SM 2

Date:

Section: 7.1

Objective: Notes on Vertex, Axis of Symmetry of Quadratics

What does quadratic mean?

Forms of Quadratic Functions:

Standard Form: $f(x) = ax^2 + bx + c$, where $a \neq 0$. There are no parentheses.

Example: $f(x) = -3x^2 + 2x - 7$

Factored Form: $f(x) = a(x-p)(x-q)$, where $a \neq 0$. Written as a multiplication problem.

Example: $f(x) = (x-4)(x+5)$

Vertex Form: $f(x) = a(x-h)^2 + k$, where $a \neq 0$. x only shows up once, as part of a perfect square.

Example: $f(x) = 2(x+7)^2 - 1$

Examples: State whether each quadratic function is in standard, factored, or vertex form.

a) $f(x) = 2(x+3)(x-5)$
$a =$ $p=$ $q=$

b) $f(x) = -(x+4)^2 - 5$
$a =$ $h=$ $k=$

c) $f(x) = x^2 + 2x + 4$
$a=$ $b=$ $c=$

d) $f(x) = -x^2 + 5x$
$a=$ $b=$ $c=$

e) $f(x) = 3x(x-2)$
$a =$ $p=$ $q=$

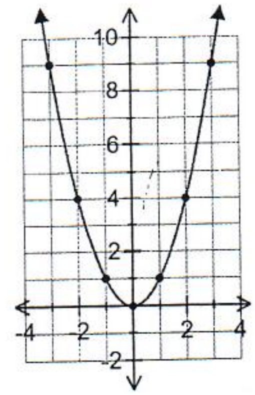
f) $f(x) = 2(x+1)^2 - 3$
$a =$ $h=$ $k=$

g) $f(x) = -(x+5)^2$
$a =$ $h=$ $k=$

h) $f(x) = -3x^2 + 4$
$a=$ $b=$ $c=$

i) $f(x) = 5x^2$

The graph of $y = x^2$:



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Vertex:

Axis of Symmetry:

Direction of Opening:

***y*-intercept:**

A) **Parabola:** The shape of the graph of a quadratic function.

B) ***Axis of Symmetry:*** A line that cuts a parabola in half. If you were to fold a parabola along its axis of symmetry, the two sides would overlap. The equation of the axis of symmetry looks like $x = \#$.

C) ***Vertex:*** The "tip" of the parabola – the point at which it changes direction.

- If the **parabola opens up** ($a > 0$), the vertex is the lowest point on the graph, or the ***minimum point.***
- If the **parabola opens down** ($a < 0$), the vertex is the highest point on the graph, or the ***maximum point.***

D) ***y-intercept:*** Where the graph touches or crosses the y-axis.

Finding the vertex in each form.

1) Vertex Form of a Quadratic Function: $y = a(x-h)^2 + k$

a is the number in front of the parentheses.
(If there isn't a number written in front, $a = 1$. If there's just a negative sign in front, $a = -1$.)

h is the *opposite* of the number with x in the parentheses.
k is the number at the end.

Vertex: (h, k)

2) *Standard Form:* $f(x) = ax^2 + bx + c$ *or* $y = ax^2 + bx + c$

- Just like with the other forms, the graph opens up if a is positive and opens down if a is negative.
- Vertex:
    - The x-coordinate of the vertex is $\frac{-b}{2a}$. (The opposite of b divided by 2 times a)
    - To find the y-coordinate, plug the x-coordinate into the original equation.

3) *Factored Form:* $f(x) = a(x-p)(x-q)$ *where* $a \neq 0$

Factored form shows the zeros or x-intercepts of the quadratic.

<u>Zeros of a Function:</u> The values of x that make $f(x)$ or y equal zero. If the zeros are real, they tell you the places where the graph crosses the x-axis, or the ***x-intercepts*** of the graph.

Other words for zeros are *solutions to* $f(x) = 0$, *roots, x-intercepts.*

<u>Vertex</u>: To find the x-value of the vertex, use $x = \frac{p+q}{2}$. To find the y-value of the vertex, plug the x-value into the original equation.

Finding the axis of symmetry, direction of opening, and y-intercept is the same in all forms.

Axis of Symmetry: Find the x-coordinate of the vertex and write the equation of the vertical line. In standard form it is $x = \frac{-b}{2a}$. In vertex form it is $x = h$. In factored form its $x = \frac{p+q}{2}$

Direction of Opening:

- Opens up if a is **positive.**
- Opens down if a is **negative.**

Finding the y-intercept:

1. Plug in 0 for x.
2. Simplify. **Don't forget to use order of operations.**

Write the form each quadratic equation is in. Find the vertex and the direction of the opening of the graph for each of the following quadratic equations. Find the y-intercept and axis of symmetry.

a) $y = (x-7)^2 + 9$

$h =$ _____ , $k =$ _____

Form: ____________________

Vertex: ____________________

Axis of Symmetry: ____________

Direction of opening: ____________

y-intercept: ________________

b) $y = 3x^2 - 12x - 10$

$a =$ ______, $b =$ ______

Form: ________________

Vertex: ________________

Axis of Symmetry: ____________

Direction of opening: ____________

y-intercept: ________________

c) $y = -(x + 4)(x - 6)$

$a =$ ______, $b =$ ______

Form: ________________

Vertex: ________________

Axis of Symmetry: ____________

Direction of opening: ____________

y-intercept: ________________

d) $y = -x^2 + 4x - 10$

$a =$ ______, $b =$ ______

Form: ________________

Vertex: ________________

Axis of Symmetry: ____________

Direction of opening: ____________

y-intercept: ________________

e) $y = -3(x + 2)^2 - 1$

$h =$ ______, $k =$ ______

Form: ________________

Vertex: ________________

Axis of Symmetry: ____________

Direction of opening: ____________

y-intercept: ________________

f) $y = 1/2(x-3)(x-7)$

$p =$ ______ , $q =$ ______

Form: ______________________

Vertex: ______________________

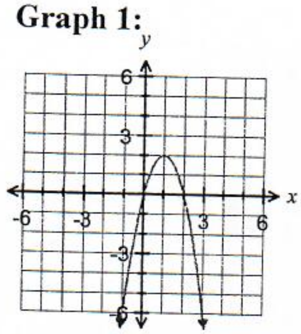
Axis of Symmetry: ____________

Direction of opening: __________

y-intercept: __________________

For each of the following graphs, find the vertex, axis of symmetry, and y-intercept.

Graph 1:



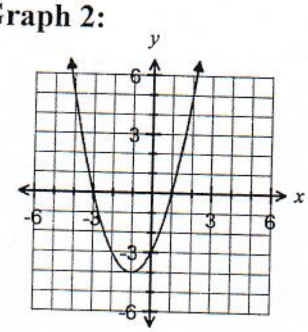
Vertex: ______________________

Axis of Symmetry: ____________

y-intercept: __________________

is the value of "a" positive or negative? _____

Graph 2:



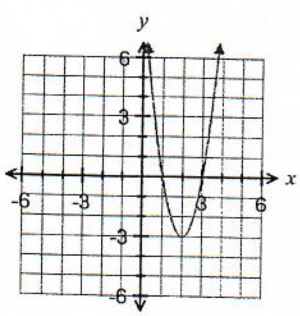
Vertex: ______________________

Axis of Symmetry: ____________

y-intercept: __________________

is the value of "a" positive or negative? ___

Graph 3:



Vertex: ______________________

Axis of Symmetry: ____________

y-intercept: __________________

is the value of "a" positive or negative? _____

SM 2

Date:

Section: 7.2

Objective: Notes graphing quadratics using the vertex

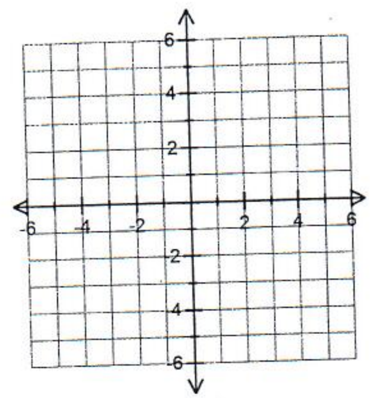
Using what we learned yesterday, we are going to graph quadratic equations.

Find the vertex of the parabola. Then use the pattern to find 4 more points, 2 on each side of the vertex. Do not forget that if a is not 1, the pattern changes.

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = x^2 + 4x + 3$ Form of the equation: ________________ $a =$ ______ $b =$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

Domain:

Range:

	x	y
Vertex		

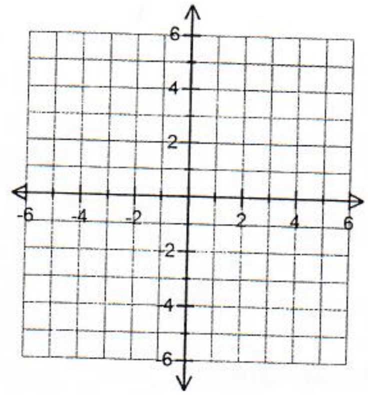
Vertical Stretch:

- a changes how wide or narrow the graph is.
  - If $|a| > 1$, the graph is *narrower* than the graph of $y = x^2$.
  - If $|a| < 1$, the graph is *wider* than the graph of $y = x^2$.
- Figure out the exact shape of the graph by making an x,y table. Always use the vertex as one point. Then choose two x-values on each side of the vertex to plug into the equation to find the corresponding y-coordinates.
- A shortcut is to use counting patterns to graph the parabola. Start at the vertex, then count:

  $\leftrightarrow 1, \uparrow a$
  $\leftrightarrow 2, \uparrow 4a$ If a is negative, count down instead of up.
  $\leftrightarrow 3, \uparrow 9a$, etc.

b) $f(x)=\frac{1}{2}(x-1)^2-4$ Form of the equation: __________ $a=$ ______ $h=$ ______ $k=$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

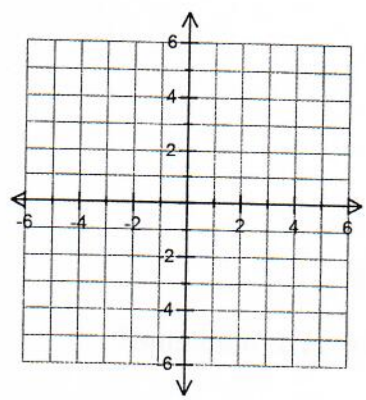
Domain:

Range:

	x	y
Vertex		

c) $f(x)=-3x^2-6x+2$ Form of the equation: ________________ $a=$ ______ $b=$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

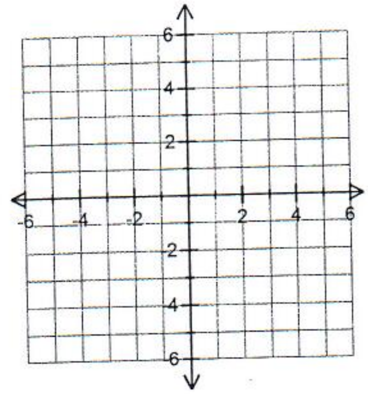
Domain:

Range:

	x	y
Vertex		

d) $f(x) = -(x+2)^2 - 1$ Form of the equation: ____________ $a =$ ______ $h =$ ______ $k =$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

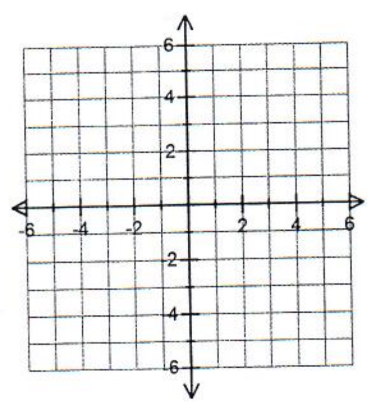
	x	y
Vertex		

Domain:

Range:

e) $f(x) = 2(x-4)^2 - 3$ Form of the equation: ____________ $a =$ ______ $h =$ ______ $k =$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

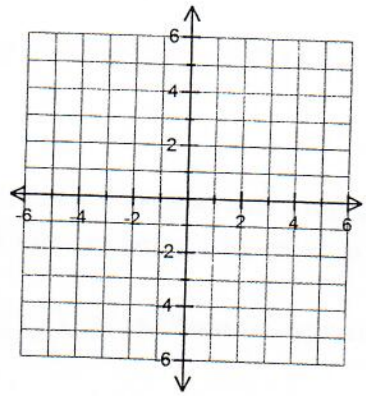
	x	y
Vertex		

Domain:

Range:

f) $f(x) = -\frac{1}{2}x^2 - x + 2$ Form of the equation: ____________ $a =$ ______ $b =$ ______

Vertex:



Axis of Symmetry:

Direction of Opening:

Is the vertex a maximum or minimum?

Maximum or minimum value:

y-intercept:

Domain:

Range:

	x	*y*
Vertex		

SM 2

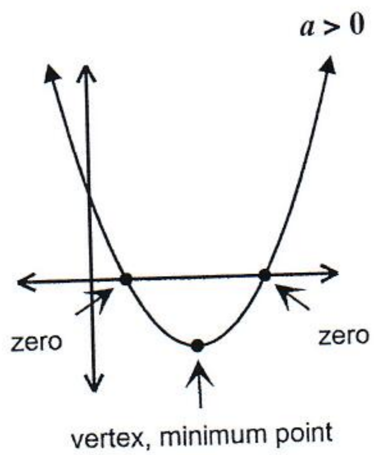
Date:

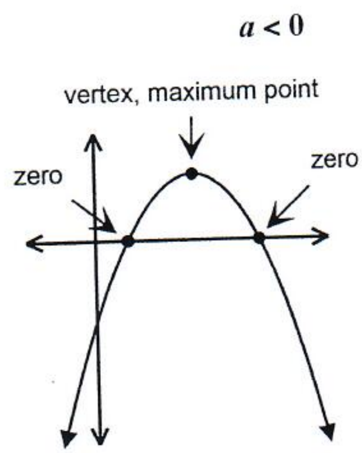
Section: 7.3

Objective: Notes on graphing quadratics using zeros

<u>Zeros of a Function:</u> The **values** of x that make *f(x)* or y equal zero. If the zeros are **real**, they tell you the places where the graph **crosses the x-axis**, or the ***x-intercepts*** of the graph.

Other words for zeros: ***solutions to*** $f(x) = 0$***, roots, x-intercepts.***





Finding zeros (x-intercepts):

1. Change y or $f(x)$ to 0.
2. Solve for x.
   - If the equation is in factored form, solving for x is easy – just think "What would x have to be to make each set of parentheses equal to 0?"
   - If the equation is in standard form, solve by factoring or by using quadratic formula
   - If the equation is in vertex form, get the perfect square by itself, take the square root of both sides (don't forget the ±), then solve for x.

★ **If your answers are imaginary** (negative under the square root), **the graph doesn't have x-intercepts.**

For each function, do the following:

1) State whether the function is in **standard**, **vertex**, or **factored** form,
2) State whether the parabola opens **up** or **down**,
3) Find the **zeros** (x-values),
4) State the ***x*-intercepts** as ordered pairs.

A. $f(x) = -(x+4)(x-1)$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ______________________

4) x-intercepts: __________________

Show work here:

B. $y = 4x^2 - 2x$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ______________________

4) x-intercepts: __________________

Show work here:

C. $y = -3(x+5)^2 + 27$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ______________________

4) x-intercepts: __________________

Show work here:

D. $f(x) = 5x^2 - 20$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ______________________

4) x-intercepts: __________________

Show work here:

E. $y = x^2 - 16x + 48$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ____________________

4) x-intercepts: ________________

Show work here:

F. $f(x) = 2(x-2)^2 + 8$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ____________________

4) x-intercepts: ________________

Show work here:

G. $f(x) = -(x+3)^2 + 50$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ____________________

4) x-intercepts: ________________

Show work here:

H. $y = -2x^2 + 4x - 10$

1) Form: ______________

2) Direction of opening: ________

3) Zeros: ____________________

4) x-intercepts: ________________

Show work here:

SM 2

Date:

Section: 7.4

Objective: Find the key features of graphs from their equations. Draw graphs from their key features. Match graphs to their equations.

For each function, fill out the requested information. Put a star by any information that can be seen just from looking at the equation. Graph the equation using its key features. Graph at least 5 points.

A. $f(x) = x^2 - 3x - 4$

1) Form: ________________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: ________

4) Zeros: ________________________

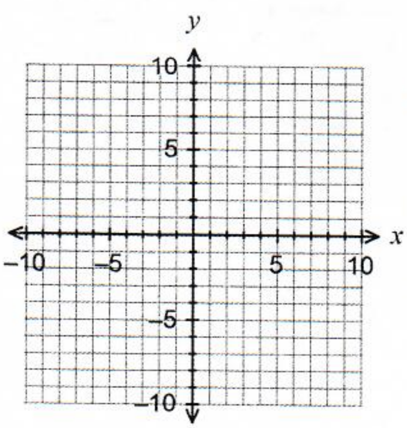
5) x-intercepts: __________________

6) y-intercept: ___________________

7) Axis of symmetry: _______________

8) Vertex: _________________

Show work here:



x	$f(x)$

B. $y = -2(x+4)^2 - 2$

1) Form: ________________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: ________

4) Zeros: ______________________

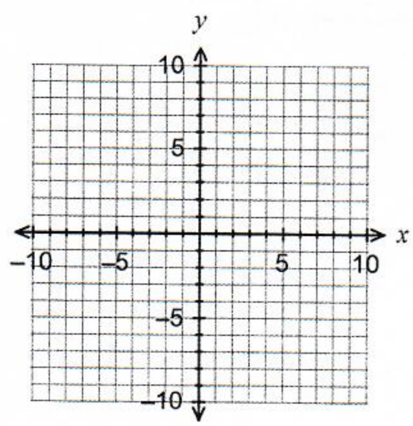
5) x-intercepts: ________________

6) y-intercept: __________________

7) Axis of symmetry: ______________

8) Vertex: _________________

Show work here:



x	y

C. $f(x) = -\frac{1}{4}(x+3)(x-6)$

1) Form: ______________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: ________

4) Zeros: ______________________

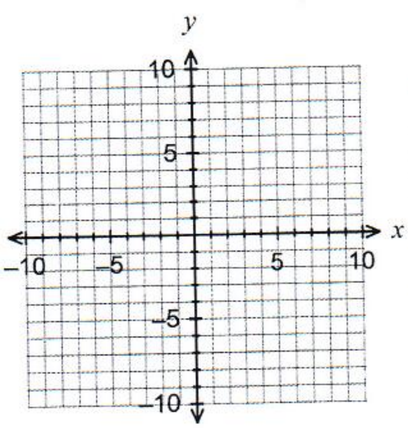
5) x-intercepts: __________________

6) y-intercept: __________________

7) Axis of symmetry: ______________

8) Vertex: ________________

Show work here:



x	$f(x)$

D. $y = x^2 - 9$

1) Form: ______________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: ________

4) Zeros: ______________________

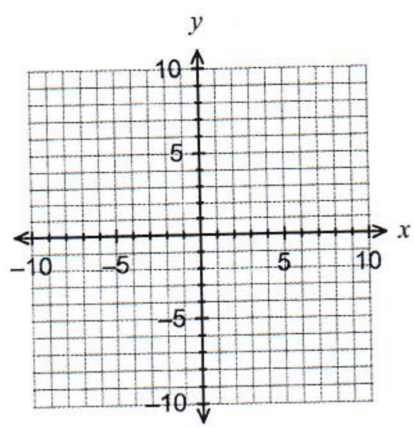
5) x-intercepts: __________________

6) y-intercept: __________________

7) Axis of symmetry: ______________

8) Vertex: ________________

Show work here:



x	y

E. $f(x) = 2x^2 - 4x - 8$

1) Form: ________________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: _________

4) Zeros: _________________________

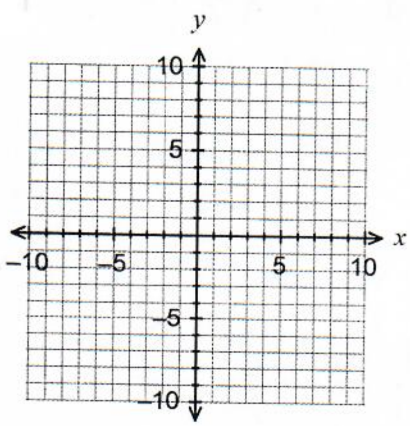
5) x-intercepts: ____________________

6) y-intercept: ____________________

7) Axis of symmetry: _______________

8) Vertex: _________________

Show work here:



x	$f(x)$

F. $y = -x(x+4)$

1) Form: ________________

2) $a =$ _____, _____ = _____, _____ = _____

3) Direction of opening: _________

4) Zeros: _______________________

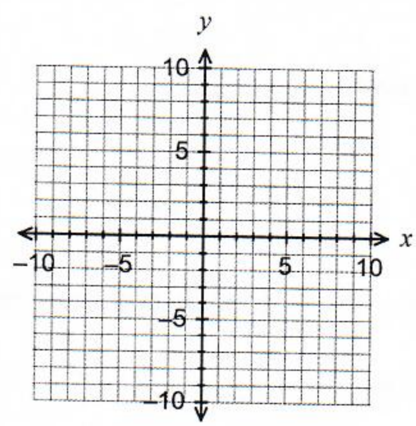
5) x-intercepts: _________________

6) y-intercept: ____________________

7) Axis of symmetry: _______________

8) Vertex: _________________

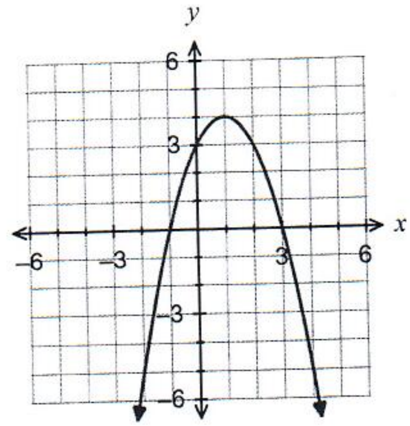
Show work here:



x	y

EXAMPLE: Given the graph, write the equation.

A. Write the equation of the graph in factored form.



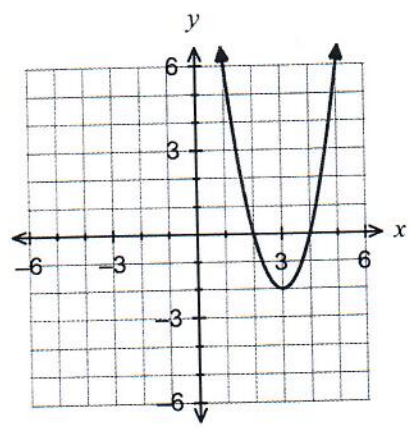
Direction of opening: ___________

Find the zeros: ________________________

$a =$ _________ $p =$ _________ $q =$ _________

Equation in factored form: __________________

B. Write the equation of the graph in vertex form.



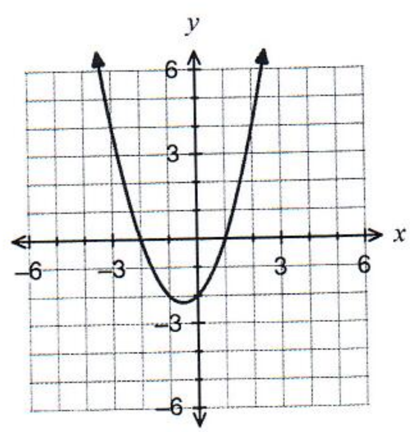
Direction of opening: ___________

Vertex: _________________

$a =$ _________ $h =$ _________ $k =$ _________

Equation in vertex form: __________________

C. Write the equation of the graph in standard form.



Direction of opening: ___________

Find the zeros: ________________________

$a =$ _________ $p =$ _________ $q =$ _________

Equation in factored form: __________________

Equation in standard form: ___________________

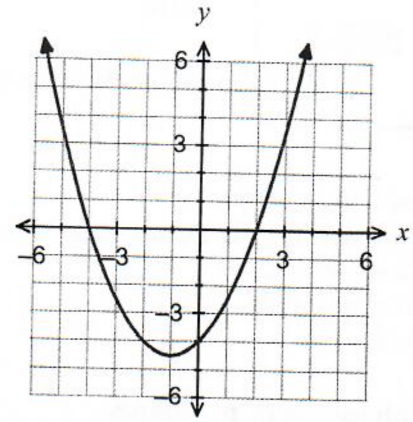
D. Write the equation of the graph in factored form.

Direction of opening: __________

Find the zeros: ______________________

$a =$_________ $p =$_________ $q =$_________

Equation in factored form: _________________



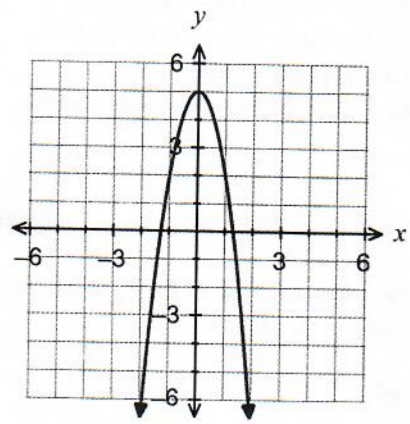
E. Write the equation of the graph in vertex form.

Direction of opening: __________

Vertex: ________________

$a =$_________ $h =$_________ $k =$_________

Equation in vertex form: _________________



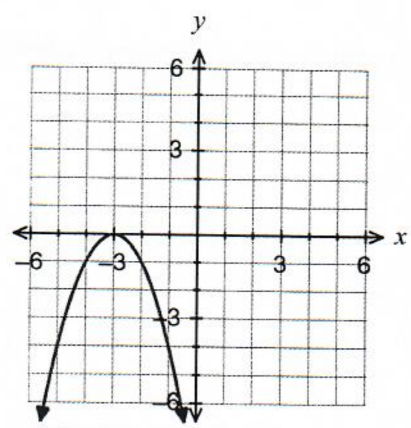
F. Write the equation of the graph in standard form.

Direction of opening: __________

Find the zeros: ______________________

$a =$_________ $p =$_________ $q =$_________

Equation in factored form: _________________



Equation in standard form: _________________

SM 2

Date:

Section: 7.5

Objective: Notes: Quadratic Story Problems

Steps for solving stories:

1. READ the story, write down the information needed and define a variable
2. Write an equation
3. Solve for variable
4. Check

Tips for solving story problems:

- Identify what you know.
- What are you trying to find out?
- Draw a picture or diagram to help you visualize the situation.
- Carefully define your variables.
- Translate the words into symbols.
- Use appropriate units.
- Make sure your answer makes sense.

Hints:

- ***Sum:*** + ***Difference:*** − ***Product:*** × ***Quotient:*** ÷

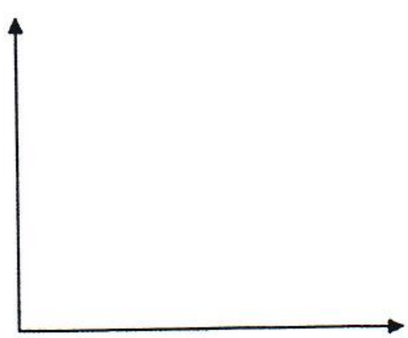
★ ***Words that tell you to look for the vertex:*** maximum, minimum, highest, lowest, biggest, littlest, largest, smallest, maximize, minimize.

EXAMPLES:

1. A ski club sells calendars to raise money. The profit, P, in dollars, from selling x calendars is given by the equation $P(x) = 120x - x^2$.

Define your variables: $x =$ ______________________, $P(x) = y =$ ______________________

Sketch a graph of the situation. Label the axes clearly.



How much profit will the club make from selling 50 calendars?

How many calendars must be sold for the club to make $2700?

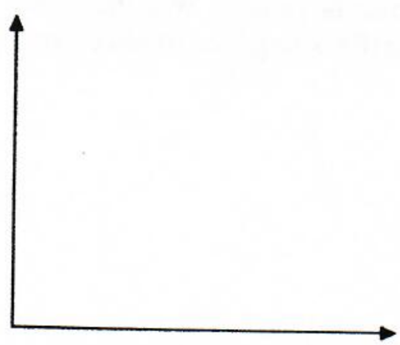
How many calendars must be sold to maximize profit?

What is the maximum profit?

2. A rock is thrown upward from the ground by the wheel of a truck. Its height in feet above the ground after t seconds is given by the function $h(t) = -16t^2 + 20t$.

Define your variables: $x = t =$ ____________________, $h(t) = y =$ ____________________

Draw a sketch of the graph representing the path of the height of the rock. Label your axes.



How long does it take the rock to reach its maximum height?

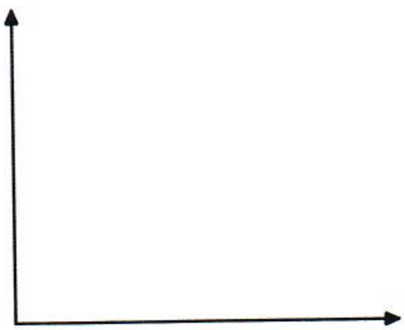
What is the maximum height of the rock?

How long will it take for the rock to return to the ground?

3. A penny is thrown upward from the observation deck on the 102^{nd} floor of the Empire State Building. It's height, h, in feet, after t seconds is given by the equation $h(t) = -16t^2 + 92t + 1250$

Define your variables: $x = t =$ ______________________, $h(t) = y =$ ______________________

Draw a sketch of the graph representing the path of the height of the penny. Label your axes.



What is the height of the observation deck? (In other words, how high is the penny at $t = 0$?)

How high is the penny after 2 seconds?

The Empire State Building has a lightning rod with a tip that is 1454 ft above the ground. Will the penny reach the top of the lightning rod? (Hint: Find the maximum height and see if it's larger or smaller than 1454 ft.)

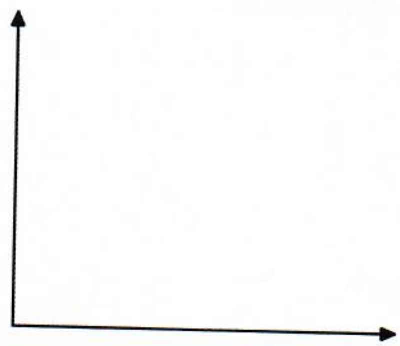
When will the penny be 1110 feet above the ground?

How long will it take for the penny to hit the ground?

4. The cost C, in dollars, of manufacturing x bikes per week at a production plant is given by the function $C(x) = 2x^2 - 800x + 92,000$.

Define your variables: $x =$ ______________________, $C(x) = y =$ ______________________

Sketch a rough graph of the cost equation. Be sure to label your axes. Use the y-intercept and the direction of opening to help draw the graph.



How much does it cost to manufacture 50 bikes per week? Show your work.

Find the number of bikes that must be manufactured each week to minimize the cost. Show your work.

Find the minimum cost. Show your work.

SM 2

Date:

Section: 7.6

Objective: Notes for Writing Quadratic Functions using the vertex or roots

Writing Quadratic Functions Given Key Features

If you know the vertex and another point on the parabola, or the roots and another point on the parabola, you can figure out the equation of the parabola.

Writing a Quadratic Equation when You Know the Vertex and Another Point

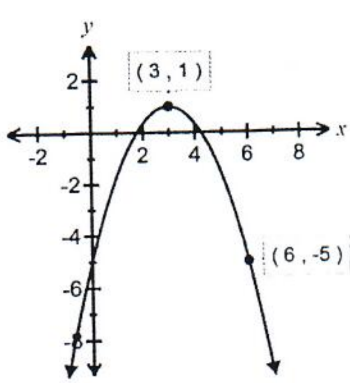
1. Use Vertex Form: $y = a(x - h)^2 + k$
2. Substitute in the vertex for h and k.
3. Substitute in the other point for x and y (or f(x))
4. Simplify and solve for a. (Don't forget to use order of operations.)
5. Write your final answer **by substituting in a, h and k into the vertex form.**

Examples: Write an equation for each parabola described below.

a) Vertex: $(-1,-2)$, passes through $(0,-1)$

b) Vertex: $(1,-3)$, passes through $(3,5)$

c)



Writing a Quadratic Equation when You Know the Roots and Another Point

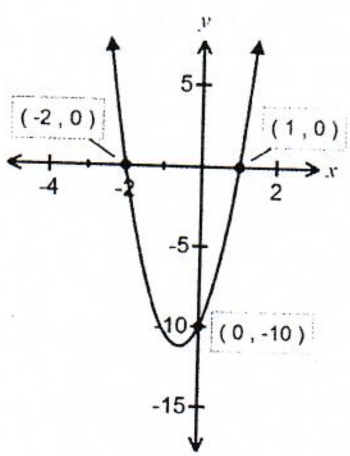
1. Use factored form: $y = a(x - p)(x - q)$
2. Substitute in the roots for p and q.
3. Substitute in the other point for x and y (or f(x)).
4. Simplify and solve for a. (Don't forget to use order of operations.)
5. Write your final answer by substituting in a, p, and q back into the factored form.

Examples: Write an equation for each parabola described below.

a) Roots: $(-1,0)$ and $(3,0)$, passes through $(2,9)$

b) Zeros: 4 & 8, passes through $(0,16)$

c)



SM 2

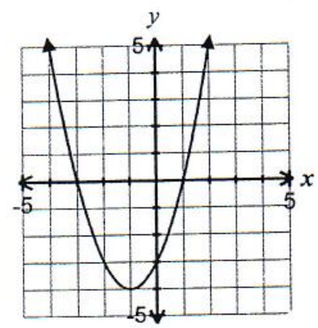
Date: **Section: 7.7**

Objective: Notes on Quadratic Inequalities

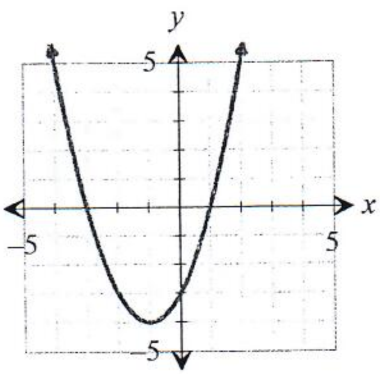
Review Example: Solve $0 = x^2 + 2x - 3$

Notice that each of these inequalities below involves the value of $x^2 + 2x - 3$, which is represented by the y-coordinate of the graph. In each case, we are trying to figure out what x-values (x-coordinates) make the inequality true. When trying to find where $x^2 + 2x - 3 > 0$, we are trying to figure out what x-coordinates have a y-coordinate that is bigger than zero—in other words, *where is the graph above the x-axis?*

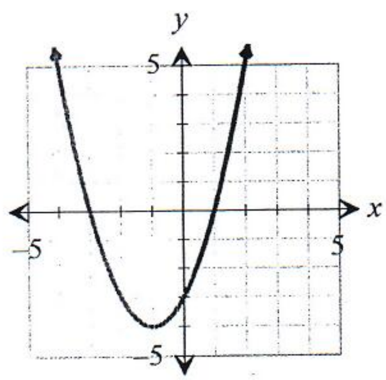
$f(x) = x^2 + 2x - 3$



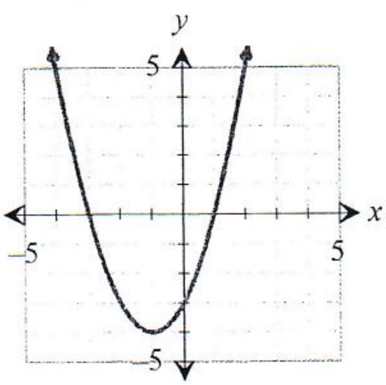
a) $x^2 + 2x - 3 > 0$



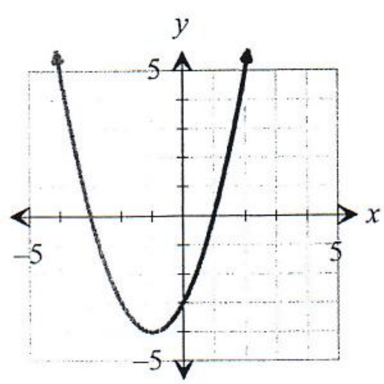
b) $x^2 + 2x - 3 \geq 0$



c) $x^2 + 2x - 3 < 0$



d) $x^2 + 2x - 3 \leq 0$



Solving a Quadratic Inequality Using the Graph:

1. Write the inequality in standard form. Replace the inequality sign with an equal sign and solve the equation $ax^2 + bx + c = 0$ by factoring, completing the square or using the quadratic formula. This gives you the x-intercepts of the graph of $y = ax^2 + bx + c$

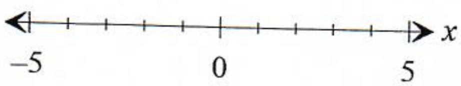
2. Graph $y = ax^2 + bx + c$. The graph does not have to be very detailed. A rough sketch of a parabola opening in the correct direction with the correct x-intercepts is all you need.

3. The solutions of $ax^2 + bx + c > 0$ are the x-values for which the graph is **above** the x-axis.
   The solutions of $ax^2 + bx + c \geq 0$ are the x-values for which the graph is **on or above** the x-axis.
   The solutions of $ax^2 + bx + c < 0$ are the x-values for which the graph is **below** the x-axis.
   The solutions of $ax^2 + bx + c \leq 0$ are the x-values for which the graph is **on or below** the x-axis.

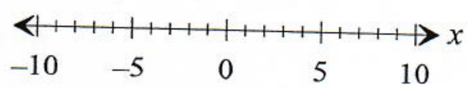
4. If the inequality involves $\leq$ or $\geq$, the x-intercepts **are included** in the solution set (use brackets).
   If the inequality involves $<$ or $>$, the x-intercepts **are not included** in the solution set (use parentheses).

Examples: Solve each inequality and graph the solution set on a number line. Write answer in interval notation.

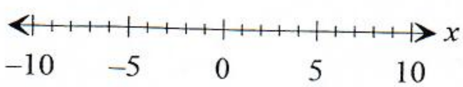
a) $(x-3)(x+1) \geq 0$



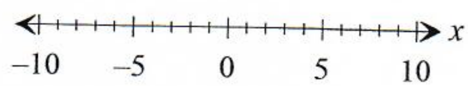
b) $(x-7)(x-5) < 0$



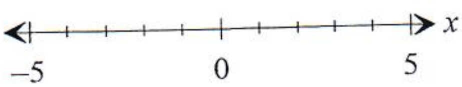
c) $x^2 + 5x > 0$



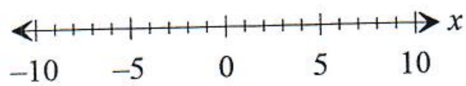
d) $x^2 - 4x - 12 \leq 0$



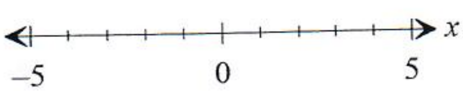
e) $x^2 - 4 < 0$



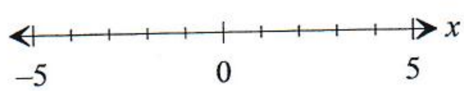
f) $x^2 + 10x \geq -9$



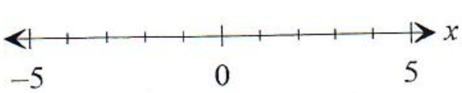
g) $x^2 - 4x + 4 > 0$



h) $x^2 - 4x + 4 \geq 0$



i) $x^2 - 4x + 4 < 0$



j) $x^2 - 4x + 4 \leq 0$