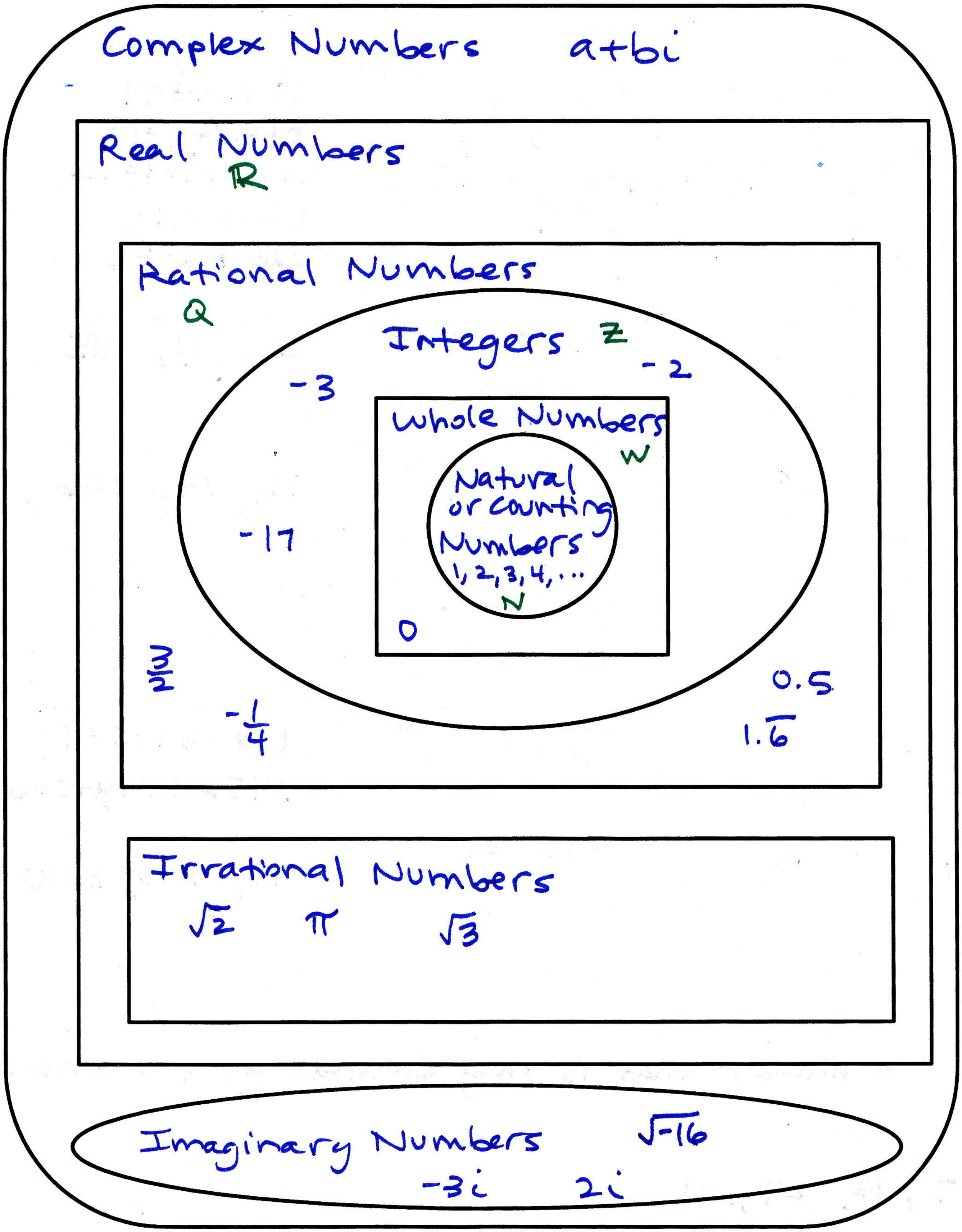
SM 2 NOTES

Date:

3.1 Number Theory



Divisibility Rules

Numbers divisible by:	If:	Example:
2	The last digit is even.	12, 26, 114
3	If you can divide the sum of the digits by 3 evenly.	18 → 1+8=9 84 → 8+4=12 147 → 1+4+7=12
4	The last 2 digits are divisible by 4.	124 → 24÷4 220 → 20÷4
5	The last digit is 0 or 5.	25, 70, 315
6	If the number is divisible by 2 and 3.	36, 84, 126
8	If the last 3 digits are divisible by 8.	
9	If you can divide the sum of the digits by 9 evenly.	108 → 1+8=9 3456 → 3+4+5+6=18
10	If the last digit is 0.	50, 200, 5380

Prime number:

a number that is only divisible by 1 and itself.

Examples:

7, 13, 29, 41

NOTES **3.2 Adding and Subtracting Polynomials**

Monomial: An expression that is a number, a variable, or numbers and variables multiplied together. Monomials only have variables with whole number exponents and never have variables in the denominator of a fraction or variables under roots.

Monomials: $5b,\ \frac{xyz}{8},\ -w,\ 23,\ x^2,\ \frac{1}{3}x^3y^4$ **Not Monomials:** $\frac{1}{x^4},\ \sqrt[3]{x},\ a^{-1},\ z^{\frac{1}{5}}$

Constant: A monomial that contains no variables, like 23 or -1.

Coefficient: The numerical part of a monomial (the number being multiplied by the variables.)

Polynomial: A monomial or several monomials joined by + or – signs.

Terms: The monomials that make up a polynomial. Terms are separated by + or – signs.

Like Terms: Terms whose variables and exponents are exactly the same.

Binomial: A polynomial with two unlike terms.

Trinomial: A polynomial with three unlike terms.

How to find the degree of a polynomial:

find the largest exponent of the polynomial.

Reasons for not a polynomial:

- No negative exponents
- No fraction exponents
- No division – no variables in the denominator

Examples: Decide whether each expression is a polynomial. If it is, state the degree of the polynomial. If it is not, explain why not.

a) $5x^4+2x^3+6x$

yes
degree = 4

b) $-\frac{4}{3}a-a^5$

yes
degree = 5

c) $\frac{12}{m+2}$

no
m is in the denominator

d) $6c^{-2}+c-1$

no
no negative exponents

e) $6z^{\frac{1}{2}}+5z^2-2$ f) 7 g) $-8n-3$ h) $3\sqrt{x+2}$

no
no fraction exponents

yes
degree = 7

yes
degree = 1

no
no fraction exponents

Adding and Subtracting Polynomials

To add or subtract polynomials, combine like terms. Add or subtract the coefficients. The variables and exponents do not change. ***Remember to subtract everything inside the parentheses after a minus sign.*** Subtract means "add the opposite," so change the minus sign to a plus sign and then change the signs of all the terms inside the parentheses.

Examples: Simplify each expression.

a) $(5n^2-2)+(7-3n^2)$

$2n^2+5$

b) $(2r^2+5r)+(r^2-4r)$

$3r^2+r$

c) $(4x^2-3x+1)+(-2x^2+5x-6)$

$2x^2+2x-5$

d) $(7z^2+12z-5)+(6z-4z^2-3)$

$3z^2+18z-8$

e) $(2w^2+3w)-(4w^2+w)$

$-4w^2-w$

$-2w^2+2w$

f) $(u^3-4u^2+u)-(2u^2-5u^3)$

$-2u^2+5u^3$

$6u^3-6u^2+u$

g) $(-6x^2-3x+2)-(-4x^2-x+3)$

$+4x^2+x-3$

$-2x^2-2x-1$

h) $(4y^2+12y-7)-(20y+5y^2-8)$

$-20y-5y^2+8$

$-y^2-8y+1$

i) $(6m^2+5m)-(4m^2-2m)+(3m^2-7m)$

$-4m^2+2m$

$\boxed{5m^2}$

j) $(-2k+5)+(k^2-3k)-(-4k^2+8)$

$+4k^2-8$

$5k^2-5k-3$

NOTES 3.3 Multiplying Polynomials

Steps for Multiplying Polynomials

To multiply two polynomials, multiply each term of one polynomial by each term of the other polynomial. Then combine any like terms. When you are multiplying two binomials, this is sometimes called the ***FOIL Method*** because you multiply **F** the *first* terms, **O** the *outside* terms, **I** the *inside* terms, and **L** the *last* terms.

Examples: Multiply.

a) $-xy\left(7x^2y+3xy-11\right)$

$-7x^3y^2-3x^2y^2+11xy$

b) $(m+3)(m-8)$

$m^2-8m+3m-24$

$m^2-5m-24$

c) $(3x+1)(5x-2)$

$15x^2-6x+5x-2$

$15x^2-x-2$

d) $(-2z+5)(-5z-8)$

$11z^2+16z-25z-40$

$11z^2-9z-40$

e) $(t^2-4)(2t+9)$

$2t^3+9t^2-8t-36$

f) $\left(2u^2-1\right)\left(-5u^2+4\right)$

$-10u^4+8u^2+6u^2-4$

$-10u^4+14u^2-4$

g) $(z+5)^2$ $(z+5)$

$z^2+5z+5z+25$

$z^2+10z+25$

h) $(2x-3)^2$ $(2x-3)$

$4x^2-6x-6x+9$

$4x^2-12x+9$

i) $(n+3)(n-3)$

$n^2\cancel{-3n}\cancel{+3n}-9$

n^2-9

j) $(5y-2)(5y+2)$

$25y^2\cancel{+10y}\cancel{-10y}-4$

$25y^2-4$

k) $(2x-3)(5x^2-6x+7)$

$10x^3-12x^2+14x$

$-15x^2+18x-21$

$10x^3-27x^2+32x-21$

l) $(4x^2+7x-3)(x^2-2x+8)$

$4x^4-8x^3+32x^2$

$+7x^3-14x^2+56x$

$-3x^2+6x-24$

$4x^4-x^3+15x^2+62x-24$

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3.4 Dividing Rational Expressions

Rational Function: A function that is the ratio of two polynomials.

Domain of a Rational Function:

- **STEP 1:** A rational function is simply a fraction and in a fraction the denominator cannot equal zero because it would be undefined. To find which numbers make the fraction undefined, create an equation where the denominator is not equal to zero.
- **STEP 2:** Solve the equation found in step 1.
- **STEP 3:** Write your answer using interval notation.

Examples: Find the domain of the following rational expressions.

a) $\dfrac{2x+1}{x-3}$

$x-3 \neq 0$
$+3 \quad +3$
$x \neq 3$

$(-\infty, 3) \cup (3, \infty)$

b) $\dfrac{x+3}{7}$

$(-\infty, \infty)$

c) $\dfrac{(2x+1)(x-4)}{(x+4)(2x+1)}$

$x+4 \neq 0$
$-4 \quad -4$
$x \neq -4$

$2x+1 \neq 0$
$-1 \quad -1$
$\frac{2x}{2} \neq \frac{-1}{2}$
$x \neq -\frac{1}{2}$

$(-\infty, -4) \cup (-4, -1/2) \cup (-1/2, \infty)$

d) $\dfrac{5x+1}{(x-4)(3x+2)}$

$x-4 \neq 0$
$+4 \quad +4$
$x \neq 4$

$3x+\cancel{2} \neq 0$
$\cancel{-2} \quad -2$
$\frac{3x}{3} \neq -\frac{2}{3}$
$x \neq -2/3$

$(-\infty, -2/3) \cup (-2/3, 4) \cup (4, \infty)$

1. Factor the numerator and the denominator. Put ( ) around the top and bottom to remind yourself not to separate the family.

2. List restricted values.

3. Divide out all common factors.

Examples: Simplify each rational expression. (Talk about domain.)

a) $\frac{2x+1}{2x-1}$ $x \neq \frac{1}{2}$

simplified

$2x-1 \neq 0$
$+1 \quad +1$
$\frac{\cancel{2}x}{\cancel{2}} \neq \frac{1}{2}$

b) $\frac{\cancel{x+2}}{\cancel{x+2}} = 1,\ x \neq -2$

$x+\cancel{2} \neq 0$
$\cancel{-2} \quad -2$
$x \neq -2$

c) $\frac{\cancel{x+2}}{(\cancel{x+2})(3x-1)} = \frac{1}{3x-1},$
$x \neq -2, \frac{1}{3}$

$x+\cancel{2} \neq 0$
$\cancel{-2} \quad -2$
$x \neq -2$

$3x\cancel{-1} \neq 0$
$\cancel{+1} \quad +1$
$\frac{\cancel{3}x}{\cancel{3}} \neq \frac{1}{3}$
$x \neq \frac{1}{3}$

d) $\frac{(4x+1)(\cancel{x-3})}{\cancel{x-3}}$

$= 4x+1;\ x \neq 3$

$x-\cancel{3} \neq 0$
$\cancel{+3} \quad +3$
$x \neq 3$

e) $\frac{(2x-7)(\cancel{3x+1})}{(\cancel{3x+1})(x-8)}$

$= \frac{2x-7}{x-8};\ x \neq -\frac{1}{3}$ or 8

$3x+1 \neq 0$
$-1 \quad -1$
$\frac{3x}{3} \neq \frac{-1}{3}$
$x \neq -\frac{1}{3}$

$x-\cancel{8} \neq 0$
$\cancel{+8} \quad +8$
$x \neq 8$

f) $\frac{\cancel{8}(x+4)}{\cancel{8}}$

$= x+4$

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3.5 Combining Functions with Arithmetic

ten, it is useful to combine two functions to make a new function. For instance, you may have a function describing the revenue from a product and a function describing the costs of producing the product. By subtracting the two functions, you can create a function describing the profit made from the product.

Steps:

1. Write each function with ( ) in given notation.
2. Distribute then add or subtract like terms.
3. Write answer in standard form.

Tips:

- Don't forget parenthesis.
- Distribute the negative when subtracting functions.
- Like terms have the same variables and exponents.

xamples: Let $f(x)=3x-5$ and $g(x)=x^2+5x-2$. Perform the indicated operations.

a) $h(x)=f(x)+g(x)$

$(3x-5)+(x^2+5x-2)$

$=x^2+8x-7$

b) $h(x)=g(x)-f(x)$

$(x^2+5x-2)-(3x-5)$

$-3x+5$

$=x^2+2x+3$

c) $h(x)=2f(x)+3g(x)$

$2(3x-5)+3(x^2+5x-2)$

$=6x-10+3x^2+15x-6$

$=3x^2+21x-16$

d) $h(x)=-f(x)+4g(x)$

$-(3x-5)+4(x^2+5x-2)$

$=-3x+5+4x^2+20x-8$

$=4x^2+17x-3$

e) $h(x)=f(x)\cdot g(x)$

$(3x-5)(x^2+5x-2)$

$3x^3+15x^2-6x$

$-5x^2-25x+10$

$3x^3+10x^2-31x+10$

f) $h(x)=f(x)\cdot f(x)$

$(3x-5)(3x-5)$

$9x^2-15x-15x+25$

$9x^2-30x+25$

Evaluating Combined Functions

1. Substitute the number into the first function for x and evaluate.

2. Substitute the number into the other function for x and evaluate.

3. Complete the arithmetic operations: +. −, •, ÷

★ **Follow order of operations!**

Examples: Let $f(x) = 2x - 7$, and let $g(x) = -x^2 + 3$. Evaluate the following.

a) $f(2) + g(1)$

$(2(2)-7) + (-1^2+3)$

$(4-7) + (-1+3)$

$-3 + 2 = -1$

b) $f(0) - g(-3)$

$f(0) = 2(0) - 7 = -7$

$g(-3) = -(-3)^2 + 3$

$= -9 + 3 = -6$

$-7 - -6 = -1$

c) $f(-2) \cdot 3g(2)$

$f(-2) = 2(-2) - 7$

$= -4 - 7 = -11$

$g(2) = -2^2 + 3 = -4 + 3 = -1$

$3g(2) = 3(-1) = -3$

$-7 \cdot -3 = \boxed{21}$

Examples: Let $f(x) = 3x - 5$ and $g(x) = (x+3)(x-1)$ Perform the indicated operations and state the domain of the new function.

a) $r(x) = \dfrac{g(x)}{f(x)} = \dfrac{(x+3)(x-1)}{3x-5}$

Domain: $3x \cancel{-5} \neq 0$
$\cancel{+5} \quad -5$
$\dfrac{\cancel{3}x}{\cancel{3}} \neq \dfrac{-5}{3}$ $x \neq -5/3$

$(-\infty, -5/3) \cup (-5/3, \infty)$

b) $r(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x-5}{(x+3)(x-1)}$

Domain: $x \neq -3, 1$

$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

c) $r(x) = \dfrac{2f(x)}{f(x)}$

$\dfrac{2\cancel{(3x-5)}}{\cancel{(3x-5)}} = 2; \ x \neq -5/3$

Domain: $x \neq -5/3$

$(-\infty, -5/3) \cup (-5/3, \infty)$

d) $r(x) = \dfrac{g(x)}{-3g(x)} = \dfrac{\cancel{(x+3)}\cancel{(x-1)}}{-3\cancel{(x+3)}\cancel{(x-1)}} = -\dfrac{1}{3}$

Domain: $x \neq -3, 1$

$(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

Examples: Let $f(x) = 3x - 5$ and $g(x) = (x+3)(x-1)$ Evaluate the following functions with the given values and functions.

a) $\frac{f(2)}{g(-2)}$ $\frac{3(2)-5}{(-2+3)(-2-1)} = \frac{6-5}{(-1)(-3)}$

$= \frac{1}{3}$

b) $\frac{-2f(5)}{g(-1)}$ $\frac{-2(3(5)-5)}{(-1+3)(-1-1)} = \frac{-2(15-5)}{(2)(-2)}$

$= \frac{-2(10)}{-4} = \frac{-20}{-4} = 5$

Story Problems Involving Combined Functions

a) A company estimates that its cost and revenue can be modeled by the functions $C(x) = 0.6x^2 + 49x + 150$ and $R(x) = 100x + 75$, where x is the number of items produced. The company's profit, P, can be modeled by $P(x) = R(x) - C(x)$. Find the profit equation and determine the profit when 60 items are produced.

$(100x+75) - (0.6x^2+49x+150)$

$-0.6x^2 - 49x + 150$

$= -0.6x^2 + 51x + 225$

$= -0.6(60)^2 + 51(60) + 225$

$= -0.6(3600) + 3060 + 225 = -2160 + 3060 + 225$

$= \$1125$

b) A service committee is organizing a fundraising dinner. The cost of renting a facility is \$250 plus \$3 per person, or $C(x) = 3x + 250$, where x represents the number of people attending the fundraiser. The committee wants to charge attendees \$20 each or $R(x) = 20x$. How many people must attend the fundraiser for the event to raise \$500?

profit $= R(x) - C(x)$

$= 20x - (3x + 250)$

$-3x - 250$

$= 20x - 3x - 250$

profit $= 17x - 250$

$500 = 17x - \cancel{250}$

$+250 \quad +\cancel{250}$

$\frac{750}{17} = \frac{17x}{17}$ $\quad x = 44.12 = 45$ people

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3.6 Imaginary Numbers

History:

For centuries, mathematicians kept running into problems that required them to take the square roots of negative numbers in the process of finding a solution. None of the numbers that mathematicians were used to dealing with (the "real" numbers) could be multiplied by themselves to give a negative. These square roots of negative numbers were a new type of number. The French mathematician René Descartes named these numbers "imaginary" numbers in 1637. Unfortunately, the name "imaginary" makes it sound like imaginary numbers don't exist. They do exist, but they seem strange to us because most of us don't use them in day-to-day life, so we have a hard time visualizing what they mean. However, imaginary numbers are extremely useful (especially in electrical engineering) and make many of the technologies we use today (radio, electrical circuits) possible.

The number i:

$i = \sqrt{-1}$

Examples: Express in terms of *i*.

a) $\sqrt{-64} = i\sqrt{64}$
$= 8i$

b) $\sqrt{-12}$
$i\sqrt{12}$
$4 \cdot 3$
$2 \cdot 2$
$= 2i\sqrt{3}$

c) $-\sqrt{-49}$
$-i\sqrt{49}$
$= -7i$

d) $-\sqrt{-18}$
$-i\sqrt{18}$
$9 \cdot 2$
$3 \cdot 3$
$= -3i\sqrt{2}$

Imaginary Number: the square root of a negative number

Complex Number: a + b*i* – a is the real part and b*i* is the imaginary part.

Steps for Adding and Subtracting Complex Numbers:

1. Add together the real parts and the imaginary parts.

2. When subtracting complex numbers, make sure the distribute the negative through both parts of the complex number that is being subtracted.

a) $(2+5i)+(1-3i)$

$3+2i$

b) $(4-3i)-(-2+5i)$

$+2-5i$

$6-8i$

c) $(-3-7i)-(-6)$

$+6$

$3-7i$

d) $5i-(1-i)$

$-1+i$

$-1+6i$

Steps for Multiplying Complex Numbers if they are written as square roots:

1. Take care of the negatives under the square roots FIRST.

2. Multiply

3. Simplify. Replace any i^2 with -1.

★ **Why is $i^2 = -1$?** $i=\sqrt{-1}$ $i^2 = (\sqrt{-1})^2 = -1$

Examples: Multiply and simplify.

a) $\sqrt{-9}\cdot\sqrt{-4}$

$3i \cdot 2i = 6i^2$

$= 6(-1) = -6$

b) $\sqrt{-3}\cdot\sqrt{-5}$

$i\sqrt{3}\cdot i\sqrt{5} = i^2\sqrt{15}$

$= -1\sqrt{15} = -\sqrt{15}$

c) $-\sqrt{-8}\cdot\sqrt{-27}$

$-i\sqrt{8}\cdot i\sqrt{27}$

$-i^2\sqrt{216}$ $= -i^2 \cdot 6\sqrt{6}$

8 27

2 4 3 9

2 2 3 3

$= 6\sqrt{6}$

Steps for Multiplying Complex Numbers if they are written *i*.

1. Think of *i* as a variable, like *x*.

2. Multiply

3. Simplify. Replace any i^2 with -1.

Examples: Multiply and simplify. If the answer is imaginary, write it in the form $a+bi$.

a) $-2i\cdot 7i$

$-14i^2$

$-14(-1) = 14$

b) $-3i\cdot i\sqrt{5}$

$-3i^2\sqrt{5}$

$-3(-1)\sqrt{5} = 3\sqrt{5}$

c) $3i(2-i)$

$6i-6i^2$

$= 6i-6(-1) = 6+6i$

d) $(7+3i)(9-8i)$

$63-56i+27i-\cancel{24i^2}$

$+24$

$= 87-29i$

e) $(2-i)^2$ $(2-i)(2-i)$

$4-2i-2i+\cancel{i^2}$

-1

$= 3-4i$

f) $(3-4i)(3+4i)$

$9+\cancel{12i}-\cancel{12i}-\cancel{16i^2}$

$+16$

$= 25$