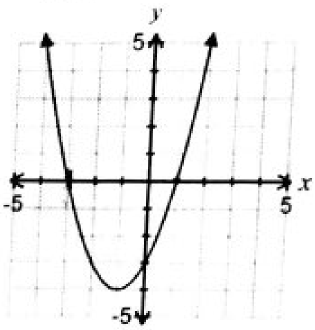
4.4 Solving Quadratic Inequalities & Systems of Equations by Graphing

Examples: Solve each inequality using the graph of $f(x) = x^2 + 2x - 3$.

Notice that each of these inequalities involves the value of $x^2 + 2x - 3$, which is represented by the y-coordinate of the graph. In each case, we are trying to figure out what x-values (x-coordinates) make the inequality true. When trying to find where $x^2 + 2x - 3 > 0$, we are trying to figure out what x-coordinates have a y-coordinate that is bigger than zero—in other words, *where is the graph above the x-axis?*

$f(x) = x^2 + 2x - 3$



a) $x^2 + 2x - 3 > 0$ positive above the x axis

$(-\infty, -3) \cup (1, \infty)$

b) $x^2 + 2x - 3 \geq 0$ above or on x-axis

$(-\infty, -3] \cup [1, \infty)$

c) $x^2 + 2x - 3 < 0$ negative below x axis

$(-3, 1)$

d) $x^2 + 2x - 3 \leq 0$ below or on x axis

$[-3, 1]$

Solving a Quadratic Inequality Using the Graph:

1. Write the inequality in standard form. Replace the inequality sign with an equal sign and solve the equation $ax^2 + bx + c = 0$ by factoring, completing the square, or using the quadratic formula. This gives you the x-intercepts of the graph of $y = ax^2 + bx + c$.

2. Graph $y = ax^2 + bx + c$. The graph does not have to be very detailed. A rough sketch of a parabola opening in the correct direction with the correct x-intercepts is all you need.

   must be 0

3. The solutions of $ax^2 + bx + c > 0$ are the x-values for which the graph is **above** the x-axis.
   The solutions of $ax^2 + bx + c \geq 0$ are the x-values for which the graph is **on or above** the x-axis.
   The solutions of $ax^2 + bx + c < 0$ are the x-values for which the graph is **below** the x-axis.
   The solutions of $ax^2 + bx + c \leq 0$ are the x-values for which the graph is **on or below** the x-axis.

4. If the inequality involves $\boxed{\leq \text{ or } \geq}$, the x-intercepts **are included** in the solution set (use brackets). [ ]
   If the inequality involves $\boxed{< \text{ or } >}$, the x-intercepts **are not included** in the solution set (use parentheses).
   ( )

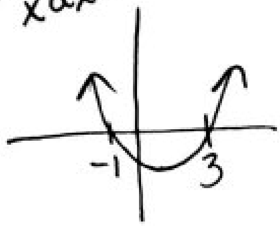
Sketch the graph / Factor First to find x int.

Examples: Solve each quadratic inequality and write the solution set in interval notation.

up x int above or on x axis

a) $(x-3)(x+1) \geq 0$

$p=3 \quad q=-1$

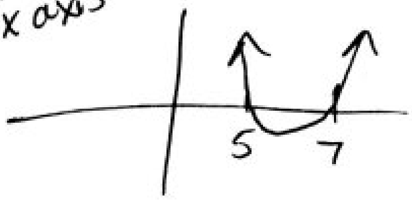


$(-\infty,-1] \cup [3,\infty)$

up below x axis

b) $(x-7)(x-5) < 0$

$p=7 \quad q=5$



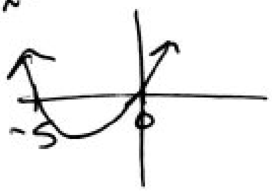
$(5,7)$

GCF Factor above x axis

c) $x^2+5x>0$

up $x(x+5) > 0$

$p=0 \quad q=-5$



$(-\infty,-5) \cup (0,\infty)$

below or on x axis

Short cut

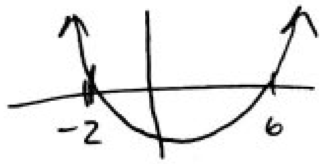
x	+
-12	-4
-6	2

up

d) $x^2-4x-12 \leq 0$

$(x-6)(x+2) \leq 0$

$p=6 \quad q=-2$



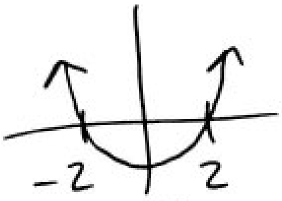
$[-2,6]$

xx 2·2 Dif. of SQ.

below x axis

e) $x^2-4<0$

$(x+2)(x-2) < 0$



$(-2,2)$

f) ~~$x^2+10x \geq -9$~~

$-x^2-10x \geq 9$

$ -9 \quad -9$

GCF

$-x^2-10x-9 \geq 0$

Short cut

x	+
9	10
9	1

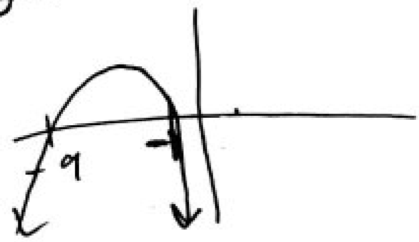
Factor

$-(x^2+10x+9) \geq 0$

above or on x axis

down $-1(x+9)(x+1) \geq 0$

$p=-9 \quad q=-1$



$[-9,-1]$

g) $x^2 + 10 \le 7x$ below or on x axis

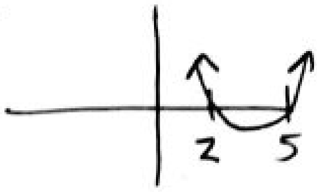
Short cut

x	+
10	−7
−5	−2

$x^2 - 7x + 10 \le 0$

$(x-5)(x-2) \le 0$

$x = 5 \quad x = 2$



$[2, 5]$

h) $x^2 > x + 6$

$-x-6 \quad -x-6$

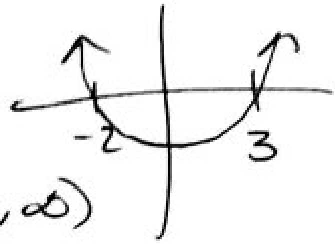
$x^2 - x - 6 > 0$ above x axis

Short cut

−6	−1
−3	2

$(x-3)(x+2) > 0$

$p = 3 \quad q = -2$



$(-\infty, -2) \cup (3, \infty)$

Solving Systems of Equations by Graphing

Solving a system of equations means finding the values of x and y that make both equations true. The solutions are usually written as ordered pairs (x, y).

Solving by graphing:

1. Solve both equations for y.
2. Graph both equations using $y = mx + b$, transformations, or x, y tables.
3. The points where the two graphs intersect (cross) are the solutions.
4. Write the solutions as ordered pairs.
5. Solve for y

Examples: Solve by graphing.

a) $x + y = 6$
$\quad -x \quad -x$
$y = x^2 + 4$

$y = -x + 6$

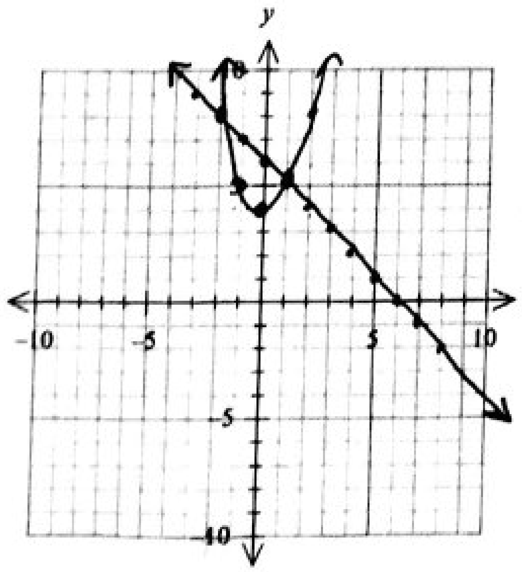
vertex form

$y = (x+0)^2 + 4$ up

vertex $(0, 4)$

$\leftrightarrow 1 \uparrow 1$

$\leftrightarrow 2 \uparrow 4$



$\boxed{(1,5) \quad (-2,8)}$

b) $6 - 2y = 4x^2$
$6x - 3y = 3$

$6 - 2y = 4x^2$
$-6 \qquad -6$
$\frac{-2y}{-2} = \frac{4x^2}{-2} \frac{-6}{-2}$

$y = -2x^2 + 3$ Graph

$y = -2(x+0)^2 + 3$ ← vertex form

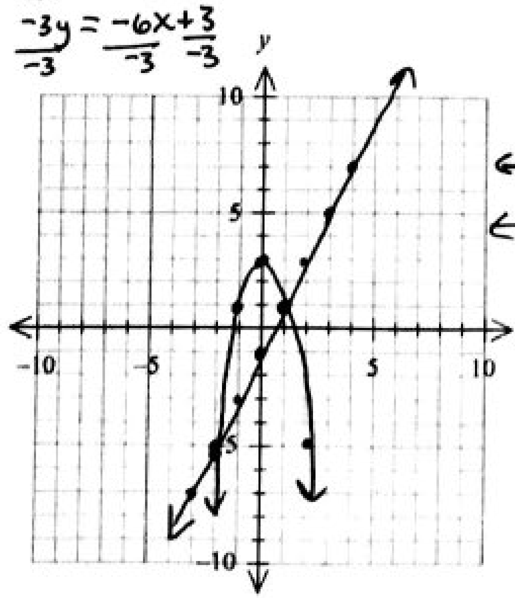
$y = 2x - 1$

$-6x \qquad -6x$
$\frac{-3y}{-3} = \frac{-6x + 3}{-3}$

vertex $(0, 3)$ down

$\leftrightarrow 1 \downarrow 1 \cdot -2 = -2$

$\leftrightarrow 2 \downarrow 4 \cdot -2 = -8$



$\boxed{(1,1) \quad (-2,-5)}$

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