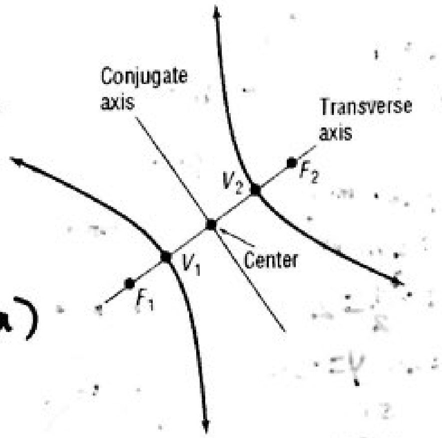
The Hyperbola

Objectives:

- **Find the equation of a hyperbola given the foci and the vertices**
- **Sketch the graph of a hyperbola, given the equation.**



Hyperbola: The collection of all points in the plane, the difference of whose distances from two fixed points, called the **foci,** is a constant.

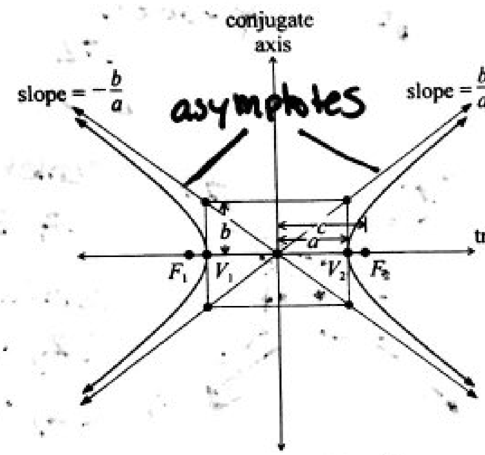
Transverse Axis: The line containing the foci and vertices. (a)

Center: The midpoint of the line segment joining the foci.

Conjugate Axis: The line through the center and perpendicular to the transverse axis. (b)

Branches: The separate curves of the hyperbola. They are symmetric with respect to the transverse axis, conjugate axis, and center.

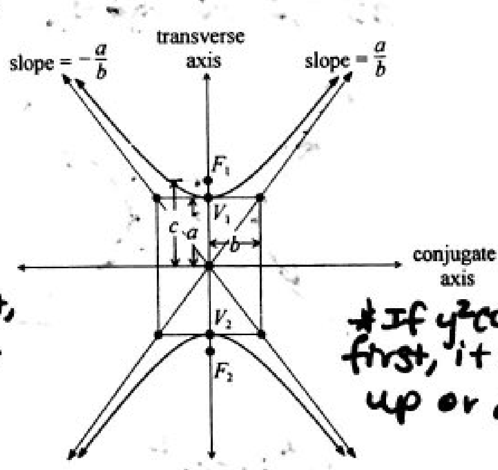
Vertices: The points of intersection of the hyperbola and the transverse axis.



*If x^2 comes first, it opens left or right

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$a^2 + b^2 = c^2$$



*If y^2 comes first, it opens up or down

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$a^2 + b^2 = c^2$$

a = distance from center to vertices — a always comes first

c = distance from center to foci

b used to find the width of branches and slope of asymptotes — $y = mx + b$ ← slope (m), ← y int (b)

When finding the equations of the asymptotes, remember that $\boxed{m = \frac{\text{change in } y}{\text{change in } x}}$ slope then use point slope form $y - y_1 = m(x - x_1)$ with the center (h, k) as (x_1, y_1).

$m = \frac{rise}{run}$

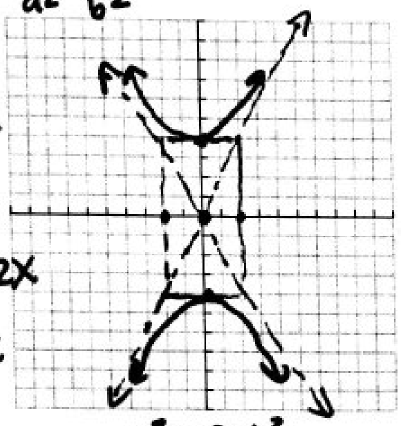
* a is distance from center to vertices.
* c is distance from center to focii

Examples: Find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.

opens up + down

a) $\frac{y^2}{16} - \frac{x^2}{4} = 1$ (a^2, b^2)

$a = 4$
$b = 2$



Asymptotes
$y = mx + b$
$m = \frac{\pm 4}{2} = 2$
$y = 2x$ $y = -2x$
or
$y = \pm 2x$

center
(0,0)

vertices
(0, 4)
(0, −4)

Foci Points
$(0, 2\sqrt{5})$
$(0, -2\sqrt{5})$

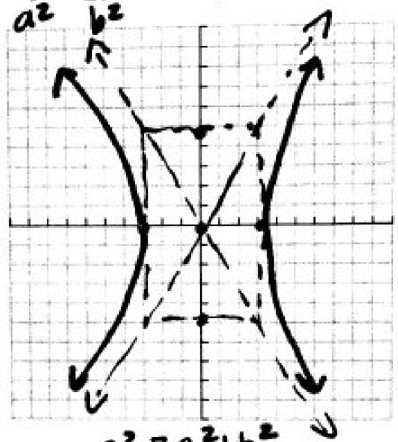
Traverse Axis
$x = 0$

$c^2 = a^2 + b^2$
$c^2 = 16 + 4$
$c^2 = 20$
$c = \pm 2\sqrt{5}$

opens left or right

b) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (a^2, b^2)

$a = 3$
$b = 5$



center
(0,0)

Vertices
(3,0) (−3,0)

Foci Points
$(\sqrt{34}, 0)$
$(-\sqrt{34}, 0)$

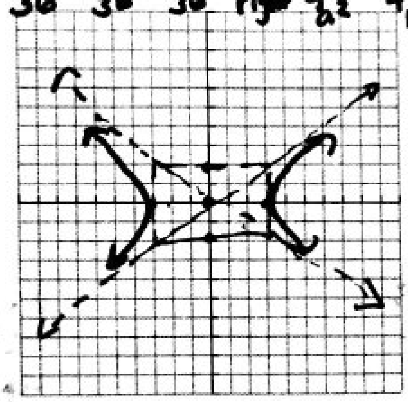
Traverse axis
$y = 0$

Asymptotes
$m = \pm \frac{5}{3}$
$y = \pm \frac{5}{3}x$

$c^2 = a^2 + b^2$
$c^2 = 9 + 25$
$c^2 = 34$
$c = \pm\sqrt{34}$

c) $\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$ left or right $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (a^2, b^2)

$a = 3$
$b = 2$



center
(0,0)

vertices
(3,0)
(−3,0)

Foci
$(\sqrt{13}, 0)$
$(-\sqrt{13}, 0)$

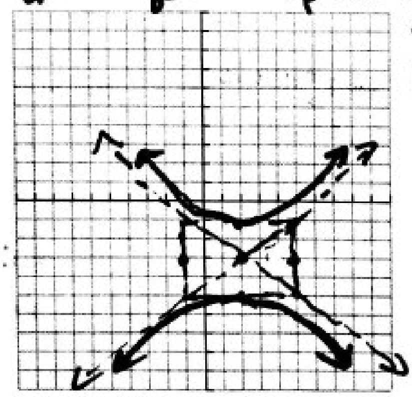
Asymptotes
$m = \pm 2/3$
$y = \pm 2/3 x$

Traverse axis
$y = 0$

$c^2 = a^2 + b^2$
$c^2 = 9 + 4$
$c^2 = 13$
$c = \pm\sqrt{13}$

d) $\frac{(y+3)^2}{4} - \frac{(x-2)^2}{9} = 1$ (a^2, b^2) opens up + down

$a = 2$
$b = 3$



center $\rightarrow \pm 2$
$(2, -3) \rightarrow \pm\sqrt{13}$

vertices
(2, −1)
(2, −5)

Foci Points
$(2, -3+\sqrt{13})$
$(2, -3-\sqrt{13})$

* add $\pm\sqrt{13}$ to y coord. of center

Traverse axis
$x = 2$

$c^2 = a^2 + b^2$
$c^2 = 4 + 9$
$c^2 = 13$
$c = \pm\sqrt{13}$

Asymptotes
$m = \pm 2/3$

Pt slope Form
$(y - y_1) = m(x - x_1)$
$\boxed{(y+3) = \pm 2/3 (x-2)}$

* plug in center
(2, −3)
x y

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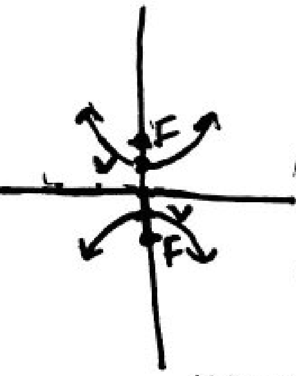
Examples: Write an equation in standard form of the hyperbola described.

a) Foci are at (0, -2) and (0, 2); Vertices are at (0,-1) and (0, 1)

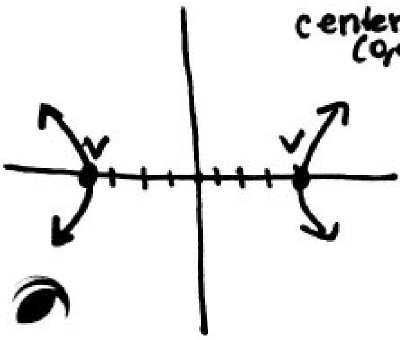


$a=1$
$a^2=1$
$c=2$
$c^2=4$

$c^2=a^2+b^2$
$4=1+b^2$
$-1 \quad -1$
$3=b^2$

$$\frac{y^2}{1}-\frac{x^2}{3}=1$$

b) Vertices are at (-4, 0) and (4, 0); Conjugate axis length is 10.



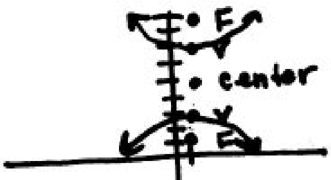
center (0,0)

$a=4$
$a^2=16$

$b=\frac{10}{2}=5$
$b^2=25$

$$\frac{x^2}{16}-\frac{y^2}{25}=1$$

c) Foci are at (1, 9) and (1, 1); Vertices are at (1, 7) and (1, 3)



Center (1,5)

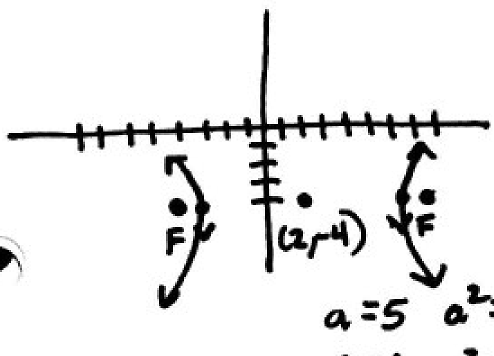
$a=2$
$a^2=4$

$c=4$
$c^2=16$

$c^2=a^2+b^2$
$16=4+b^2$
$12=b^2$

$$\frac{(y-5)^2}{4}-\frac{(x-1)^2}{12}=1$$

d) Foci are at (8, -4) and (-4, -4); Vertices are at (7,-4) and (-3, -4)



center
$\frac{7+-3}{2}=\frac{4}{2}=2$

center
(2,-4)

$a=5 \quad a^2=25$
$c=6 \quad c^2=36$

$c^2=a^2+b^2$
$36=25+b^2$
$11=b^2$

$$\frac{(x-2)^2}{25}-\frac{(y+4)^2}{11}=1$$