SM2H Name:_____ Period: Key

4.1 Graphing Quadratic Functions: Vertex and Axis of Symmetry

Find the vertex and the direction of the opening of the graph for each of the following quadratic equations. Find the *y*-intercept and axis of symmetry.

1. $y = (x-4)^2 + 3$

yint
$y = (0-4)^2+3$
$= (-4)^2+3$
$= 16+3 = 19$
$(0, 19)$

Vertex: $(4, 3)$
Axis of Symmetry: $x = 4$
Direction of opening: up
y-intercept: $(0, 19)$

2. $y = -2(x+3)^2$

yint
$y = -2(0+3)^2$
$= -2(3)^2$
$= -18$

Vertex: $(-3, 0)$
Axis of Symmetry: $x = -3$
Direction of opening: down
y-intercept: $(0, -18)$

3. $y = x^2 - 2x - 11$ yint

$x = \frac{-b}{2a} = \frac{2}{2(1)} = \frac{2}{2} = 1$ $(1, -12)$

$y = 1^2 - 2(1) - 11$
$= 1 - 2 - 11$
$= -12$

yint
$y = x^2 - 2x - 11$

Vertex: $(1, -12)$
Axis of Symmetry: $x = 1$
Direction of opening: up
y-intercept: $(0, -11)$

4. $f(x) = -2x^2 + 8x - 58$ yint

$x = \frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2$

$f(2) = -2(2)^2 + 8(2) - 58 = -50$

Vertex: $(2, -50)$
Axis of Symmetry: $x = 2$
Direction of opening: down
y-intercept: $(0, -58)$

5. $y = (x-3)(x-7)$

$x = \frac{3+7}{2} = 5$

$y = (5-3)(5-7)$
$(2)(-2)$
-4

yint
$y = (0-3)(0-7)$
$(-3)(-7)$
21

Vertex: $(5, -4)$
Axis of Symmetry: $x = 5$
Direction of opening: up
y-intercept: $(0, 21)$

6. $f(x) = (x+2)(x-6)$
$x = -2 \quad x = 6$

$x = \frac{-2+6}{2} = \frac{4}{2} = 2$

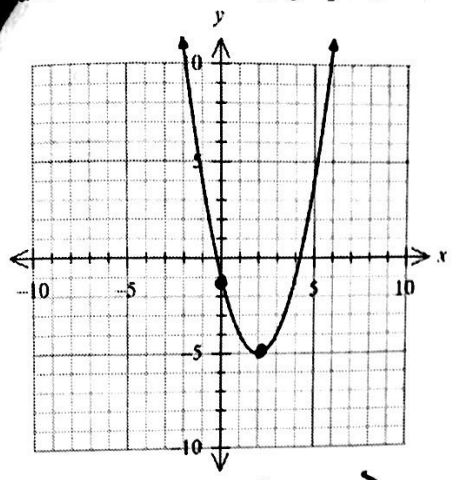
$f(2) = (2+2)(2-6)$
$(4)(-4)$
-16

yint
$y = (0+2)(0-6)$
$y = 2(-6)$
$y = -12$

Vertex: $(2, -16)$
Axis of Symmetry: $x = 2$
Direction of opening: up
y-intercept: $(0, -12)$

6a. What do the vertex and axis of symmetry always have in common? the x values are the same

ach of the following graphs, find the vertex, axis of symmetry, and y-intercept.



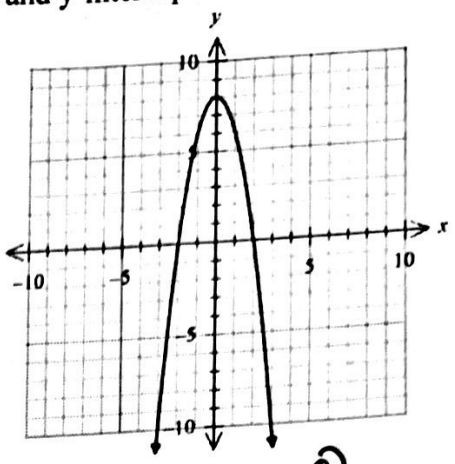
Vertex: (2, -5)

Axis of Symmetry: x = 2

y-intercept: (0, -1)

is the value of "a" positive or negative? positive

8.



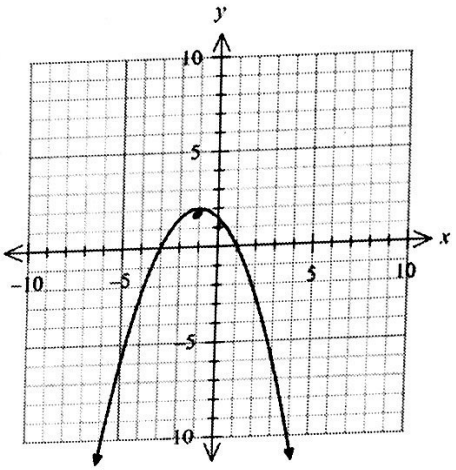
Vertex: (0, 8)

Axis of Symmetry: x = 0

y-intercept: (0, 8)

is the value of "a" positive or negative? negativ

9.



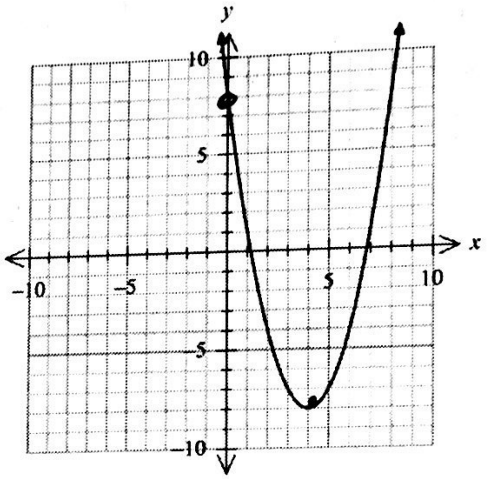
Vertex: (1, 2)

Axis of Symmetry: x = 1

y-intercept: about (0, 1) or rounded (0, 1.3)

is the value of "a" positive or negative? negative

10.



Vertex: (4, -8)

Axis of Symmetry: x = 4

y-intercept: (0, 8)

is the value of "a" positive or negative? positive

Solve.

11. $(x+3)(2x-5)=0$

$x+3=0$ $2x-5=0$

$x=-3$ $x=5/2$

12. $-3(x-7)^2+45=0$

$\frac{-3(x-7)^2}{-3} = \frac{-45}{-3}$

$\sqrt{(x-7)^2} = \pm\sqrt{15}$

$x = 7 \pm \sqrt{15}$

13. $4x^2-11=3x$

$a=4$ $b=-3$ $c=-11$

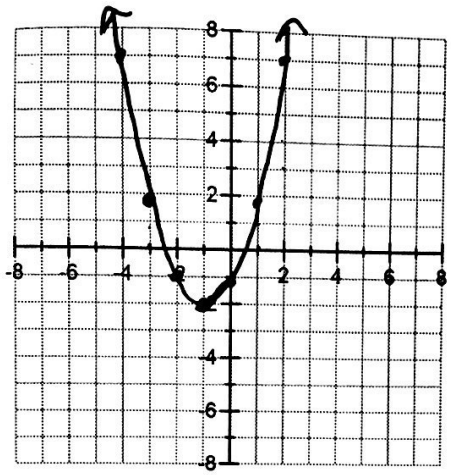
$4x^2-3x-11=0$

$x = \frac{3 \pm \sqrt{9-4(4)(-11)}}{2(4)}$ $9+176$

$x = \frac{3 \pm \sqrt{185}}{8}$

…e the vertex and graph each parabola. Clearly mark the vertex and four other points on the graph.

14. $y = x^2 + 2x - 1$ Vertex: $(-1, -2)$



$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$y = (-1)^2 + 2(-1) - 1$

$1 - 2 - 1 = -2$

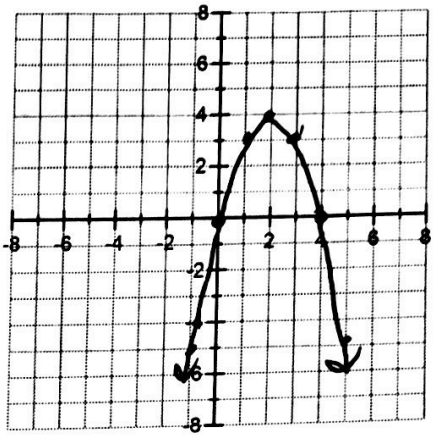
$y = x^2$

$\leftrightarrow 1 \uparrow 1$

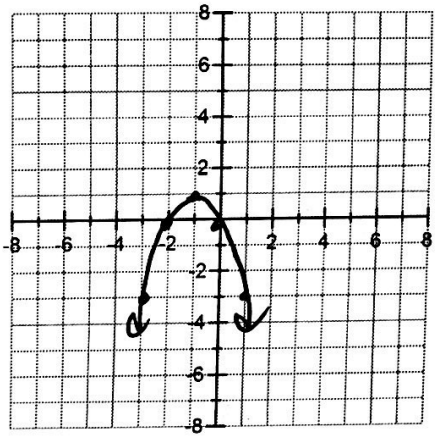
$\leftrightarrow 2 \uparrow 4$

$\leftrightarrow 3 \uparrow 9$

15. $y = -(x-2)^2 + 4$ Vertex: $(2, 4)$



16. $f(x) = -x^2 - 2x$ Vertex: $(-1, 1)$



$x = \frac{-b}{2a} = \frac{2}{2(-1)} = -1$

$f(-1) = -(-1)^2 - 2(-1)$

$= -1 + 2 = 1$

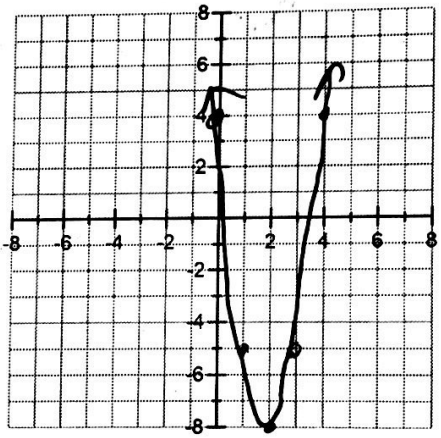
17. $y = 3(x-2)^2 - 8$ Vertex: $(2, -8)$

Increases

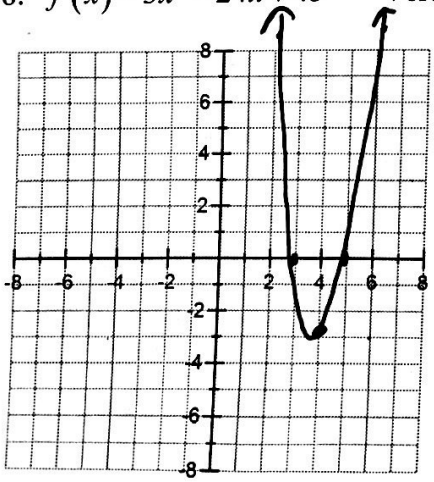
$\leftrightarrow 1 \updownarrow 1 \cdot 3$

$\leftrightarrow 2 \updownarrow 4 \cdot 3$

$\leftrightarrow 3 \updownarrow 9 \cdot 3$



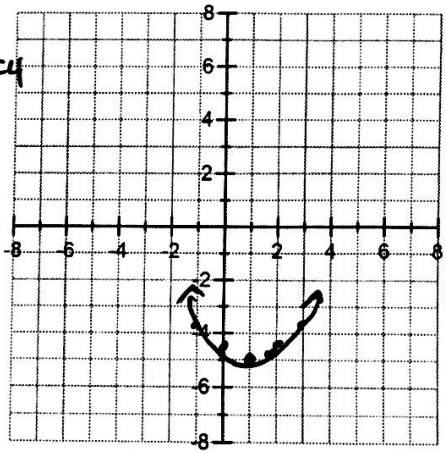
18. $f(x) = 3x^2 - 24x + 45$ Vertex: $(4, -3)$



$x = \frac{-b}{2a} = \frac{+24}{2(3)} = \frac{24}{6} = 4$

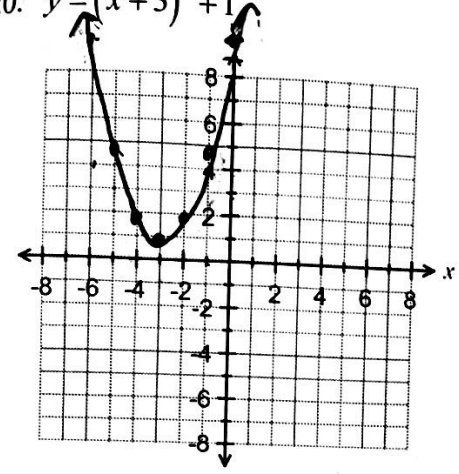
$3(4)^2 - 24(4) + 45$

19. $y = \frac{1}{3}(x-1)^2 - 5$ Vertex: $(1, -5)$



…l in the requested information for each function. Draw the graph. You need AT LEAST 5 POINTS!

20. $y=(x+3)^2+1$



Vertex: (-3, 1)

Axis of Symmetry: x = -3

Direction of Opening: up

Is the vertex a maximum or a minimum? min (-3, 1)

Maximum or minimum value: 1

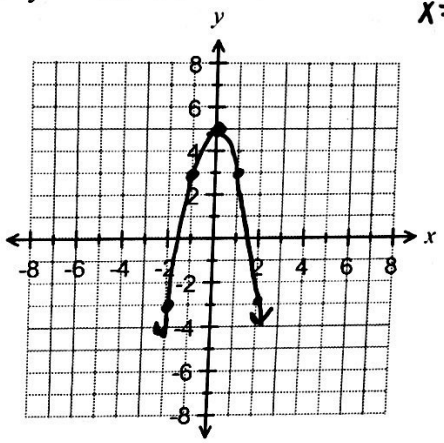
y-intercept: (0, 10)

Domain: (-∞, ∞)

Range: [1, ∞)

21. $y=-2x^2+5$

$x=\frac{-0}{2(-2)}=0$



Vertex: (0, 5)

Axis of Symmetry: x = 0

Direction of Opening: down

Is the vertex a maximum or a minimum? max (0, 5)

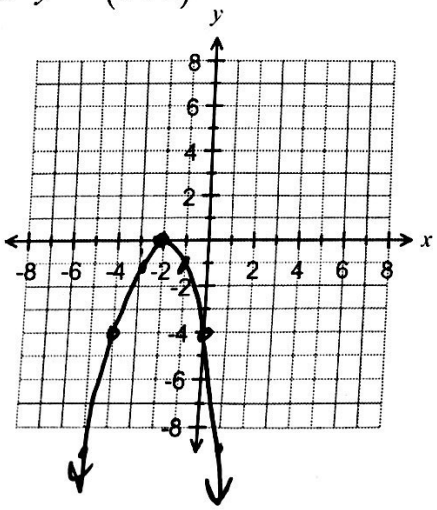
Maximum or minimum value: 5

y-intercept: (0, 5)

Domain: (-∞, ∞)

Range: (-∞, 5]

22. $y=-(x+2)^2$



Vertex: (-2, 0)

Axis of Symmetry: x = -2

Direction of Opening: down

Is the vertex a maximum or a minimum? max (-2, 0)

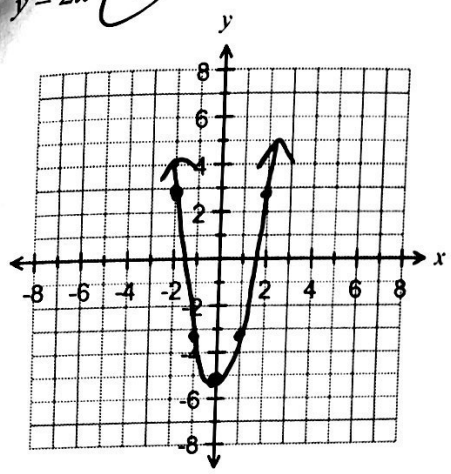
Maximum or minimum value: 0

y-intercept: (0, -4)

Domain: (-∞, ∞)

Range: (-∞, 0]

23. $y = 2x^2 - 5$ (-5) yint $x = \frac{-b}{2a} = \frac{0}{2(2)} = 0$



Vertex: (0, -5)

Axis of Symmetry: x = 0

Direction of Opening: up

Is the vertex a maximum or a minimum? min (0, -5)

Maximum or minimum value: -5

y-intercept: (0, -5)

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

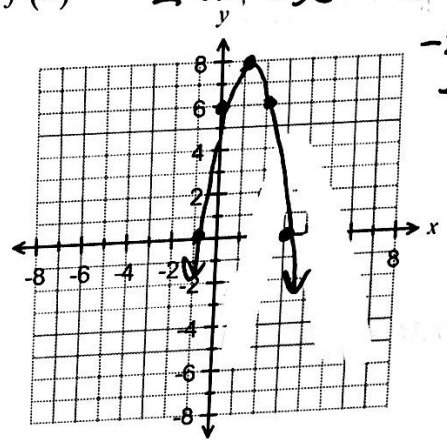
24. $f(x) = -2(x-3)(x+1)$ a p q $x = \frac{3+-1}{2} = \frac{2}{2} = 1$

$-2(1-3)(1+1)$

$-2(-2)(2)$

8

(1, 8)



Vertex: (1, 8)

Axis of Symmetry: x = 1

Direction of Opening: down

Is the vertex a maximum or a minimum? maximum

Maximum or minimum value: 8

y-intercept: (0, 6)

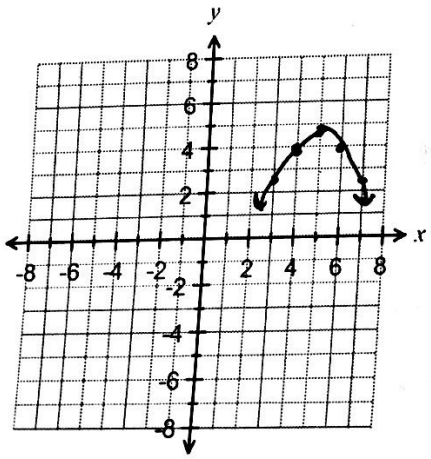
Domain: $(-\infty, \infty)$

Range: $(-\infty, 8]$

$-12.5 \quad 25$

$-\frac{1}{2}(5)^2 + 5(5) - 8$

25. $y = -\frac{1}{2}x^2 + 5x$ (-8) yint $x = \frac{-b}{2a} = \frac{-5}{2(-\frac{1}{2})} = \frac{-5}{-1} = 5$



Vertex: (5, 4.5)

Axis of Symmetry: x = 5

Direction of Opening: down

Is the vertex a maximum or a minimum? (5, 4.5)

Maximum or minimum value: 4.5

y-intercept: (0, -8)

Domain: $(-\infty, \infty)$

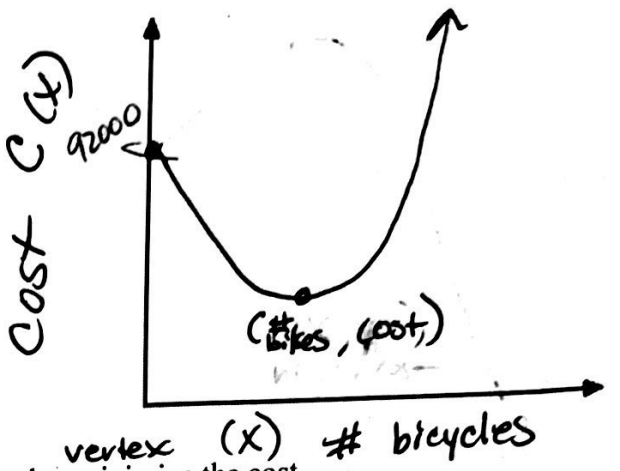
Range: $(-\infty, 4.5]$

Range: _____

ch problem, draw a rough sketch of a graph representing the situation. Determine which variable gs on each axis. SHOW ALL YOUR WORK!

The cost C in dollars of manufacturing x bicycles at a production plant is given by the function $C(x) = 2x^2 - 800x + 92{,}000$.

a. Sketch of graph, label what each axis represents.



b. Find the number of bicycles that must be manufactured to minimize the cost.

$\frac{-b}{2a}$ $\quad x = \frac{800}{2(2)} = \frac{800}{4} =$ **200 bicycles**

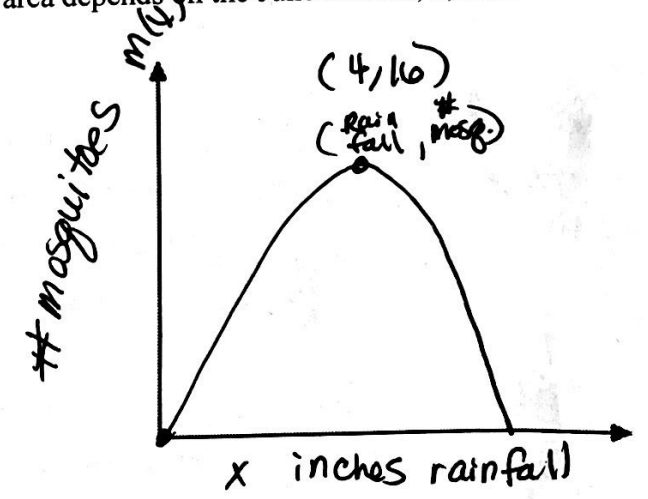
c. Find the minimum cost.

$C(200) = 2(200)^2 - 800(200) + 92000$

$= \$12{,}000$

27. The number of mosquitoes, M, in millions, in a certain area depends on the June rainfall, x, in inches: $M(x) = -x^2 + 8x$.

a. Sketch of graph, label what each axis represents.



b. How much rain results in the maximum number of mosquitoes?

vertex

$x = \frac{-b}{2a} = \frac{-8}{2(-1)} = \frac{-8}{-2} =$ **4 inches rain**

c. What is the maximum number of mosquitoes?

$M(4) = -(4)^2 + 8(4)$

$-16 + 32 =$ 16 million mosquitoes