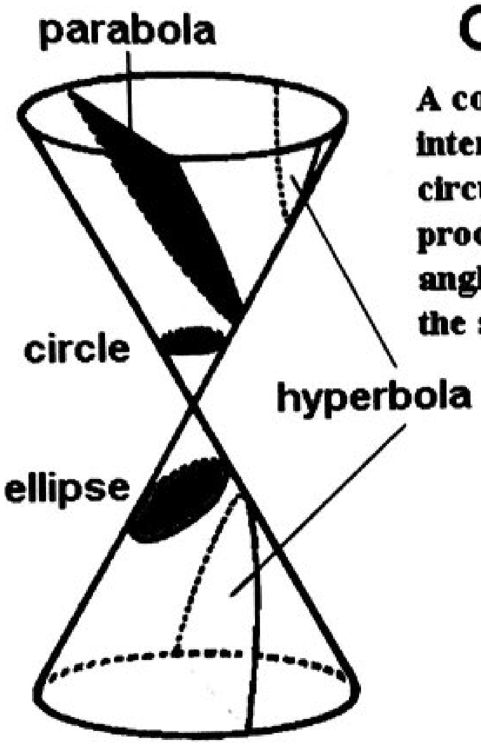
Name:____ Period:________

Sec Math 2H – Unit 5 – Conic Sections Notes

Intro to Conic Sections



Conic Sections

A conic section is formed by the intersection of a plane with a right circular cone. The "kind" of curve produced is determined by the angle at which the plane intersects the surface.

Some online resources for learning about Conic Sections:

http://www.mathplanet.com/education/algebra-2/conic-sections/equations-of-conic-sections

https://www.khanacademy.org/math/precalculus/conics-precalc

http://www.purplemath.com/modules/conics.htm

CONIC SECTIONS

Squared comes first (left side)

Parabolas with Vertex (h, k)		
~~Transformation~~ Standard Equation	$(x-h)^2 = 4a(y-k)$	$(y-k)^2 = 4a(x-h)$
Opens	Down if $a < 0$ Up if $a > 0$	Left if $a < 0$ Right if $a > 0$
Axis of Symmetry	$x = h$	$y = k$
a = distance from vertex to focus a = distance from vertex to directrix		

*If only one squared, must be parabola

Circles with Center (h, k)	
Standard Equation	$(x-h)^2 + (y-k)^2 = r^2$
Radius	r
Distance Formula	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint Formula	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

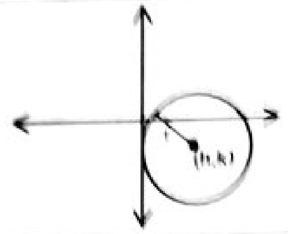
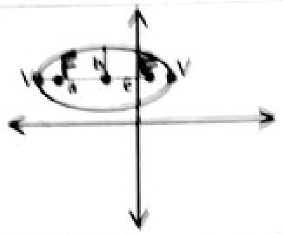
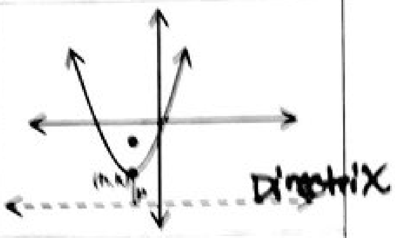
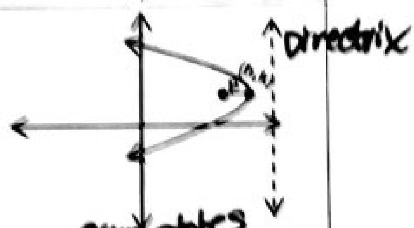
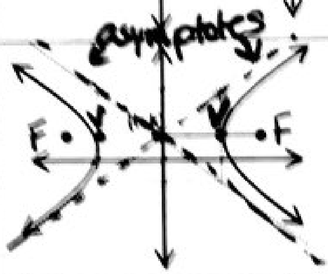
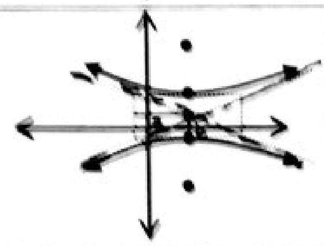
Ellipses with Center (h, k)	★ a is always the bigger number;	$c^2 = a^2 - b^2$
Standard Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Major Axis	horizontal	vertical
a = distance from center to vertices b = distance from center to co-vertices c = distance from center to foci		

Hyperbolas with Center (h, k)	★ a^2 is always first;	$c^2 = a^2 + b^2$
Standard equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transverse Axis	$y = k$	$x = h$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Asymptotes from the center	$m = \frac{rise}{run}$	$y - k = \pm m(x - h)$
Opens	Left and Right	Up and Down
	horizontal x^2 comes first	vertical y^2 comes first

Classifying Conic Sections

General Equation	Circles	Parabolas	Ellipse	Hyperbola
$Ax^2 + Cy^2 + Dx + Ey + F = 0$	$A = C$	$AC = 0$, both are not 0 (will have an x^2 term or a y^2 term, but not both)	$AC > 0$ positive #	$AC < 0$ negative #

Conic Sections

Circle	$(x-h)^2 + (y-k)^2 = r^2$	*Centered at (h, k) with a radius of r.*	
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	*Centered at (h, k).* *Distance from center to the edge of the ellipse:* *a in the x-direction b in the y-direction* *Vertices are the points on the ends of the major (longer) axis.* *Focal points (foci) c units from the center on the major axis: $c^2 = a^2 - b^2$ (always put the larger of a^2 or b^2 first)*	
Parabola	$(x-h)^2 = 4a(y-k)$	*Vertex at (h, k)* *If parabola opens vertically:* *Up if p is positive Down if p is negative* *Focus: p units from the vertex in the direction it opens* *Directrix: the **line** p units from the vertex in the opposite direction as the focus.*	
	$(y-k)^2 = 4a(x-h)$	*Vertex at (h, k)* *If parabola opens horizontally:* *Right if p is positive Left if p is negative* *Focus: p units from the vertex in the direction it opens* *Directrix: the **line** p units from the vertex in the opposite direction as the focus.*	
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	*Centered at (h, k).* *Transverse axis is horizontal.* *Vertices are a units in the x-direction from the center.* *Focal points (foci) c units from the center in the x-direction: $c^2 = a^2 + b^2$* *Slopes of Asymptotes: $\pm\frac{b}{a}$*	
	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	*Centered at (h, k).* *Transverse axis is vertical.* *Vertices are b units in the y-direction from the center.* *Focal points (foci) c units from the center in the y-direction: $c^2 = a^2 + b^2$* *Slopes of Asymptotes: $\pm\frac{b}{a}$*	

Completing the Square with Conic Sections

When the equation of a conic section isn't written in its standard form, completing the square is the only way to convert the equation to its standard form. The steps of the process are as follows:

1. Add/subtract any constant to the opposite side of the given equation, away from all the variables.
2. Factor the leading coefficient out of all terms in front of the set of parentheses.
3. Divide the remaining linear coefficient by two, but only in your head.
4. Square the answer from Step 3 and add that inside the parentheses. Don't forget that if you have a coefficient from Step 2, you must multiply the coefficient by the number you get in this step and add *that* to both sides.
5. Factor the quadratic polynomial as a perfect square trinomial.

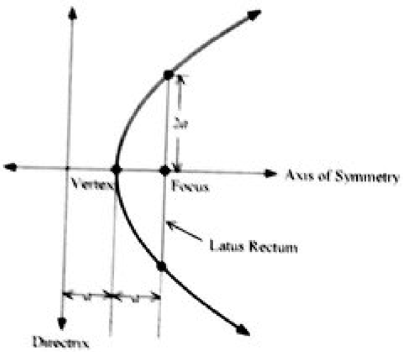
Distance Formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

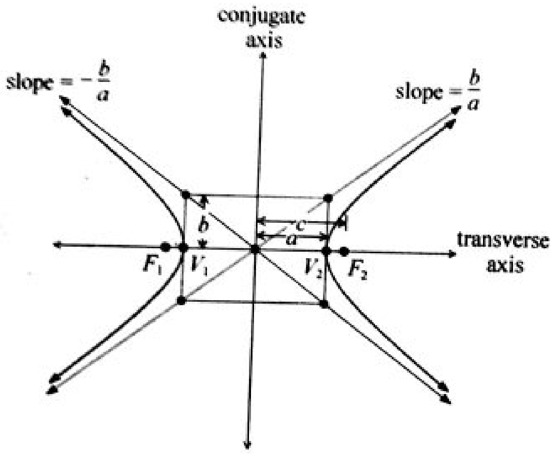
Midpoint Formula

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Parabolas

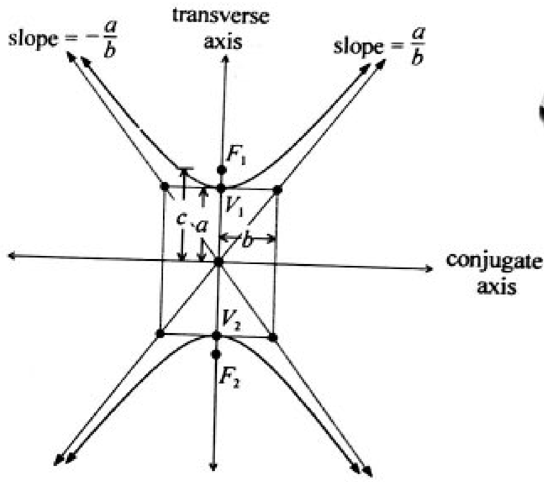


Hyperbola



$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$$a^2+b^2=c^2$$



$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$$a^2+b^2=c^2$$

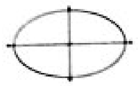
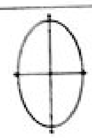
a = distance from center to vertices

c = distance from center to foci

b used to find the width of branches and slope of asymptotes

Ellipse

Standard Form of the Equation of an Ellipse with Center at (h,k)

Equation	Description	Picture
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to x-axis	
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to y-axis	

a = Distance from center to vertices
b = Distance from center to covertices
c = Distance from center to foci