SM 2

Name: ______________________________

Period: __________

Unit 5 – Solving Quadratic Equations

5.1 Solving Quadratic Equations by Factoring

Quadratic Equation: Any equation that can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Zero Product Property: If the product of several factors is equal to zero, then at least one of the factors is equal to zero.

- The only way to end up with zero when you multiply is if one of the numbers being multiplied is zero.
- If a and b are real numbers and $a \cdot b = 0$, then $a = 0$ or $b = 0$ or both.

★ **This is only true if <u>one side of the equation is zero.</u>**

Solving Quadratic Equations by Factoring:

1. Get a zero on one side of the equation.
2. Factor completely.
3. Set each factor *containing a variable* equal to 0.
4. Solve the resulting equations.

amples: Solve each equation by factoring.

a) $(x-3)(x+5)=0$

$x-3=0$, $+3\ +3$, $\boxed{x=3}$ $\qquad$ $x+5=0$, $-5\ -5$, $\boxed{x=-5}$

b) $3x(x+4)=0$

$\frac{3x}{3}=\frac{0}{3}$, $\boxed{x=0}$ $\qquad$ $x+4=0$, $-4\ -4$, $\boxed{x=-4}$

c) $2(x+5)(3x-4)=0$

$x+5=0$, $-5\ -5$, $\boxed{x=-5}$ $\qquad$ $3x-4=0$, $+4\ +4$, $\frac{3x}{3}=\frac{4}{3}$, $\boxed{x=4/3}$

d) $(x+7)^2=0$

$(x+7)(x+7)=0$

$x+7=0$, $-7\ -7$, $\boxed{x=-7}$ $\qquad$ $x+7=0$, $-7\ -7$, $x=-7$

e) $3x^2=0$

$\frac{3x^2}{3}=\frac{0}{3}$

$x^2=0$

$\boxed{x=0}$

f) $x^2-8x=0$

$x(x-8)=0$

$\boxed{x=0}$ $\qquad$ $x-8=0$, $+8\ +8$, $\boxed{x=8}$

* factor first !

g) $x^2 + 7x + 6 = 0$ 6 1·6

$(x+1)(x+6)=0$

$\boxed{x=-1,\ -6}$

h) $x^2 + 21 = 10x$

$x^2-10x+21=0$ -7·-3

$(x-7)(x-3)=0$

$\boxed{x=7,\ 3}$

i) $x^2 - 4x = 12$

$x^2-4x-12=0$ -6·2

$(x-6)(x+2)=0$

$\boxed{x=6,\ -2}$

j) $-x^2 - 10x = 25$

$0=x^2+10x+25$ 5·5

$0=(x+5)(x+5)$

$\boxed{x=-5}$

k) $-x^2 = -4x - 32$

$0=x^2-4x-32$ -8·4

$0=(x-8)(x+4)$

$\boxed{x=8,\ -4}$

l) $2x^2 = x$

$2x^2-x=0$

$x(2x-1)$

$\boxed{x=0}$ $2x-\cancel{1}=0$ $+1\ +1$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{1}{2}$ $\boxed{x=1/2}$

m) $x^2 - 36 = 0$

$(x+6)(x-6)=0$

$\boxed{x=-6,\ 6}$

n) $4x^2 = 9$

$4x^2-9=0$

$(2x+3)(2x-3)=0$

$2x+\cancel{3}=0$ $-\cancel{3}\ -3$ $2x-\cancel{3}=0$ $+\cancel{3}\ +3$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{-3}{2}$ $\frac{\cancel{2}x}{\cancel{2}}=\frac{3}{2}$

$\boxed{x=-3/2,\ 3/2}$

o) $3x^2 + 15x + 18 = 0$

$3(x^2+5x+6)=0$ 2·3

$3(x+2)(x+3)=0$

$\boxed{x=-2,\ -3}$

p) $-2x^2 + 14x = 24$

$0=2x^2-14x+24$

$0=2(x^2-7x+12)$ -4·-3

$0=2(x-4)(x-3)$

$\boxed{x=4,\ 3}$

q) $4x^2 + 5x - 6 = 0$ -24 8·-3

$4x^2+8x-3x-6=0$

$4x(x+2)-3(x+2)=0$

$(x+2)(4x-3)=0$

$\boxed{x=-2}$

$4x-\cancel{3}=0$ $+\cancel{3}\ +3$

$\frac{4x}{4}=\frac{3}{4}$

$\boxed{x=3/4}$

r) $2x^2 - 21x = 11$

$2x^2-21x-11=0$ -22 -22·1

$2x^2-22x+x-11=0$

$2x(x-11)+1(x-11)=0$

$(x-11)(2x+1)=0$

$\boxed{x=11}$

$2x+\cancel{1}=0$ $-\cancel{1}\ -1$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{-1}{2}$

$\boxed{x=-1/2}$

5.2 Solving Quadratic Equations by Taking Square Roots

Example: How many numbers can be squared to get 9? In other words, how many solutions are there to the equation $x^2 = 9$? What are they? What about the equation $x^2 = -9$?

$\sqrt{x^2} = \sqrt{9}$

$x = 3 \rightarrow 3^2 = 9$

$x = -3 \rightarrow (-3)^2 = 9$

$x = \pm 3$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

★ **All numbers except zero have two square roots, a positive square root and a negative square root. The $\sqrt{\ }$ symbol means the positive square root. Both roots must be considered when solving an equation by taking square roots, so we use the ± symbol to include both roots.**

Square Root Property: If b is a real number and if $a^2 = b$, then $a = \pm\sqrt{b}$.

Solving Equations by Taking Square Roots: Do this when the equation has a perfect square and no other variables.

1. Get the perfect square alone on one side of the equation.
2. Use the square root property.
3. Simplify all square roots. Write the square roots of negative numbers in terms of i.
4. Solve for the variable, if necessary.

Examples: Solve each equation using the square root property. Include both real and imaginary solutions. Write your solutions in simplest radical form. Write imaginary solutions in the form $a + bi$.

a) $\sqrt{x^2} = \sqrt{50}$

$x = \pm\sqrt{50}$ $<^{25\ 5\ 5}_{2}$

$\boxed{x = \pm 5\sqrt{2}}$

b) $\frac{2z^2}{2} = \frac{-48}{2}$

$\sqrt{z^2} = \sqrt{-24}$

$z = \pm i\sqrt{24}$ $<^{4\ 2\ 2}_{6\ 2\ 3}$

$\boxed{z = \pm 2i\sqrt{6}}$

c) $\sqrt{16} = \sqrt{(y+1)^2}$

$\pm 4 = y + \cancel{1}$
$-1 \qquad\quad \cancel{-1}$

$-1 \pm 4 = y$

$y = -1 + 4 = \boxed{3}$

$y = -1 - 4 = \boxed{-5}$

d) $\sqrt{(2m-5)^2} = \sqrt{-25}$

$2m - \cancel{5} = \pm 5i$
$\quad\ \cancel{+5} \qquad +5$

$\frac{\cancel{2}m}{\cancel{2}} = \frac{5 \pm 5i}{2}$

$\boxed{m = \frac{5 \pm 5i}{2}}$

e) $3(t-2)^2 = 54$

$\frac{\cancel{3}(t-2)^2}{\cancel{3}} = \frac{54}{3}$

$\sqrt{(t-2)^2} = \sqrt{18}$

$t-2 = \pm\sqrt{18}$ $\quad 18 < 9,\ 2$; $\quad 9 < 3,\ 3$

$t-2 = \pm 3\sqrt{2}$

$+2 \quad +2$

$\boxed{t = 2 \pm 3\sqrt{2}}$

f) $(r+4)^2 - 10 = 26$

$+10 \quad +10$

$\sqrt{(r+4)^2} = \sqrt{36}$

$r+4 = \pm 6$

$-4 \quad -4$

$r = -4 \pm 6$

$\boxed{r = 2, -10}$

g) $-10 = \frac{1}{2}(n-7)^2$

$2 \cdot -10 = 2 \cdot \frac{1}{2}(n-7)^2$

$\sqrt{-20} = \sqrt{(n-7)^2}$

$\pm i\sqrt{20} = n-7$

$20: 2,\ 10$; $\quad 10: 2,\ 5$

$\pm 2i\sqrt{5} = n-7$

$+7 \quad +7$

$\boxed{7 \pm 2i\sqrt{5} = n}$

h) $-4(w+3)^2 + 6 = 86$

$-6 \quad -6$

$\frac{-4(w+3)^2}{-4} = \frac{80}{-4}$

$\sqrt{(w+3)^2} = \sqrt{-20}$

$w+3 = \pm i\sqrt{20}$ $\quad 20 < 2,\ 10$; $\quad 10 < 2,\ 5$

$w+3 = \pm 2i\sqrt{5}$

$-3 \quad -3$

$\boxed{w = -3 \pm 2i\sqrt{5}}$

i) $0 = -x^2 + 8$

$+x^2 \quad +x^2$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm\sqrt{8}$ $\quad 8 < 4,\ 2$; $\quad 4 < 2,\ 2$

$\boxed{x = \pm 2\sqrt{2}}$

j) $5(x+10)^2 = 0$

$\frac{\cancel{5}(x+10)^2}{\cancel{5}} = \frac{0}{5}$

$\sqrt{(x+10)^2} = \sqrt{0}$

$x+10 = \pm 0$

$-10 \quad -10$

$x = -10 \pm 0$

$\boxed{x = -10}$

k) $-2(x-3)^2 = -32$

$\frac{\cancel{-2}(x-3)^2}{\cancel{-2}} = \frac{-32}{-2}$

$\sqrt{(x-3)^2} = \sqrt{16}$

$x-3 = \pm 4$

$+3 \quad +3$

$x = 3 \pm 4$

$\boxed{x = 7, -1}$

l) $16 = -\frac{1}{3}(x-2)^2$

$-3 \cdot 16 = -3 \cdot -\frac{1}{3}(x-2)^2$

$\sqrt{-48} = \sqrt{(x-2)^2}$

$\pm i\sqrt{48} = x-2$

$48: 2,\ 24$; $\quad 24: 2,\ 12$; $\quad 12: 2,\ 6$; $\quad 6: 2,\ 3$

$\pm 4i\sqrt{3} = x-2$

$+2 \quad +2$

$\boxed{2 \pm 4i\sqrt{3} = x}$

5.3 The Quadratic Formula

We've learned how to solve quadratic equations by factoring, but what do we do if we have an equation with something that can't be factored, like $x^2 + 5x + 2 = 0$?

The Quadratic Formula: A quadratic equation written in the form $ax^2 + bx + c = 0$, where $a \neq 0$, has the solutions

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$

Solving a Quadratic Equation Using the Quadratic Formula:

1. Write the equation in standard form: $ax^2 + bx + c = 0$.
2. Identify a, b, and c. Plug them into the equation. Be careful with parentheses.
3. Simplify. Be careful to follow order of operations and deal with negatives correctly.

Examples: Solve each equation using the quadratic formula.

a) $x^2 + 4x + 7 = 0$

$a = 1$, $b = 4$, $c = 7$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 28}}{2} = \frac{-4 \pm \sqrt{-12}}{2} \quad \begin{matrix} 2 \\ 6 \\ 2 \quad 3 \end{matrix}$$

$$= \frac{-4 \pm 2i\sqrt{3}}{2} = \boxed{-2 \pm i\sqrt{3}}$$

b) $3m^2 + 16m + 5 = 0$

$a = 3$, $b = 16$, $c = 5$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{-16 \pm \sqrt{256 - 60}}{6} = \frac{-16 \pm \sqrt{196}}{6}$$

$$= \frac{-16 \pm 14}{6} = \frac{-16 + 14}{6} = -\frac{2}{6} = \boxed{-1/3}$$

$$= \frac{-16 - 14}{6} = -\frac{30}{6} = \boxed{-5}$$

c) $2w^2 - 4w = 3$

$2w^2 - 4w - 3 = 0$

$a = 2$, $b = -4$, $c = -3$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4} \quad \begin{matrix} 2 \\ 20 \\ 2 \quad 10 \end{matrix}$$

$$= \frac{4 \pm 2\sqrt{10}}{4} = \boxed{\frac{2 \pm \sqrt{10}}{2}}$$

d) $-n^2 + 4n - 4 = 0$

$a = -1$, $b = 4$, $c = -4$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-4)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{16 - 16}}{-2} = \frac{-4 \pm \sqrt{0}}{-2}$$

$$= \frac{-4 \pm 0}{-2} = \frac{-4}{-2} = \boxed{2}$$

e) $r^2 + 9 = 0$

$a = 1$, $b = 0$, $c = 9$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{0 \pm \sqrt{-36}}{2} = \frac{0 \pm 6i}{2}$$

$$= \frac{\pm 6i}{2} = \boxed{\pm 3i}$$

f) $6u^2 - 2u = 0$

$a = 6$, $b = -2$, $c = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(6)(0)}}{2(6)}$$

$$= \frac{2 \pm \sqrt{4 - 0}}{12} = \frac{2 \pm \sqrt{4}}{12} = \frac{2 \pm 2}{12}$$

$$= \frac{2 + 2}{12} = \frac{4}{12} = \boxed{1/3}$$

$$= \frac{2 - 2}{12} = \frac{0}{12} = \boxed{0}$$

g) $z = -3z^2 - 3$

$-3z^2 - z - 3 = 0$

$a = -3$
$b = -1$
$c = -3$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-3)(-3)}}{2(-3)}$

$= \frac{1 \pm \sqrt{1 - 36}}{-6} = \frac{1 \pm \sqrt{-35}}{-6}$

$= \boxed{\frac{-1 \pm i\sqrt{35}}{6}}$

h) $\frac{1}{4}y^2 - y + \frac{1}{2} = 0$

$a = 1/4$
$b = -1$
$c = 1/2$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1/4)(1/2)}}{2(1/4)}$

$= \frac{1 \pm \sqrt{1 - 1/2}}{1/2}$

$\boxed{= \frac{1 \pm \sqrt{+1/2}}{1/2}} = 2 \pm 2\sqrt{1/2}$

Discriminant: The radicand of the quadratic equation, $b^2 - 4ac$.

The discriminant tells us about the number and types of solutions of a quadratic equation without actually solving it. It also tells us how many x-intercepts the graph of a function has.

Discriminant: $b^2 - 4ac$	**Solutions of** $ax^2 + bx + c = 0$
Positive	Two real solutions
Zero	One real solution
Negative	Two imaginary solutions

$b^2 - 4ac$

Examples: Find the discriminant of each quadratic equation and state the number and type (real or imaginary) of solutions.

a) $4x^2 - 20x + 25 = 0$

$a = 4$
$b = -20$
$c = 25$

$(-20)^2 - 4(4)(25)$

$= 400 - 400$

$= 0$
one real solution

b) $x^2 + 2x + 4 = 0$

$a = 1$
$b = 2$
$c = 4$

$2^2 - 4(1)(4)$

$= 4 - 16$

$= -12$
2 imaginary solutions

c) $3x^2 + 5 = -7x$

$3x^2 + 7x - 5 = 0$

$a = 3$
$b = 7$
$c = -5$

$7^2 - 4(3)(-5)$

$= 49 + 60$

$= 109$
2 real solutions

d) $x^2 - 5x = 14$

$x^2 - 5x - 14 = 0$

$a = 1$
$b = -5$
$c = -14$

$(-5)^2 - 4(1)(-14)$

$= 25 + 56$

$= 81$
2 real solutions