10.5 Probabilities from Venn Diagrams

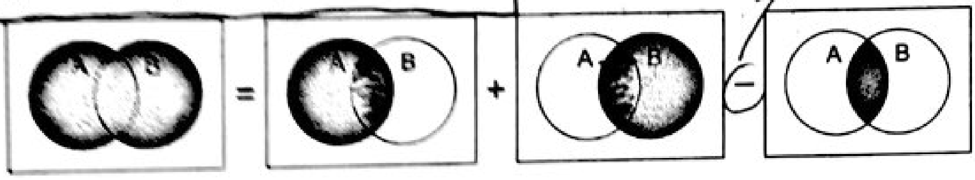
Probability: A value that represents the likelihood of an event. It can be expressed as a fraction, decimal, or a percentage. A probability of 0 means that the event is impossible and a probability of 1 (or 100%) means that the event is certain to occur.

$$\text{probability} = \frac{\text{total \# of favorable outcomes in the category of interest}}{\text{total \# of possible outcomes}}$$

overlap or

Remember, $(A \cap B)$ means "A and B" and $(A \cup B)$ or means "A or B (or both)". With "or" probabilities, makes sure you don't count the individuals who fall in both categories twice!

* $\boxed{P(A \overset{or}{\cup} B) = P(A) + P(B) - P(A \cap B)}$ middle part got counted twice



TOTAL   * write answer as reduced fraction

Example: In the Math Club, there are 34 students. Eleven of the students are seniors, including 7 of the 20 girls. A student is chosen at random from the club. Fill in the table and find the following probabilities:

	Seniors	Non-Seniors	Total
Boys	4	10	14
Girls	7	13	20
Total	11	23	34

a) $P(\text{boy}) = \frac{\text{TOT boys}}{\text{Total}} = \frac{14 \div 2}{34 \div 2} = \boxed{\frac{7}{17}}$

b) $P(\text{senior})$ $\boxed{\frac{11}{34}}$

overlaps and

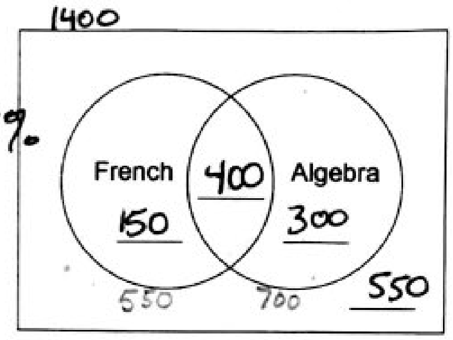
c) $P(\text{boy} \cap \text{senior})$ $\frac{4 \div 2}{34 \div 2} = \boxed{\frac{2}{17}}$

d) $P(\text{girl} \overset{or}{\cup} \text{non-senior})$ $\frac{7+13+10}{34} = \frac{30 \div 2}{34 \div 2} = \boxed{\frac{15}{17}}$

$\frac{20+23-13}{34} = \frac{30}{34} = \boxed{\frac{15}{17}}$

write percent rounded to nearest tenth.

Example: The number of students in a high school is 1400. Of those students, 550 take French, 700 take algebra, and 400 take both French and algebra. Fill in the Venn diagram, then find the following probabilities.



a) P(does not take French) $\frac{300+550}{1400} = \frac{850}{1400} = 60.7\%$

b) P(algebra $\cap$ French) $= \frac{400}{1400} = 28.6\%$ and overlap

c) P(algebra, but not French) $\frac{300}{1400} = 21.4\%$

d) P(algebra $\cup$ French) $= \frac{150+400+300}{1400} = \frac{850}{1400} = 60.7\%$ or

Conditional Probability: The probability of an event occurring when we already know that another event has occurred.

Example: $P(\text{lung cancer}|\text{smoke})$ would mean the probability of a person getting lung cancer given that the person smokes. given TOTAL (denominater in fraction)

Conditional Probability Formula: $P(A|B) = \frac{P(A\cap B)}{P(B)}$ or $\frac{\text{total \# in } A\cap B}{\text{total \# in } B}$

★ **"And" and "or" probabilities are fractions of the entire sample, but with conditional probabilities, the condition becomes the denominator of the fraction!**

Examples:

An ice cream shop keeps track of whether people order vanilla or chocolate ice cream and whether they order a sugar cone or a waffle cone. Fill in the marginal totals and find the requested probabilities.

	Sugar Cone	Waffle Cone	Total
Vanilla	35	26	61
Chocolate	51	47	98
Total	86	73	159

* leave answers as decimals rounded to the nearest thousandth. (3 dec. places)

a) $P(\text{vanilla})$

$\frac{61}{159} = .384$

b) $P(\text{waffle})$

$\frac{73}{159} = .459$

c) $P(\text{sugar})$

$\frac{86}{159} = .541$

d) $P(\text{chocolate})$ $\frac{98}{159} = .616$

e) $P(\text{vanilla} \cap \text{sugar})$ and

$\frac{35}{159} = .220$

f) $P(\text{vanilla} \cap \text{waffle})$ and

$\frac{26}{159} = .164$

g) $P(\text{chocolate} \cap \text{sugar})$ and

$\frac{51}{159} = .321$

h) $P(\text{chocolate} \cap \text{waffle})$ and

$\frac{47}{159} = .296$

i) $P(\text{vanilla} \cup \text{sugar})$

$\frac{35+26+51}{159} = .704$ (112)

j) $P(\text{vanilla} \cup \text{waffle})$

$\frac{35+26+47}{159} = .679$

k) $P(\text{chocolate} \cup \text{sugar})$

$\frac{35+51+47}{159} = .836$

l) $P(\text{chocolate} \cup \text{waffle})$

$\frac{51+47+26}{159} = .780$

Conditional Prob.

m) $P(\text{vanilla}|\underline{\text{sugar}})$ total

$\frac{35}{86} = .407$

n) $P(\text{vanilla}|\underline{\text{waffle}})$ tot

$\frac{26}{73} = .356$

o) $P(\text{chocolate}|\underline{\text{sugar}})$ tot

$\frac{51}{86} = .593$

p) $P(\text{chocolate}|\underline{\text{waffle}})$ tot

$\frac{47}{73} = .644$

q) $P(\text{sugar}|\underline{\text{vanilla}})$ tot

$\frac{35}{61} = .574$

r) $P(\text{sugar}|\underline{\text{chocolate}})$ tot

$\frac{51}{98} = .520$

s) $P(\text{waffle}|\text{vanilla})$

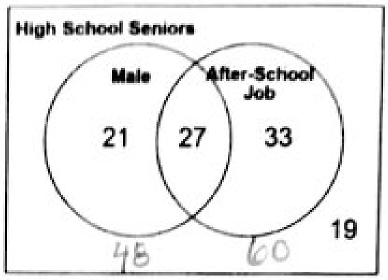
$\frac{26}{61} = .426$

t) $P(\text{waffle}|\text{chocolate})$

$\frac{47}{98} = .480$

* reduced fraction

Examples: Use the Venn diagram to find the following probabilities.



21
27
33
19
100
TOT senior

a) $P(\text{job}|\text{male})$ TOT

$\frac{27}{48} = \boxed{\frac{9}{16}}$

b) $P(\text{female}|\text{job})$ TOT

$\frac{33}{60} = \boxed{\frac{11}{20}}$

c) $P(\text{male}|\text{no job})$ TOT

$\frac{21}{21+19} = \boxed{\frac{21}{40}}$

d) $P(\text{no job}|\text{female})$ TOT

$\frac{19}{33+19} = \boxed{\frac{19}{52}}$

e) A student from the sample works at McTaco. What is the probability that the student is male?

JOB

$P(\text{male}|\text{JOB}) = \frac{27}{60} = \boxed{\frac{9}{20}}$

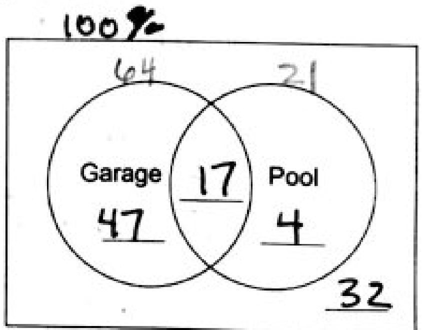
f) Is a student from the sample more likely to have a job if he is a male? Justify your answer using conditional probability.

$P(\text{job}|\text{male}) = \frac{9}{16} = 56.3\%$

$P(\text{job}|\text{female}) = \frac{33}{33+19} = \frac{33}{52} = 63.5\%$

% to nearest %

Examples: Real-estate ads suggest that 64% of homes have a garage, 21% have a pool, and 17% have both a garage and a pool. Fill in the Venn diagram, then answer the following questions.



a) Find $P(\text{garage} \text{ or } \text{pool})$

$\frac{47+17+4}{100} = \frac{68}{100} = 68\%$

b) Find $P(\text{garage}|\text{pool})$ TOT

$\frac{17}{21} = .809$ $\boxed{81\%}$

c) Find $P(\text{pool}|\text{garage})$ TOT

$\frac{17}{64} = 27\%$

d) Find $P(\text{pool}|\text{no garage})$ TOT

$\frac{4}{4+32} = \frac{4}{36} = \boxed{11\%}$

e) Find $P(\text{no pool}|\text{garage})$ TOT

f) Find $P(\text{no garage}|\text{no pool})$ TOT