Foci $(0, 2\sqrt{5})$ $(0, -2\sqrt{5})$



$c^2 = a^2 - b^2$

$c^2 = 36 - 16$

$\sqrt{c^2} = \pm\sqrt{20}$ 2.2.5

$c = \pm 2\sqrt{5}$ or 4.5

c) $\frac{(x+1)^2}{81} + \frac{(y-2)^2}{49} = 1$

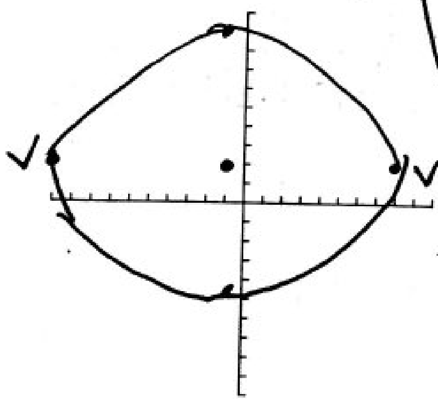
$a^2 \quad b^2$

$a = 9 \quad b = 7$

center $(-1,2)$

vertices $(8,2)$ $(-10,-2)$

Foci $(-1 + 4\sqrt{2}, 2)$ $(-1 - 4\sqrt{2}, 2)$



$c^2 = a^2 - b^2$

$c^2 = 81 - 49$ 2.2.2.2.2

$\sqrt{c^2} = \pm\sqrt{32}$

$c = \pm 4\sqrt{2}$ or 5.6

* Adding c onto x coord. of the center

d) $\frac{9(x-3)^2}{18} + \frac{(y+2)^2}{18} = \frac{18}{18}$

$\frac{(x-3)^2}{2} + \frac{(y+2)^2}{18} = 1$

$b^2 \quad a^2$

$b = \pm\sqrt{2}$ 1.4

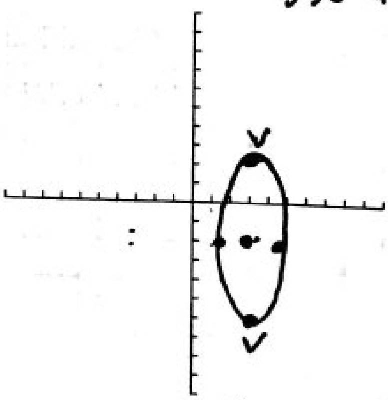
$a = \sqrt{18} = \pm 3\sqrt{2}$ or 4.2 (3.3.2)

center $(3,-2)$

vertices $(3, -2+3\sqrt{2})$ $(3, -2-3\sqrt{2})$

* add $\pm 3\sqrt{2}$ to y coord. of center

Foci $(3, -2+4)$ $(3, -2-4)$



$c^2 = a^2 - b^2$

$c^2 = 18 - 2$

$\sqrt{c^2} = \pm\sqrt{16}$

$c = \pm 4$

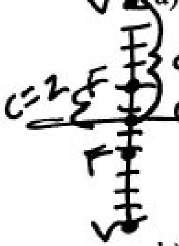
$(3, 2)$ $(3, -6)$

* Add ±4 to y coord. of center

Scanned with CamScanner

Examples: Write the standard equation of the ellipse having the given characteristics.

a) Foci at (0,-2) and (0, 2); Vertices at (0,–6) and (0, 6)



$a=6 \quad a^2=36$

$c=2 \quad c^2=4$

$c^2=a^2-b^2$

$4=36-b^2$

$-36 \quad -36$

$\frac{-32}{-1}=\frac{-b^2}{-1}$

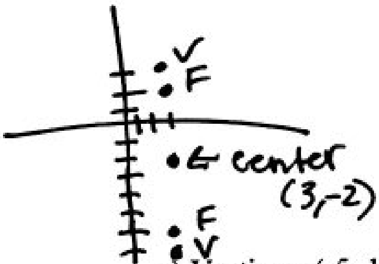
$b^2=32$

$$\frac{(x-0)^2}{32}+\frac{(y-0)^2}{36}=1$$

b^2 a^2

$$\boxed{\frac{x^2}{32}+\frac{y^2}{36}=1}$$

b) Foci at (3, -6) and (3, 2); Vertices at (3, -7) and (3, 3)



$a=5 \quad a^2=25$

$c=4 \quad c^2=16$

$c^2=a^2-b^2$

$16=25-b^2$

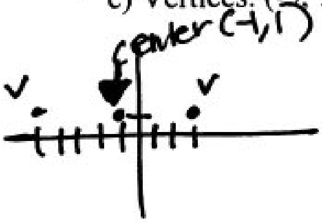
$-9=-b^2$

$b^2=9$

$$\boxed{\frac{(x-3)^2}{9}+\frac{(y+2)^2}{25}=1}$$

b^2 a^2

c) Vertices: (-5, 1) and (3, 1) ; Minor axis length is 6.



$a=4 \quad a^2=16$

$b=\frac{6}{2}=3$

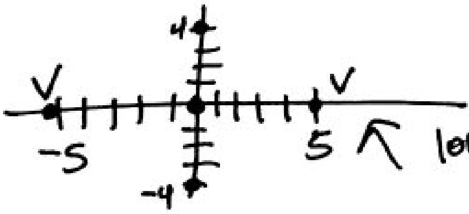
$b^2=9$

$$\boxed{\frac{(x+1)^2}{16}+\frac{(y-1)^2}{9}=1}$$

a^2 b^2

d) Endpoints of axes are: (-5, 0), (5, 0), (0, -4) and (0, 4)

center (0,0)



longest one has vertices

$a=5$

$a^2=25$

$b=4$

$b^2=16$

$$\boxed{\frac{x^2}{25}+\frac{y^2}{16}=1}$$

a^2 b^2