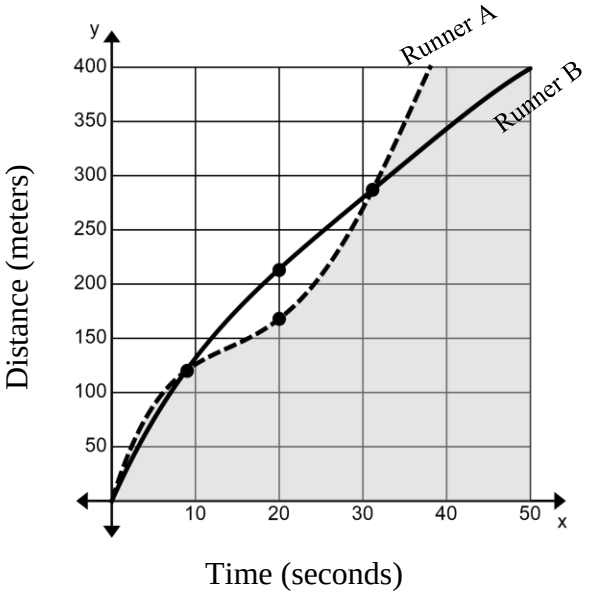
SM2H Name:_____ Period:_____

SM2H 1.6 HW-Average Rate of Change

Core Alignment: F.IF.4, F.IF.6, F.IF.9

The **average rate of change** between two points is essentially the slope of the line that connects the two points. For example, if the function $f(t)$ represents the distance in miles a car travels at any time $t \geq 0$ (hours), then finding the slope between $t = 1$ and $t = 4$ will give the average speed (mph) the car was traveling during those three hours. Recall that slope is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. In this example y is the distance traveled and x is the time interval that elapses. The slope or rate of change shows the change in distance divided by the change in time which results in the average speed.

Below is the graph of two runners as they run a 400 meter hurdles race. Runner A is the dashed line and Runner B is the solid line.



Time	Runner A's Distance	Runner B's Distance
0	0	0
9	120	120
20	168	213
31	287	287

A. Which runner has a faster average speed for the first 9 seconds? Explain how you know.

B. Which runner has a faster average speed for 9 to 20 seconds? Explain how you know.

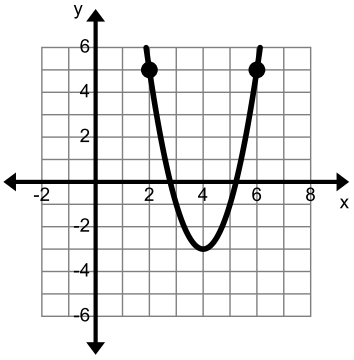
C. Which runner has a faster average speed for 20 to 31 seconds? Explain how you know.

D. Which runner has a faster average speed for 9 to 31 seconds? Explain how you know.

E. Which runner wins the race? Use mathematical evidence to support your answer.

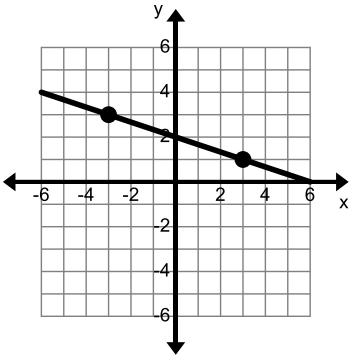
For each of the following, draw the line that connects the two points. Write the coordinates of the two points then calculate the average rate of change on the specified interval.

1. $f(x) = 2x^2 - 16x + 29$ on [2, 6]



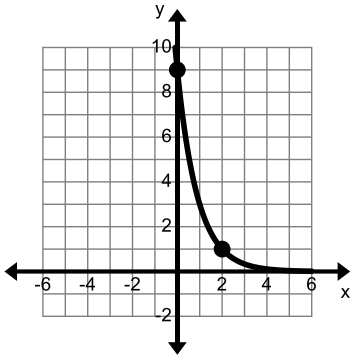
Average rate of change: _____

2. $f(x) = -\frac{1}{3}x + 2$ on [-3, 3]



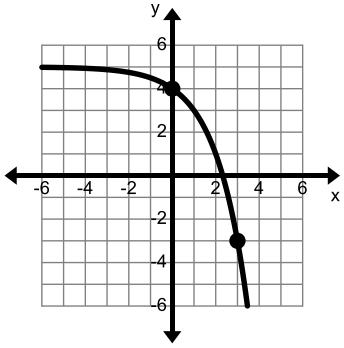
Average rate of change: _____

3. $f(x) = 3^{-x+2}$ on [0, 2]



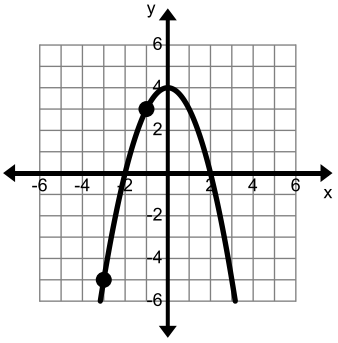
Average rate of change: _____

4. $f(x) = -2^x + 5$ on [0, 3]



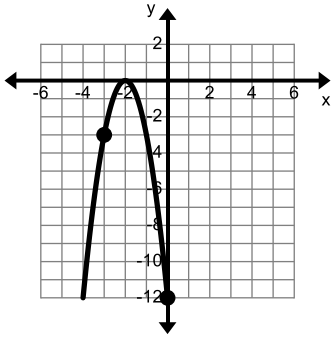
Average rate of change: _____

5. $f(x) = -x^2 + 4$ on [-3, -1]



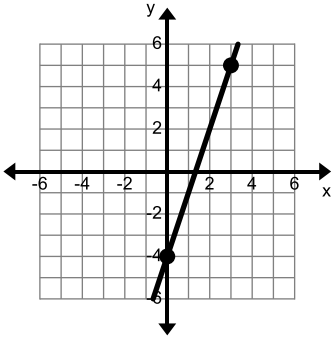
Average rate of change: _____

6. $f(x) = -3x^2 - 12x - 12$ on [-3, 0]



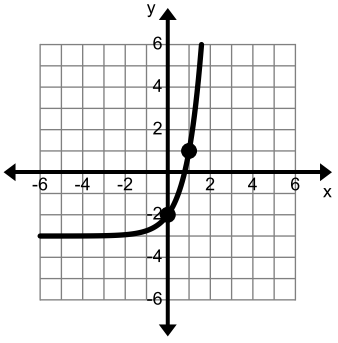
Average rate of change: _____

7. $f(x) = 3x - 4$ on [0, 3]



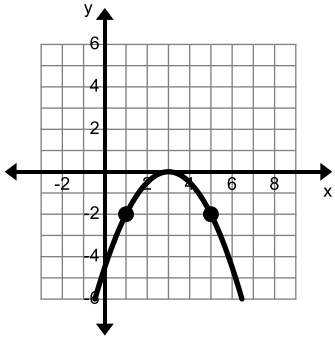
Average rate of change: _____

8. $f(x) = 4^x - 3$ on [0, 1]



Average rate of change: _____

9. $f(x) = -\frac{1}{2}(x-3)^2$ on [1, 5]



Average rate of change: _____

10. ***Population Growth:*** Suppose 25 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles in size every week. The equation $P(x) = 25 \cdot 2^x$ can be used to determine the number of beetles after *x* weeks. Complete the table below.

Week	Population
0	
1	
2	
3	
4	
5	

a. Calculate the average growth rate between weeks 1 and 3.

b. Calculate the average growth rate for the first five weeks [0, 5].

c. Which average growth rate is higher? Why do you think it is higher?

11. ***Gravity and the Moon***: The gravitational constant on the moon is very different from the gravitational constant on the earth. If a rocket were launched from the moon's surface with an initial velocity of 12 meters per second, then the equation $h(t) = -0.8t^2 + 12t$ would model the rocket's height, in meters, at any time *t*, in seconds. Complete the table below.

Time	Height
0	
3	
6	
9	
12	
15	

a. Calculate the average velocity of the rocket for the first 3 seconds of flight.

b. Calculate the average velocity of the rocket for 6 to 9 seconds of flight.

c. Calculate the average velocity of the rocket for 9 to 15 seconds.

d. Which interval had the greatest average rate of the change? Why do you think this interval had the greatest average rate of change?

Write the coordinates of the endpoints of the interval then find the average rate of change over the specified interval.

12. $f(x) = 2^{x+1} + 1$ on [-3, 2]

Average rate of change: _____

13. $f(x) = -x^2 + 6x - 4$ on [-1,3]

Average rate of change: _____

14. $f(x) = -3^x + 7$ on [0, 2]

Average rate of change: _____

15. $f(x) = x^2 + 5x + 1$ on [-6, -2]

Average rate of change: _____

16. $f(x) = \frac{1}{4}(x+2)^2 - 5$ on [-4, 0]

Average rate of change: _____

17. $f(x) = 0.5 \cdot 2^{-x+1}$ on [-3, 0]

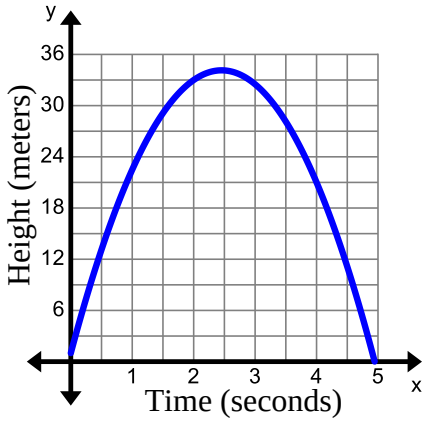
Average rate of change: _____

18. A rocket is launched with an initial velocity of 27 meters per second from a platform that is 1 meter above the earth's surface. The height of the rocket was recorded at 0.5 second intervals. Scientists wondered how this launch would compare to a similar launch on the planet of Saturn. Based on the gravitational constants of each planet the scientists came up with the following models for Earth and Saturn.

Earth

Time (Seconds)	Height (Meters)
0	1
0.5	13.275
1	23.1
1.5	30.475
2	35.4
2.5	37.875
3	37.9
3.5	35.475
4	30.6
4.5	23.275
5	13.5
5.5	1.275

Saturn



a. What is the domain for Earth? What is the domain for Saturn?

b. What is the range for Earth? What is the range for Saturn?

c. Would the rocket reach the greatest height on Earth or Saturn? Explain.

d. On which planet would the rocket reach its maximum height the fastest?

e. On which planet would the rocket have the greatest average rate of change for the first three seconds?

f. On which planet would the rocket have the greatest average rate of change between 2 and 3 seconds?

g. On which planet would the rocket be in the air the longest? Explain.

h. On which planet would the rocket hit the ground in the shortest amount of time? Explain.

Review is the precalculus analyzing functions with graphs assignment. 1.6 review page.