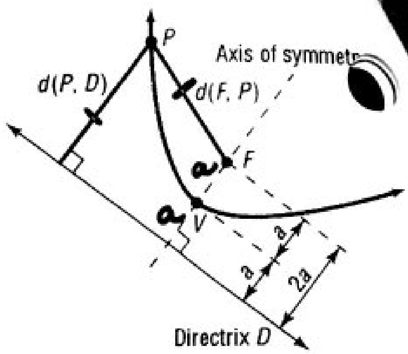
The Parabola



Objectives:
- **Find the equation of a parabola given the focus and the directrix; where the directrix is parallel to either of the coordinate axes.**
- **Sketch the graph of a parabola given the equation.**

Parabola: The collection of all points P in the plane that are the same distance from a fixed point F, called the **focus** of the parabola, as they are from a fixed line D, called the **directrix** of the parabola.

Axis of Symmetry: The line through the focus F and perpendicular to the directrix D.

Vertex: The point of intersection of the parabola with its axis of symmetry.

General Forms of the Equation of a Parabola with Vertex (h,k)

a = **Distance from Focus to Vertex** a = **Distance from Vertex to Directrix**

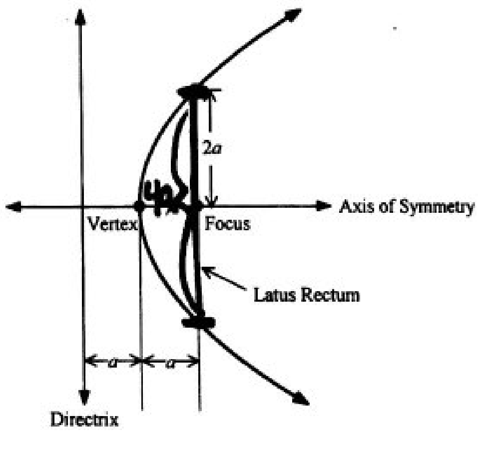
If y is squared, opens left or right

If x is squared opens up or down

Equation	Description	Picture
$(y-k)^2=4a(x-h)$	Opens Right, 4a positive Axis of Symmetry parallel to x-axis	D
$(y-k)^2=-4a(x-h)$	Opens Left, −4a negative Axis of Symmetry parallel to x-axis	D
$(x-h)^2=4a(y-k)$	Opens Up, 4a positive Axis of Symmetry parallel to y-axis	D
$(x-h)^2=-4a(y-k)$	Opens Down, −4a negative Axis of Symmetry parallel to y-axis	D

Focal width

Latus Rectum: The line segment with endpoints on the parabola that passes through the focus and is perpendicular to the axis of symmetry. Each of the endpoints is at a distance of $2a$ from the focus.

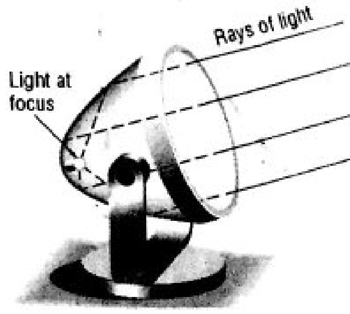


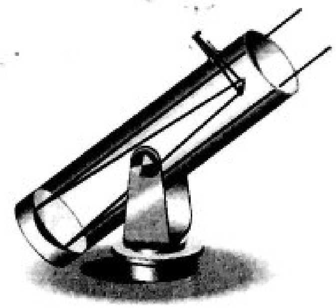
4a is focal width

Paraboloid of Revolution: A surface formed by rotating a parabola about its axis of symmetry.

Suppose a mirror is shaped like a paraboloid of revolution. If a light (or other radiation source) is placed at the focus of the parabola, all the rays emanating from the light will reflect off the mirror in lines parallel to the axis of symmetry.

Conversely, when rays of light from a distant source strike the surface of a parabolic mirror, they are reflected to a single point at the focus. This fact is used in the design of telescopes and other optical devices.





Review Problems

Complete the square and write in factored form.

1. $y^2 - 14y + \underline{7^2}$ $\quad \frac{14}{2} = 7$

$(y-7)^2$

2. $y^2 + \frac{2}{3}y + \underline{\frac{1}{3}^2}$ $\quad \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$\left(y + \frac{1}{3}\right)^2$

Write the equations of the parabola in vertex form by completing the square.

3. $x = y^2 - 8y + 5$ $\quad \frac{8}{2} = 4$

$x - 5 + 4^2 = y^2 - 8y + 4^2$

$x - 5 + 16 = (y-4)^2$

$x + 11 = (y-4)^2$

or

$\boxed{(y-4)^2 = (x+11)}$

4. $y = x^2 - 5x + \frac{1}{4}$ $\quad \frac{5}{2}$

$y - \frac{1}{4} + \left(\frac{5}{2}\right)^2 = x^2 - 5x + \left(\frac{5}{2}\right)^2$

$y - \frac{1}{4} + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$

$y + \frac{24}{4} = \left(x - \frac{5}{2}\right)^2$

$y + 6 = \left(x - \frac{5}{2}\right)^2$

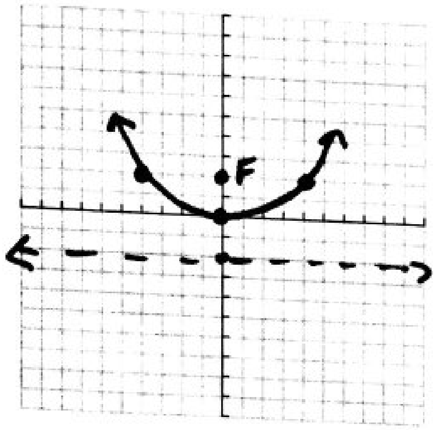
or

$\boxed{\left(x - \frac{5}{2}\right)^2 = (y+6)}$

Examples: Graph the following parabolas. State the vertex, focus, axis of symmetry, directrix, length of latus rectum, and direction of opening.

FW

a) $x^2 = 8y$



vertex
(0,0)

Opens up

$4a = 8$
$a = 2$

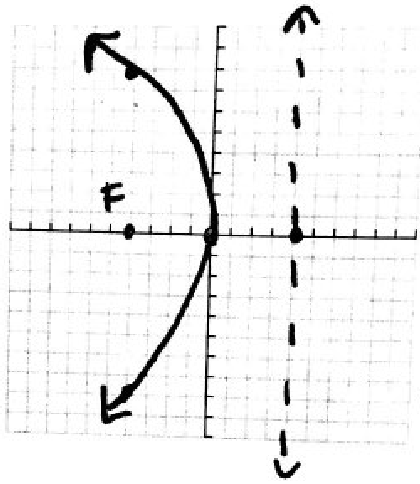
Focus
(0,2)

$y = -2$
Directrix

FW = 8

Axis of sym
$x = 0$

b) $y^2 = -16x$



vertex
(0,0

opens le

$-4a = -1$
$a = 4$

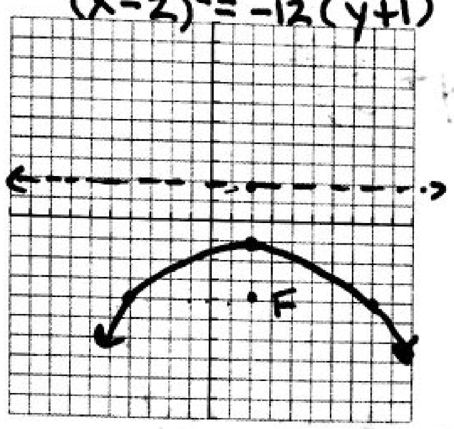
Focus
(−4,0)

Directrix
$x = 4$

FW = 16

c) $x^2 - 4x + 4 = -12y - 12$

$(x-2)^2 = -12(y+1)$



vertex
(2,−1)

opens down

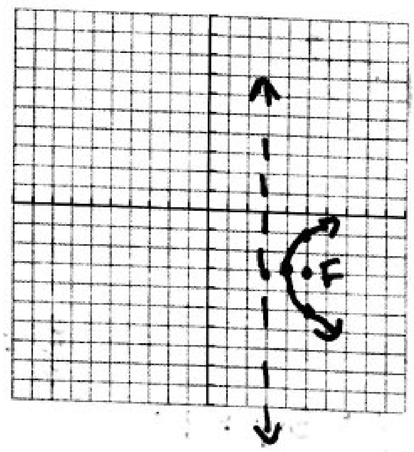
$-4a = -12$
$a = 3$

Focus
(2,−4)

FW = 12

Axis of sym
$x = 2$

d) $(y+3)^2 = 4(x-4)$



vertex
(4,−3)

opens rig

$4a = 4$
$a = 1$

Focus
(5,−3)

Directrix
$x = 3$

FW = 4

Axis of Sym
$y = -3$

Examples: Write each equation in standard form.

a) $y = x^2 + 2x + 2$

$\frac{2}{2} = 1$

$y - 2 + 1^2 = x^2 + 2x + 1^2$

$(y-1) = (x+1)^2$

or

$\boxed{(x+1)^2 = (y-1)}$

b) $x + y^2 = 2y - 3$

$\frac{2}{2} = 1$

$y^2 - 2y + 1^2 = -x - 3 + 1^2$

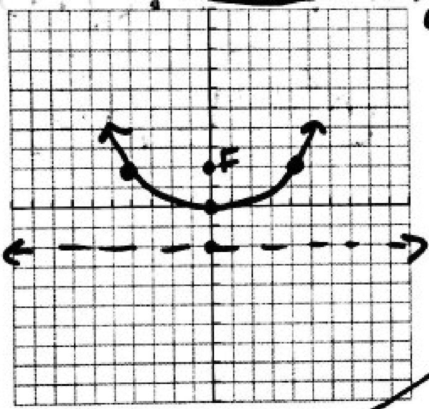
$(y-1)^2 = -x - 2$

$\boxed{(y-1)^2 = -1(x+2)}$

Examples: Find the equation of the parabola described. Find the two points that define the latus rectum and graph the equation.

a) Vertex: (0,0); Focus: (0,2)

± 4



opens up

$a = 2$

$Fw = 4a$
$= 4 \cdot 2$
$= 8$

$(x-0)^2 = 4a(y-0)$

$(x-0)^2 = 4 \cdot 2\,(y-0)$

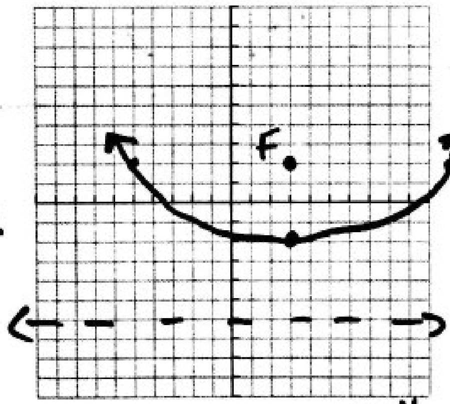
$\boxed{x^2 = 8y}$

Endpoints of LR

(4,2) (−4,2)

b) Vertex: (3,-2); Focus: (3,2)

± 8



opens up

$a = 4$

$Fw = 4a = 4$

LR/$Fw = 16$

Endpoints of

(11,2)

(−5,2)

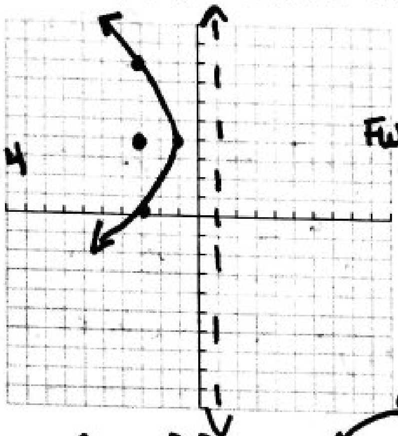
$a = 4$

$(x-3)^2 = 4a(y+2)$

$\boxed{(x-3)^2 = 16(y+2)}$

c) Vertex: (-1,4); Directrix: $x = 1$ opens left

Focus ±4
(-3,4)



a=2

$Fw = -4a = -4 \cdot 2$

$Fw = -8$

a=2

$(y-4)^2 = -4a(x+1)$

$\boxed{(y-4)^2 = -8(x+1)}$

End points of LR/FW

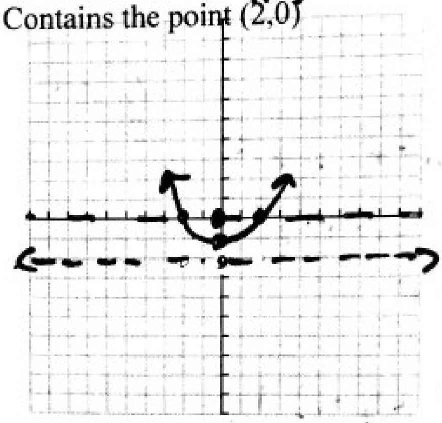
(-3, 8)

(-3, 0)

d) Vertex: (0,-1); Axis of Symmetry: y-axis; Contains the point (2,0)

x y

opens up



$(x-0)^2 = 4a(y+1)$

$(2-0)^2 = 4a(0+1)$

$4 = 4a$

$1 = a$

$(x-0)^2 = 4 \cdot 1(y+1)$

$\boxed{x^2 = 4(y+1)}$

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