4.2 Graphing using zeroes, solutions, roots, and x-intercepts

Zeros **_of a Function:_** The values of x that make $f(x)$ or y equal zero. If the zeros are real, they tell you the places where the graph crosses the x-axis, or the ***x-intercepts*** of the graph.

Other words for zeros: ***solutions to*** $f(x)=0$, ***roots, x-intercepts.*** Zeros = x int = roots

To find x int, plug in zero for y and solve.

Finding zeros (x-intercepts):

1. Change y or $f(x)$ to 0.
2. Solve for x.
   - If the equation is in factored form, solving for x is easy – just think "What would x have to be to make each set of parentheses equal to 0?"
   - If the equation is in standard form, solve by factoring or by using quadratic formula
   - If the equation is in vertex form, get the perfect square by itself, take the square root of both sides (don't forget the ±), then solve for x.

★ If your answers are imaginary (negative under the square root), the graph doesn't have x-intercepts.

Examples: For each equation, find the zeros and state whether the graph opens up or down. Then match the equation to the correct graph.

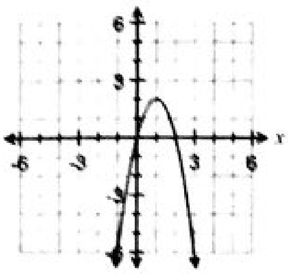
a) $y = 1(x-1)(x+3)$ up

b) $f(x) = -\frac{1}{2}x(x+2)$ down

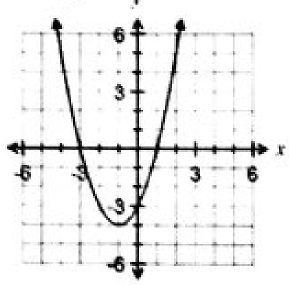
c) $y = 3(x-3)(x-1)$ up

d) $f(x) = -2x(x-2)$ down

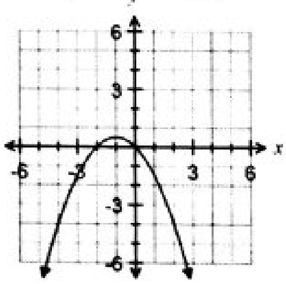
Graph 1: D



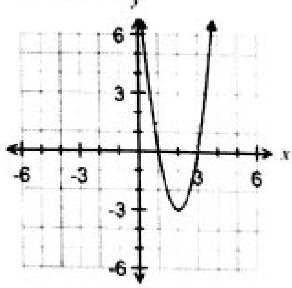
Graph 2: A



Graph 3: B



Graph 4: C



Examples: For each function, do the following: 1) State whether the parabola has a maximum or minimum. 2) State whether the parabola opens up or down. 3) Find the x-intercept(s). 4) Find the y-intercept. 5) Draw a rough sketch of the graph.

down

a) $f(x) = -(x+4)(x-1)$ Max/Min: Maximum Direction: down

Zeros / x-intercept(s):

Plug in zero for y + solve

$0 = -(x+4)(x-1)$

$x+4=0 \quad x-1=0$

$\boxed{(-4,0)}$ $x=-4$ $\quad$ $x=1$ $\boxed{(1,0)}$

y-intercept: Plug in 0 for x.

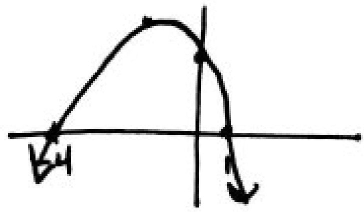
$f(0) = -(0+4)(0-1)$

$= -(4)(-1)$

$= 4$

$\boxed{(0,4)}$

sketch graph:



Graphing from Factored Form:

1. Determine whether the parabola will open up or open down.
2. Find the zeros or x-intercepts.
   - Let $f(x) = 0$ and solve the equation ← Plug in 0 for y + solve
   - Mark the x-intercepts on the graph.
3. Find the y-intercept but substituting in $x = 0$. Mark this point on the graph.
4. Find the axis of symmetry and the vertex. Use the method from 4.1
   - If there is only one zero, then the axis of symmetry will run vertically through that point and that x-intercept will also be the vertex.
5. Use the pattern from Section 4.1 to find other points on the graph now that you have the location of the vertex.

**Remember that a parabola is a smooth curve. Do not draw straight lines!

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = (x-1)(x+3)$ $\quad p = 1 \quad q = -3$

up

Direction of Opening: up

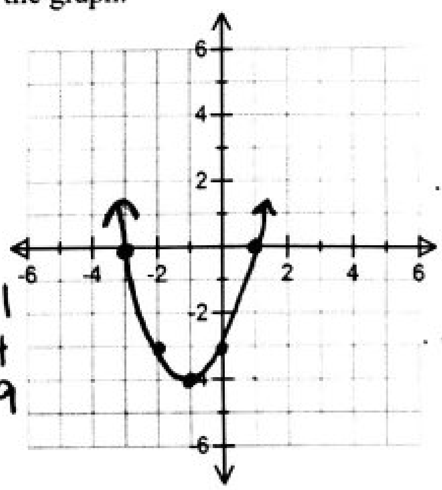
Vertex: $x = \frac{p+q}{2} = \frac{1+-3}{2} = \frac{-2}{2} = -1$ $\quad (-1,-4)$

$f(-1) = (-1-1)(-1+3) = (-2)(2) = -4$

↔1 ↑1

↔2 ↑4

↔3 ↑9



Is it a maximum or minimum point? minimum

Axis of Symmetry: $x = -1$

Zeros (x-intercepts): $(1,0)$ $(-3,0)$

y-intercept: $(0,-3)$

Domain: $(-\infty,\infty)$

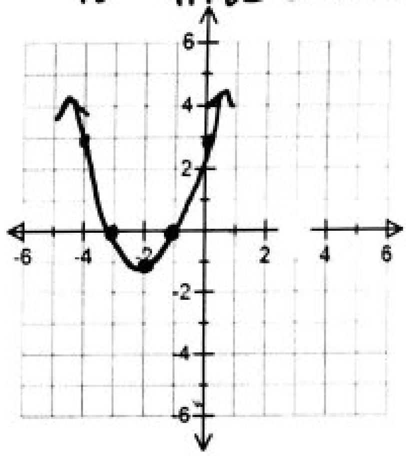
Range: $[-4,\infty)$

up short cut $\begin{array}{c|c} \times & + \\ 3 & 4 \\ 1 & 3 \end{array}$

* can use factored form or standard form to find vertex

b) $f(x) = x^2 + 4x + 3$ ← y int

Factored Form: $f(x) = (x+1)(x+3)$ ← x int



Direction of Opening: up

Vertex: $x = \frac{-1+-3}{2} = \frac{-4}{2} = -2$

$(-2,-1)$

$f(-2) = (-2+1)(-2+3)$

$(-1)(1) = -1$

Is it a maximum or minimum point? minimum

Axis of Symmetry: $x = -2$

Zeros (x-intercepts): $(-1,0)$ $(-3,0)$

$f(x) = (x+1)(x+3)$

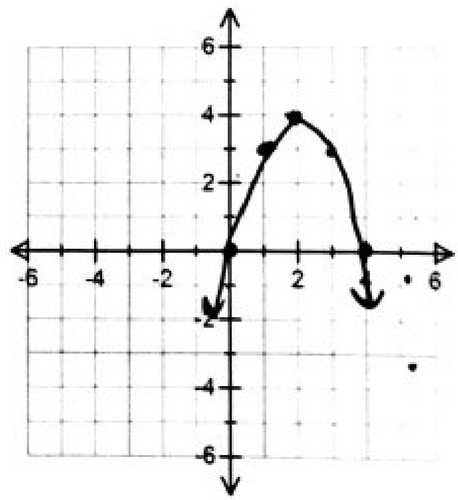
$x+1=0$, $-1\ -1$, $x=-1$

$x+3=0$, $-3\ -3$, $x=-3$

y-intercept: $(0,3)$

c) $f(x) = -x^2 + 4x$

Factored Form: $f(x) = -x(x-4)$ ← x int.



Direction of Opening: down

Vertex: $x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$(2,4)$

$f(2) = -(2)^2 + 4(2) = -4+8 = 4$

Is it a maximum or minimum point? maximum

Axis of Symmetry: $x = 2$

Zeros (x-intercepts): $\frac{-x}{-1} = \frac{0}{-1}$, $x=0$ $(0,0)$

$x-4=0$, $x=4$ $(4,0)$

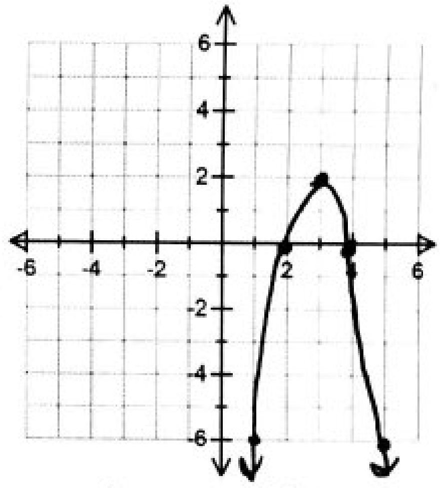
y-intercept: $(0,0)$

d) $f(x) = -2x^2 + 12x - 16$ ← yint

GCF $-2(x^2 - 6x + 8)$ ← shortcut
$-2(x-4)(x-2)$

x +
8 -6
-4 -2

Factored Form: $f(x) = \underline{-2(x-4)(x-2)}$ ← xint



Direction of Opening: down

Vertex: $x = \frac{4+2}{2} = \frac{6}{2} = 3$ (3,2)

$f(3) = -2(3)^2 + 12(3) - 16$
$-18 + 36 - 16 = 2$

Is it a maximum or minimum point?

Axis of Symmetry: $x = 3$

Zeros (*x*-intercepts):

$x - 4 = 0$
$+4 \ +4$
$x = 4$

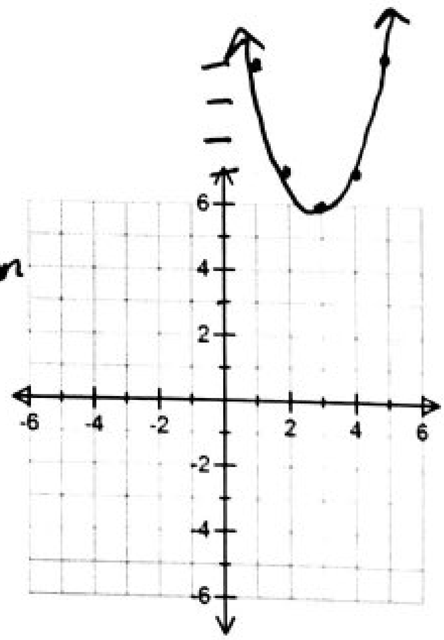
$x - 2 = 0$
$+2 \ +2$
$x = 2$

⟷ 1 ↓ 1·2 = 2
⟷ 2 ↓ 4·2 = 8

y-intercept: (0, -16)

e) $f(x) = x^2 - 6x + 15$ ← shortcut
yint
$a=1$ $b=-6$ $c=15$

x +
15 -6
1 15
3 5



Factored Form: $f(x) = \underline{\text{no factored form}}$
* must use standard form

Direction of Opening: up

Vertex: $x = \frac{-b}{2a} = \frac{6}{2(1)} = \frac{6}{2} = 3$ (3,6)

$f(3) = 3^2 - 6(3) + 15$
$9 - 18 + 15 = 6$

Is it a maximum or minimum point?

Axis of Symmetry: $x = 3$

Zeros (*x*-intercepts): no x intercepts

y-intercept: (0,15)