4.1 Writing equations from a graph or from set of information

Examples: Write a quadratic equation or function in **Vertex Form**: $f(x) = y = a(x-h)^2 + k$ ← opposite

If you know the vertex of a parabola, (h,k), then you still need at least one other point on the parabola in order to write an equation.

- Use the vertex form and fill in all the information you have.
- Then use the point on the parabola and substitute in for x and y.
- Solve for a.
- Write your final equation

a) Vertex: $(2,1)$, passes through $(4,13)$ — h k, x y

Vertex: $y = a(x-2)^2 + 1$

Plug in point: $13 = a(4-2)^2 + 1$

$a(2)^2 + 1$

$13 = 4a + 1$

$12 = 4a$

$3 = a$

Plug in original equation

$y = 3(x-2)^2 + 1$

b) Vertex: $(-5,3)$, passes through $(-1,-29)$ — h k, x y

$y = a(x+5)^2 + 3$ plug in vertex

$-29 = a(-1+5)^2 + 3$ plug in point

$-3 \quad -3$

$-32 = a(4)^2$

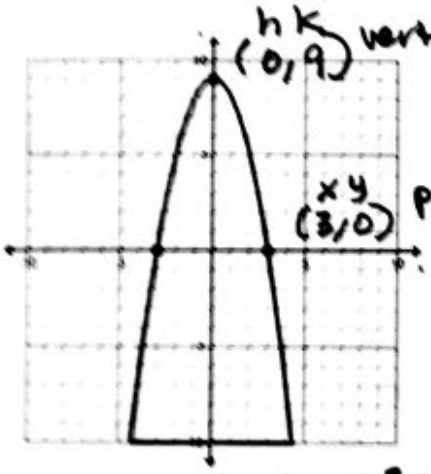
$-32 = 16a$

$-2 = a$

$y = -2(x+5)^2 + 3$

Examples: Write the equation of each parabola based on the information in the graph. Follow the steps outlined above. Leave the equations in **Vertex Form**.

a)



h k (0,9) vertex

x y (3,0) point

$y = a(x-0)^2 + 9$

$0 = a(3-0)^2 + 9$

$a \cdot 3^2 + 9$

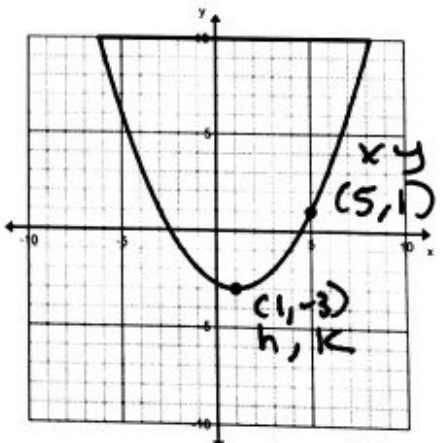
$0 = 9a + 9$

$\frac{-9}{9} = \frac{9a}{9}$

$a = -1$

$y = -(x-0)^2 + 9$

$y = -x^2 + 9$



x y (5,1)

(1,-3) h, k

$y = a(x-1)^2 - 3$

$1 = a(5-1)^2 - 3$

$1 = a \cdot 4^2 - 3$

$1 = 16a - 3$

$4 = 16a$

$\frac{4}{16} = \frac{16a}{16}$

$\frac{1}{4} = a$

$y = \frac{1}{4}(x-1)^2 - 3$

Examples: Write a quadratic equation or function in **Factored Form:** $f(x) = y = a(x-p)(x-q)$ (opposite)

Use the factored form if you know the roots (a.k.a. solutions, x-intercepts, zeros). You will still need to know at least one other point on the parabola in order to write an equation.

- Use the factored form and fill in all the information you have.
- Then use the point on the parabola and substitute in for x and y.
- Solve for a.
- Write your final equation

a) Roots: $(3,0)$ & $(-2,0)$, goes through $(2,-4)$ (p, q; x, y)

$y = a(x-3)(x+2)$
$-4 = a(2-3)(2+2)$
$-4 = a(-1)(4)$
$-4 = -4a$
$1 = a$

$y = (x-3)(x+2)$

b) x-intercept: $(3,0)$ & $(-2,0)$, goes through $(0,12)$ (p, q; x, y)

$y = a(x-3)(x+2)$
$12 = a(0-3)(0+2)$
$12 = a(-3)(2)$
$\frac{12}{-6} = \frac{-6a}{-6}$
$-2 = a$

$y = -2(x-3)(x+2)$

c) Zeros: $x = -1$ & $x = 3$, goes through $(6,-10)$ (p, q; x, y)

$y = a(x+1)(x-3)$
$-10 = a(6+1)(6-3)$
$-10 = a(7)(3)$
$-10 = 21a$
$\frac{-10}{21} = a$

$y = \frac{-10}{21}(x+1)(x-3)$

d) Roots: $x = \sqrt{7}$ & $x = -\sqrt{7}$, goes through $(-6,29)$ (p, q; x, y)

$y = a(x-\sqrt{7})(x+\sqrt{7})$
$29 = a(6-\sqrt{7})(6+\sqrt{7})$ FOIL
$36 + 6\sqrt{7} - 6\sqrt{7} - \sqrt{49}$
$29 = 29a$
$1 = a$

$y = (x-\sqrt{7})(x+\sqrt{7})$

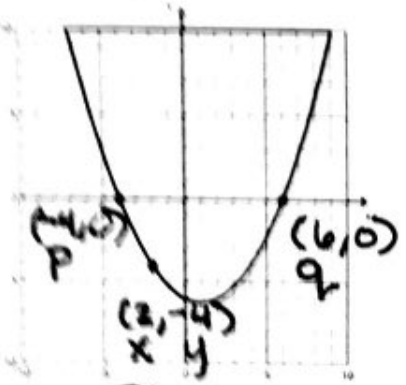
e) Solutions: $x = 8i$ & $x = -8i$, passes through $(-2,-204)$ (p, q; x, y)

$y = a(x-8i)(x+8i)$
$-204 = a(-2-8i)(-2+8i)$ FOIL
$a(4 - 16i + 16i - 64i^2)$
$4 + 64$
$-204 = 68a$
$-3 = a$

$y = -3(x-8i)(x+8i)$

Examples: Write the equation of each parabola based on the information in the graph. Leave the equations in Factored Form.

a)



$y = a(x+4)(x-6)$

$-4 = a(2+4)(2-6)$

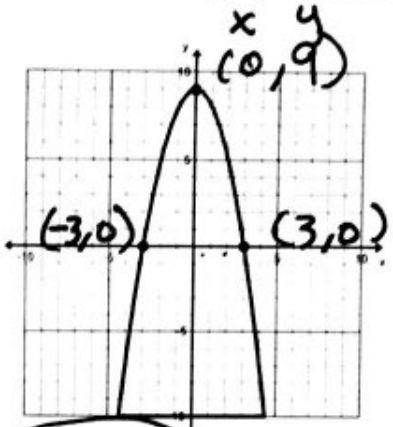
$a(6)(-4)$

$-4 = -24a$

$\frac{1}{6} = a$

$y = \frac{1}{6}(x+4)(x-6)$

b)



$y = a(x+3)(x-3)$

$9 = a(0+3)(0-3)$

$9 = a(3)(-3)$

$9 = -9a$

$-1 = a$

$y = -(x+3)(x-3)$

Examples: Write a quadratic equation or function in **Standard Form**: $f(x) = ax^2 + bx + c$

- First write the equation in either Vertex Form or Factored Form (whichever seems easier)
- Then use correct order of operations to multiply/distribute in order to get rid of parenthesis
- List the three terms in correct order: x^2, then x, then the constant term

a) Use the graph just above to write the equation in **Standard Form**

$y = -(x+3)(x-3)$

$y = -(x^2 - 3x + 3x - 9)$

$y = -(x^2 - 9)$

$y = -x^2 + 9$

b) Write one of the vertex form equations in **Standard Form**

$y = 3(x-2)^2 + 1$

$y = 3(x-2)(x-2) + 1$

$y = 3(x^2 - 2x - 2x + 4) + 1$

$y = 3x^2 - 12x + 12 + 1$

$y = 3x^2 - 12x + 13$

c) Write the equation with the following characteristics in **Factored Form**, **Vertex Form** and **Standard Form** (yes, we'll need to complete the square to get it into vertex form ☺)

roots at $(-3, 0)$ and $(5, 0)$ and goes through the point $(2, -15)$

Factored

$y = a(x+3)(x-5)$

$-15 = a(2+3)(2-5)$

$-15 = a(5)(-3)$

$-15 = -15a$

$1 = a$

$y = (x+3)(x-5)$

standard

$y = (x+3)(x-5)$

$y = x^2 - 5x + 3x - 15$

$y = x^2 - 2x - 15$

vertex

$y = x^2 - 2x - 15$

$y + 15 + 1^2 = x^2 - 2x + 1^2$

$y + 16 = (x-1)^2$

$y = (x-1)^2 - 16$