SM2H

Secondary Math 2 Honors
Unit 4 Graphing Quadratic Functions

0 Forms of Quadratic Functions

standard **Form:** $f(x) = ax^2 + bx + c$, where $a \neq 0$. There are no parentheses. ← y int

Example: $f(x) = -3x^2 + 2x - 7$ ← y int.

factored **Form:** $f(x) = a(x - p)(x - q)$, where $a \neq 0$. Written as a multiplication problem. opposite

p and q are x intercepts

Also known as **intercept form**.

Example: $f(x) = (x-4)(x+5)$ ← x intercepts

vertex **Form:** $f(x) = a(x - h)^2 + k$, where $a \neq 0$. x only shows up once, as part of a perfect square. opposite

(h, k) vertex

Example: $f(x) = 2(x+7)^2 - 1$

Conic Form of a parabola: $4p(y-k) = (x-h)^2$ or $4p(x-h) = (y-k)^2$

Examples: $4(y-2) = (x+5)^2$ or $-8(x+6) = (y-1)^2$

Examples: State whether each quadratic function is in standard, factored, or vertex form.

a) $f(x) = 2(x+3)(x-5)$

Factored

p = -3 q = 5

b) $f(x) = -(x+4)^2 - 5$

vertex

c) $f(x) = x^2 + 2x + 4$ ← y int

standard

d) $f(x) = -x^2 + 5x$

standard

e) $f(x) = 3x(x-2)$

Factored

f) $f(x) = 2(x+1)^2 - 3$

vertex

g) $f(x) = -(x+5)^2$

vertex

same as

OR

$-(x+5)(x+5)$

Factored

h) $f(x) = -3x^2 + 4$

standard

OR

$-3(x+0)^2 + 4$

vertex

i) $f(x) = 5x^2$

standard

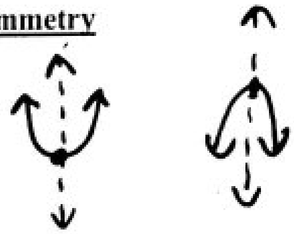
or

vertex

or

factored

4.1 Graphing Quadratic Functions: Vertex and Axis of Symmetry



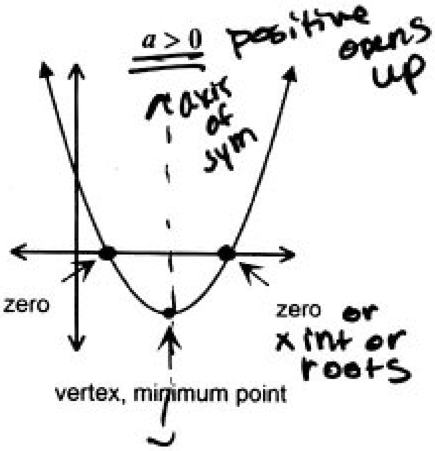
Vocabulary:

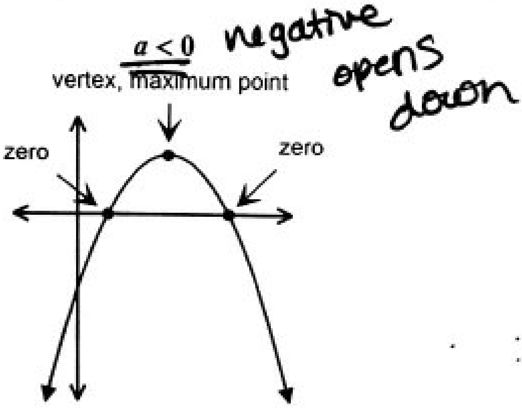
Parabola: The shape of the graph of a quadratic function.

axis of symmetry: A line that cuts a parabola in half. If you were to fold a parabola along its axis of symmetry, the two sides would overlap. The equation of the axis of symmetry looks like $x = \#$.

vertex: The "tip" of the parabola – the point at which it changes direction.

- If the parabola opens up ($a > 0$), the vertex is the lowest point on the graph, or the ***minimum point.***
- If the parabola opens down ($a < 0$), the vertex is the highest point on the graph, or the ***maximum point.***





Finding the y-intercept:

1. Plug in 0 for x.
2. Simplify. Don't forget order of operations.

opposite

Vertex Form of a Quadratic Function: $y = a(x-h)^2 + k$

axis of sym

Vertex: (h,k)

Axis of Symmetry: $x = h$

- The sign of h is the *opposite* of the sign in the equation. h moves the graph of $y = x^2$ right and left in the *opposite* direction as the sign in the equation (but the *same* direction as the sign of h itself).
- The sign of k is the *same* as the sign in the equation. k moves the graph of $y = x^2$ up and down in the *same* direction as the sign in the equation.
  - For $y = (x-2)^2 + 5$, $h = 2$ and $k = 5$. The vertex is $(2,5)$ and the axis of symmetry is $x = 2$. The graph of $y = x^2$ moved *right* 2 and *up* 5.

    axis of sym
  - For $y = (x+3)^2 - 7$, $h = -3$ and $k = -7$. The vertex is $(-3,-7)$ and the axis of symmetry is $x = -3$. The graph of $y = x^2$ moved *left* 3 and *down* 7.

Scanned with CamScanner

vertex Form

$$f(x) = a(x-h)^2 + k$$

Direction of Opening:

- Opens up if a is positive.
- Opens down if a is negative.

Examples: For each function, do the following: 1) State the coordinates of the vertex. 2) State the direction of the opening, that is, whether the parabola opens up or down. 3) Find the y-intercept. 4) Draw a rough sketch of the graph. 5) Find the Domain and Range

down

a) $y = -3(x+2)^2 + 27$

Vertex: $(-2, 27)$

Direction: down

axis of symmetry:

$\boxed{x = -2}$

y-intercept:

Plug in zero for x

$y = -3(0+2)^2 + 27$

$-3 \cdot 2^2 + 27$

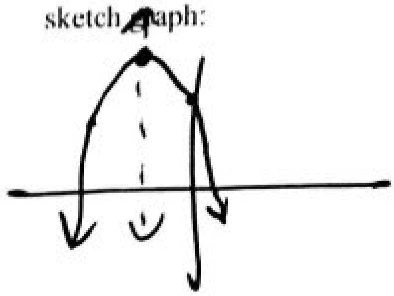
$-3 \cdot 4 + 27$

$-12 + 27$

$y = 15$

$\boxed{(0, 15)}$

sketch graph:



Domain: $(-\infty, \infty)$ Range: $(-\infty, 27]$

c) $f(x) = \frac{1}{2}(x+4)^2 - 6$

Vertex: $(-4, -6)$

Direction: up

axis of symmetry:

$x = -4$

y-intercept:

Plug in 0 for x

$\frac{1}{2}(0+4)^2 - 6$

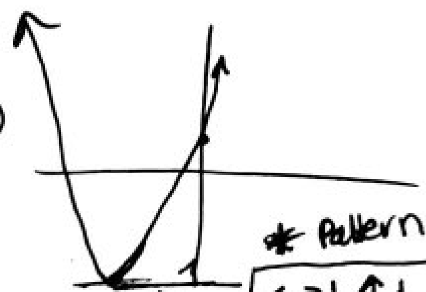
$\frac{1}{2} \cdot 4^2 - 6$

$\frac{1}{2} \cdot 16 - 6$

$8 - 6$ $y = 2$

$(0, 2)$

sketch graph:

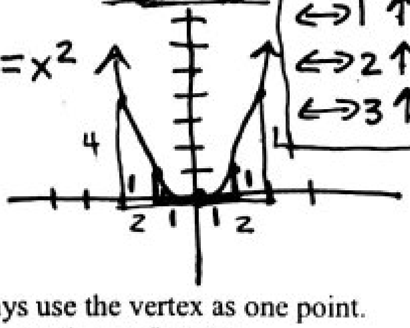


Domain: $(-\infty, \infty)$ Range: $[-6, \infty)$

x	y
1	1
2	4

x	y
-2	4
-1	1
0	0

$y = x^2$



* Pattern

↔1 ↑1

↔2 ↑4

↔3 ↑9

Vertical Stretch:

- a changes how wide or narrow the graph is.
  - If $|a| > 1$, the graph is *narrower* than the graph of $y = x^2$.
  - If $|a| < 1$, the graph is *wider* than the graph of $y = x^2$.
- Figure out the exact shape of the graph by making an x,y table. Always use the vertex as one point. Then choose two x-values on each side of the vertex to plug into the equation to find the corresponding y-coordinates.
- A shortcut is to use counting patterns to graph the parabola. Start at the vertex, then count:

From the vertex

↔1, ↑a

↔2, ↑$4a$ If a is negative, count down instead of up.

↔3, ↑$9a$, etc.

  - For $y = 2(x-3)^2 - 4$, $a = 2$.

    Start at the vertex $(3, -4)$, and count ↔1, ↑2; ↔2, ↑8; ↔3, ↑18…

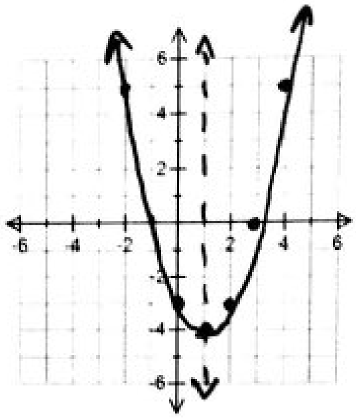
Examples: Fill in the requested information for each function. Then draw the graph.

opposite

a) $f(x)=(x-1)^2-4$

$a =$ 1 $h =$ 1 $k =$ −4

From Vertex
↔1 ↑1
↔2 ↑4
↔3 ↑9



Direction of Opening: up (positive)

Vertex: $(1,-4)$

Is it a maximum or minimum point? minimum

Axis of Symmetry: $x=1$

y-intercept: $f(0)=(0-1)^2-4=(-1)^2-4=1-4=-3$ $(0,-3)$

Domain: $(-\infty,\infty)$

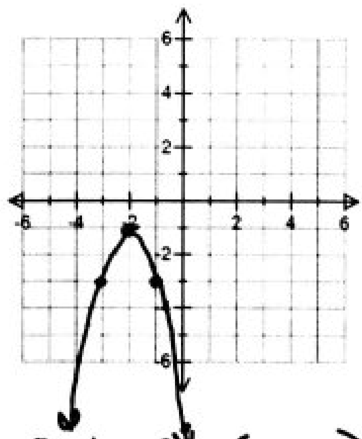
Range: $[4,\infty)$

down opposite multiply pattern by a

b) $f(x)=-2(x+2)^2-1$

$a =$ −2 $h =$ −2 $k =$ −1

From vertex
↔1 ↓ 1·2 = 2
↔2 ↓ 4·2 = 8



Direction of Opening: down

Vertex: $(-2,-1)$

Is it a maximum or minimum point? maximum

Axis of Symmetry: $x=-2$

y-intercept: $f(0)=-2(\cancel{0+2})^2-1$ $-2\cdot 2^2$ $=-8-1=-9$ $(0,-9)$

Domain: $(-\infty,\infty)$

Range: $(-\infty,-1]$

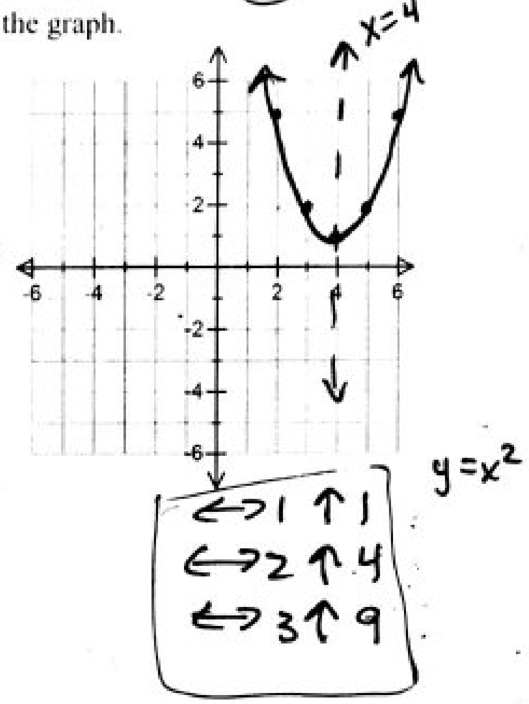
Standard Form: $f(x)=ax^2+bx+c$

- Just like with the other forms, the graph opens up if a is positive and opens down if a is negative.
- **Vertex:**
  - The x-coordinate of the vertex is $\frac{-b}{2a}$. (The opposite of b divided by 2 times a)
  - To find the y-coordinate, plug the x-coordinate into the original equation.

Standard Form $f(x)=ax^2+bx+c$ yint

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = x^2 - 8x + 17$ ← yint $a =$ 1 $b =$ −8 $c =$ 17



Direction of Opening: up

Vertex: How to find vertex in standard form

(4, 1)

$x = \frac{-b}{2a} = \frac{8}{2(1)} = \frac{8}{2} = 4$

$f(4) = 4^2 - 8(4) + 17$

$16 - 32 + 17 = 1$

Is it a maximum or minimum point? min

What is the maximum/minimum **value**? 1

Axis of Symmetry: $x = 4$

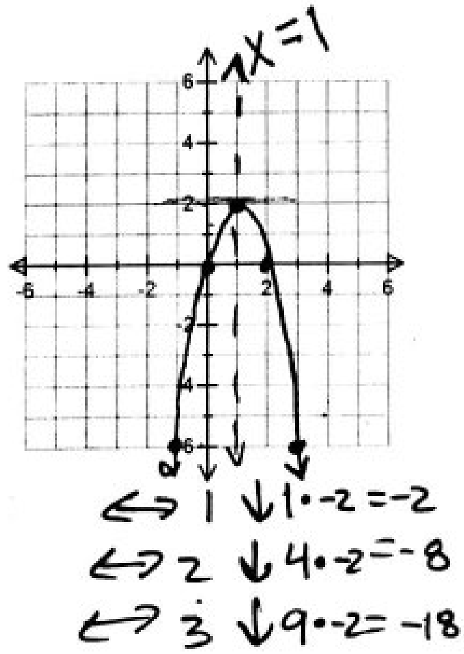
y-intercept: (0, 17)

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

← mult. pattern by

b) $y = -2x^2 + 4x + 0$ ← down ← yint $a =$ −2 $b =$ 4 $c =$ 0



Direction of Opening: down

Vertex: $x = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$

(1, 2)

$y = -2(1)^2 + 4(1)$

$-2 + 4 = 2$

Is it a maximum or minimum point? max

What is the maximum/minimum **value**? 2

Axis of Symmetry: $x = 1$

y-intercept: (0, 0)

Factored Form: $f(x) = a(x-p)(y-q)$

- Like other forms, a is the vertical stretch
- $(p, 0)$ and $(q, 0)$ are the x-intercepts (zeroes). The x-value of the vertex is exactly half-way between them
- Evaluate the function at $x = \frac{p+q}{2}$ to find the y value of the vertex.

Factors tell me the x int. $f(x) = a(x-p)(x-q)$

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = 1(x-5)(x+1)$ positive opposites $p = $ 5 $q = $ −1 x int.

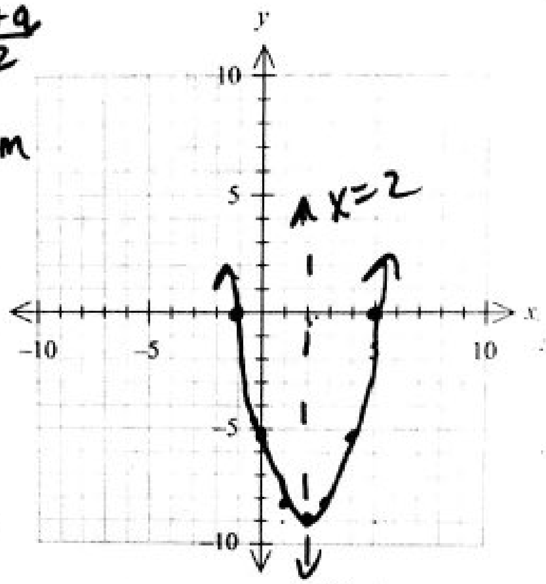
direction of opening: up $x = \frac{p+q}{2}$

vertex: $x = 2$ How to find vertex in factored form

$\boxed{(2,-9)}$

$x = \frac{5+-1}{2} = \frac{4}{2} = 2$

$f(2) = (2-5)(2+1) = (-3)(3) = -9$



Is it max or min: Minimum

Axis of Symmetry: $x = 2$

y-intercept: Plug in zero for x or (0, −5) look at graph

Domain: $(-\infty, \infty)$

Range: $[-9, \infty)$

↔1 ↑1
↔2 ↑4
↔3 ↑9

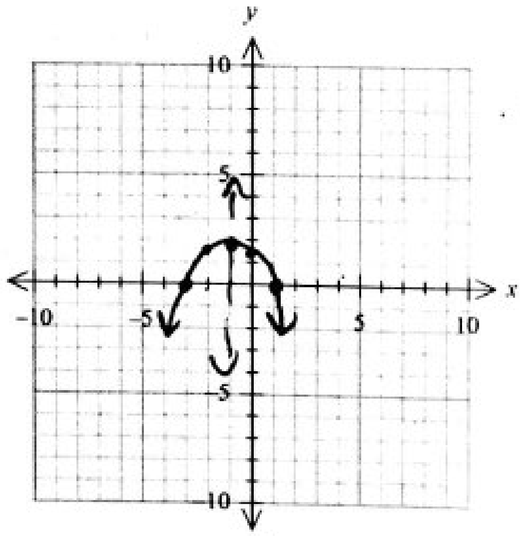
b) $y = -\frac{1}{2}(x+3)(x-1)$ $p = $ −3 $q = $ 1 x int

direction of opening: down

vertex: $x = -1$ (1/2 way between x int)

$x = \frac{-3+1}{2} = \frac{-2}{2} = -1$ $(-1, 2)$

$y = -\frac{1}{2}(-1+3)(-1-1) = 2$



Is it max or min: max

Axis of Symmetry: $x = -1$

y-intercept: $(0, 1.5)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

↔1 ↓1 · −1/2 = −1/2
↔2 ↓4 · −1/2 = −2