SM 2

NOTES

Name: ______________________________ **Period:** ________

Date: ____________________

3.1 Rules of Exponents

The following properties are true for all real numbers *a* and *b* and all integers *m* and *n*, provided that no denominators are 0 and that 0^0 is not considered.

1 as an exponent:	$a^1 = a$	e.g.) $7^1 = 7,\ \pi^1 = \pi,\ (-10)^1 = -10$
0 as an exponent:	$a^0 = 1$	e.g.) $2^0 = 1,\ 27^0 = 1,\ \left(-\frac{5}{8}\right)^0 = 1$
The Product Rule:	$a^m \cdot a^n = a^{m+n}$	e.g.) $x^2 \cdot x^5 = x^{2+5} = \underline{\underline{x^7}}$ $xx \cdot xxxxx$
The Quotient Rule:	$\frac{a^m}{a^n} = a^{m-n}$	e.g.) $\frac{x^5}{x^2} = x^{5-2} = x^3$ $\frac{\cancel{x}\cancel{x}.x.x.x.}{\cancel{x}.\cancel{x}} = x^3$
The Power Rule:	mult. exp. $\left(a^m\right)^n = a^{mn}$	e.g.) $\left(x^2\right)^5 = x^{(2)(5)} = x^{10}$
Raising a product to a power:	$(ab)^n = a^n b^n$	e.g.) $(2k)^4 = 2^4 \cdot k^4 = 16k^4$
Raising a quotient to a power:	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	e.g.) $\left(\frac{p^1}{q^2}\right)^3 = \frac{p^3}{\left(q^2\right)^3} = \frac{p^3}{q^6}$
Negative exponents:	$a^{-n} = \frac{1}{a^n}$	e.g.) $2^{-3} = \frac{1}{2^3}$, $7x^3y^{-4} = \frac{7x^3}{y^4}$ $= \frac{1}{8}$
	$\frac{1}{a^{-n}} = a^n$	e.g.) $\frac{1}{x^{-9}} = x^9$, $\frac{b}{c^{-3}d} = \frac{bc^3}{d}$
	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$	e.g.) $\left(\frac{2}{v}\right)^{-3} = \left(\frac{v}{2}\right)^3 = \frac{v^3}{2^3} = \frac{v^3}{8}$

To *simplify* an expression containing powers means to rewrite the expression without parentheses or gative exponents.

* no negative exponents

Examples Simplify the following expressions.

a) $m^5 \cdot m^7$

$m^{5+7} = \boxed{m^{12}}$

b) $(5a^2b^3)(3a^4b^5)$

$5a^2b^3 \cdot 3a^4b^5$

$\boxed{15a^6b^8}$

c) $\frac{r^{9-3}}{\cancel{r^3}\,1} = \boxed{r^6}$

d) $\frac{\cancel{p^3}\,1}{p^{7-3}}$

$\boxed{\frac{1}{p^4}}$

e) $\frac{\overset{5}{\cancel{10}}x^{11}\overset{4}{\cancel{y^5}}}{\underset{1}{\cancel{2}}\cancel{x^4}y^{7-5}}$

$\boxed{\frac{5x^7}{1y^2}}$

f) $\frac{\overset{2}{\cancel{6}}\cancel{x^3}y^{2-1}}{\underset{3}{\cancel{9}}x^7\cancel{y}}$

$\boxed{\frac{2y}{3x^4}}$

g) $(-2)^4$

$-2 \cdot -2 \cdot -2 \cdot -2$

$\boxed{16}$

h) -2^4

$-1 \cdot 2^4$

$-1 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$-1 \cdot 16$

$\boxed{-16}$

$x^{-3} \cdot x^8 = x^5$

i) $5x^{-4}y^3 \cdot x^2y^{-1}$

$5x^{-2}y^2$

$\boxed{\frac{5y^2}{x^2}}$

j) $\frac{1}{6^{-2}} = 6^2$

$= \boxed{36}$

k) $9^{-3} \cdot 9^8$

no not multiply to 81

9^5

$\boxed{59049}$

l) $\frac{\overset{1}{\cancel{3}}x^{2+3}}{\underset{5}{\cancel{15}}\cancel{x^{-3}}\cancel{y^{-4}}}\,y^4$

$\boxed{\frac{x^5y^4}{5}}$

m) $(3^5)^4$ mult.

3^{20}

3486784401

n) $\frac{\cancel{y^{-5}}\,1}{y^{-4+5}} = \frac{1}{y}$

o) $(y^{-5})^7$ mult.

y^{-35}

$\frac{1}{y^{35}}$

p) $(a^{-3})^{-7}$ mult

a^{21}

mult. exp

q) $(-2x)^3$

$(-2)^3x^3$

$-2 \cdot -2 \cdot -2x^3$

$-8x^3$

r) $\left(\frac{x^2}{2}\right)^4 = \frac{x^8}{2^4}$

$= \boxed{\frac{x^8}{16}}$

s) $(3x^5y^{-1})^{-2}$

$3^{-2}x^{-10}y^2$

$\frac{y^2}{3^2x^{10}}$

$\boxed{\frac{y^2}{9x^{10}}}$

mult. exp.

t) $\left(\frac{y^2z^3}{5}\right)^{-3}$

$\frac{y^{-6}z^{-9}}{5^{-3}}$

$\frac{5^3}{y^6z^9}$

$\boxed{\frac{125}{y^6z^9}}$

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3.2 Simplifying Radicals

Square Root: The **square root** of a number is a value that, when multiplied by itself, gives the number.

$$\sqrt{25} = \sqrt[2]{\cancel{5 \cdot 5}} = 5$$

Radical Sign: $\sqrt{\ }$

Radicand: The number under the $\sqrt{\ }$.

25 is radicand or $\sqrt{6}$
6 is radicand

Perfect squares: a whole number which is the square of another whole number

0^2	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2
0	1	4	9	16	25	36	49	64	81	100	121	144	169

List of common perfect squares:

14^2	15^2
196	225

Perfect cubes: a whole number which is the **cube** of another whole number

List of common perfect cubes:

0^3	1^3	2^3	3^3	4^3	5^3	....
0	1	8	27	64	125	...

Examples: Simplify each of the following:

exp ÷ by index

a) $\sqrt{121} = \sqrt[2]{\cancel{11 \cdot 11}}$
121
^
11 11
$\boxed{11}$

b) $\sqrt{81}$
81
^
$\cancel{9}$ 9
$\boxed{9}$

c) $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$
$= \boxed{\frac{2}{3}}$

d) $\sqrt[2]{y^4}$
$\sqrt[2]{\cancel{y y y y}}$
$y \cdot y$
$\boxed{y^2}$ 4÷2

e) $\sqrt[2]{z^{14}}$ 14÷2
$\boxed{z^7}$

Index: $\sqrt[x]{\ }$ x is the index

Examples: Simplify each expression, if possible.

a) $\sqrt[3]{125}$
$\sqrt[3]{125}$
$\sqrt[3]{\cancel{5 \cdot 5 \cdot 5}}$
125
^
25 ⑤
^
⑤⑤
$\boxed{5}$

b) $\sqrt[4]{81}$
$\sqrt[4]{\cancel{3 \cdot 3 \cdot 3 \cdot 3}}$
81
^
9 9
^ ^
③③③③
$\boxed{3}$

c) $\sqrt[5]{32}$
$\sqrt[5]{\cancel{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}}$
32
^
8 4
^ ^
4 ② ②②
^
②②
$\boxed{2}$

d) $\sqrt[3]{8x^6y^3}$
$\sqrt[3]{\cancel{2 \cdot 2 \cdot 2}\ \cancel{xxxxxx}\ \cancel{yyy}}$
$2xxy = \boxed{2x^2y}$

Steps To Simplify a Radical Expression with Index *n* Using a Factor Tree:

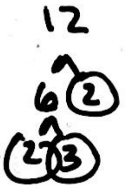
1. Use a factor tree to find the prime factorization of the number inside the radical.
2. Determine the index of the radical. Look for groups equal in size to the index of the same prime number.
3. Move each group of numbers or variables from inside the radical to outside the radical.
4. Simplify the expressions both inside and outside the radical by multiplying.

Examples: Simplify each expression.

a) $\sqrt{12}$

$\sqrt{\cancel{2}\cdot\cancel{2}\cdot 3}$

$\boxed{2\sqrt{3}}$



b) $\sqrt[2]{40}$

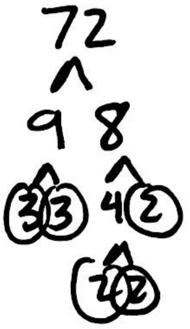
$\sqrt{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 5}$

$\boxed{2\sqrt{10}}$

c) $5\sqrt{72}$

$3\cdot 2\cdot 5\sqrt{\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{3}}$

$\boxed{30\sqrt{2}}$



d) $\sqrt{20x^2y^3}$

$\sqrt{\cancel{2}\cdot\cancel{2}\cdot 5\cancel{x}\cancel{x}\cancel{y}\cancel{y}y}$

$\boxed{2xy\sqrt{5y}}$

e) $2xy^2\sqrt{300x^3y^5}$

$y\cdot y\cdot x\cdot\cancel{5}\cdot\cancel{2}\cdot\cancel{2}xy^2\sqrt{\cancel{2}\cdot\cancel{2}\cdot 3\cdot\cancel{5}\cdot\cancel{5}\cancel{x}\cancel{x}x\;\cancel{y}\cancel{y}\cancel{y}\cancel{y}y}$

$\boxed{20x^2y^4\sqrt{3xy}}$

f) $\sqrt[3]{54}$

$\sqrt[3]{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot 2}$

$\boxed{3\sqrt[3]{2}}$

DONT FORGET INDEX

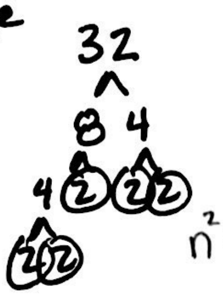
g) $7\sqrt[3]{40}$

$$2\cdot 7\sqrt[3]{\cancel{2\cdot 2\cdot 2}\cdot 5}$$

$$\boxed{14\sqrt[3]{5}}$$

h) $\sqrt[3]{32t^7u^9}$

rewrite



$$\sqrt[3]{\cancel{2\cdot 2\cdot 2}\cdot 2\cdot 2\cdot \cancel{t^6}\cdot t\, \cancel{u^9}}$$

$$\boxed{2t^2u^3\sqrt[3]{4t}}$$

i) $3m\sqrt[3]{40mn^6}$

$$n^2\cdot 2\cdot 3m\sqrt[3]{\cancel{2\cdot 2\cdot 2}\cdot 5m\cancel{n^6}}$$

$$\boxed{6mn^2\sqrt[3]{5m}}$$

j) $\sqrt[4]{240}$

$$\sqrt[4]{\cancel{2\cdot 2\cdot 2\cdot 2}\cdot 3\cdot 5}$$

$$\boxed{2\sqrt[4]{15}}$$

k) $\sqrt[4]{x^6y^9z^3}$

rewrite

$$\sqrt[4]{\cancel{x^4}x^2\cancel{y^8}yz^3}$$

$$\boxed{xy^2\sqrt[4]{x^2yz^3}}$$

l) $pr\sqrt[5]{p^7q^{23}r^{14}}$

$$q^4\cdot p^1\cdot pr^1\sqrt[5]{\cancel{p^5}\cdot p^2\cancel{q^{20}}q^3\cancel{r^{10}}r^4}$$

$$r^2$$

$$\boxed{p^2r^3q^4\sqrt[5]{p^2q^3r^4}}$$

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3.3 Multiplying, Adding, and Subtracting Radicals

The Product Rule for Radicals:
For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}$.

Caution: The product rule doesn't work if you are trying to multiply the even roots of <u>negative</u> numbers, because those roots are not real numbers. For example, $\sqrt{-2}\cdot\sqrt{-8}\neq\sqrt{16}$.

Caution: The product only applies when the radicals have the same index: $\sqrt[3]{5}\cdot\sqrt[4]{6}\neq\sqrt[12]{30}$.

Examples: Multiply.

a) $\sqrt{7}\cdot\sqrt{5}$

$\boxed{\sqrt{35}}$

b) $5\sqrt{2}\cdot\sqrt{8}$

$5\sqrt{2\cdot 8}$

$5\sqrt{16}$

$5\cdot 4=\boxed{20}$

c) $2\sqrt{5}\cdot 7\sqrt{15}$

60
6 10
2 3 5 2

$14\sqrt{60}$

$14\sqrt{\cancel{2}\cancel{2}\cdot 3\cdot 5}$

$2\cdot 14\sqrt{15}=\boxed{28\sqrt{15}}$

d) $\sqrt{3}\cdot\sqrt{3}$ $=\sqrt{9}=\boxed{3}$

e) $\left(\sqrt{8}\right)^2$ $=\sqrt{8}\cdot\sqrt{8}$

$=\sqrt{64}$

$=\boxed{8}$

f) $\left(3\sqrt{11}\right)^2$

$3\sqrt{11}\cdot 3\sqrt{11}$

$9\sqrt{121}$

$11\cdot 9\sqrt{\cancel{11\cdot 11}}=\boxed{99}$

g) $\sqrt[3]{3}\cdot\sqrt[3]{9}$

$\sqrt[3]{27}$

$\sqrt[3]{\cancel{3\cdot 3\cdot 3}}$

$\boxed{3}$

h) $2\sqrt[3]{10}\cdot 6\sqrt[3]{25}$

$12\sqrt[3]{250}$

$12\sqrt[3]{\cancel{5\cdot 5\cdot 5}\cdot 2}$

$5\cdot 12\sqrt[3]{2}$

$\boxed{60\sqrt[3]{2}}$

Question: Can you add and subtract radicals the same way you multiply and divide them?
e.g.) Since $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$, does $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$? **NO!!!!!!!!!!**

Don't make the following mistakes:

- $\sqrt{2}+\sqrt{5}\neq\sqrt{7}$
- $\sqrt{9+16}\neq 3+4$
- $\sqrt{m}-\sqrt{n}\neq\sqrt{m-n}$
- $\sqrt{x^2-4}\neq x-2$
- $\left(\sqrt{x}+\sqrt{y}\right)^2\neq x+y$

Like Radicals: Radicals with the same index *and* the same radicand.

Examples: Determine whether the following are like radicals. If they are not, explain why not.

a) $\sqrt{3}$ and $\sqrt{2}$

no
different
radicands

b) $4\overset{2}{\sqrt{5}}$ and $-3\overset{2}{\sqrt{5}}$

yes
same radicands
same indexes

c) $2\sqrt{x}$ and $\sqrt[3]{x}$

no
different
indexes

Adding and Subtracting Radicals:

1. Simplify each radical completely.
2. Combine like radicals. When you add or subtract radicals, you can *only* combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.

Examples:

a) $5\sqrt{3x}-7\sqrt{3x}$

$\boxed{-2\sqrt{3x}}$

b) $4\sqrt{11}+8\sqrt{11}$

$\boxed{12\sqrt{11}}$

c) $10\sqrt{6}+3\sqrt{2}-8\sqrt{6}$

$\boxed{2\sqrt{6}+3\sqrt{2}}$

d) $\sqrt{20}-\sqrt{50}+\sqrt{45}$

2 10 5 10 5 9

2 5 5 2 3 3

$2\sqrt{5}-5\sqrt{2}+3\sqrt{5}$

$=\boxed{5\sqrt{5}-5\sqrt{2}}$

e) $2\sqrt{50}+4\sqrt{500}-6\sqrt{125}$

5 10 10 50 5 25

5 2 5 2 5 2 10 5 5

5 2

$10\sqrt{2}+40\sqrt{2}-30\sqrt{5}$

$=\boxed{50\sqrt{2}-30\sqrt{5}}$

f) $\sqrt[3]{54}-5\sqrt[3]{16}+\sqrt[3]{2}$

9 6 4 4

3 3 3 2 2 2 2 2

$3\sqrt[3]{2}-10\sqrt[3]{2}+\sqrt[3]{2}$

$=\boxed{-6\sqrt[3]{2}}$

ultiplying Radical Expressions: Use the Product Property. Use the Distributive Property and FOIL to ultiply radical expressions with more than one term.

Examples: Multiply.

a) $\sqrt{3}\left(5+\sqrt{30}\right)$

$5\sqrt{3}+\sqrt{90}$

9 10

3 3 2 5

$=\boxed{5\sqrt{3}+3\sqrt{10}}$

b) $\sqrt{2}\left(\sqrt{6}-3\sqrt{2}\right)$

$\sqrt{12}-3\sqrt{4}$

$=\sqrt{12}-(3\cdot 2)$

$\boxed{=\sqrt{12}-6}$

c) $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{7}+1\right)$

$\boxed{\sqrt{35}+\sqrt{5}-\sqrt{42}-\sqrt{6}}$

d) $\left(5\sqrt{3}-4\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$

$5\sqrt{9}+5\sqrt{6}-4\sqrt{6}-4\sqrt{2}$

$5\cdot 3$ $4\cdot 2$

$=15+\sqrt{6}-8$

$=\boxed{7+\sqrt{6}}$

e) $\left(4\sqrt{3}-1\right)^{2}\left(4\sqrt{3}-1\right)$

$16\sqrt{9}-4\sqrt{3}-4\sqrt{3}+1$

$16\cdot 3$

$48-8\sqrt{3}+1$

$=\boxed{49-8\sqrt{3}}$

f) $\left(\sqrt{2}+5\right)\left(\sqrt{2}-5\right)$

$\sqrt{4}-\cancel{5\sqrt{2}}+\cancel{5\sqrt{2}}-25$

$=2-25$

$=\boxed{-23}$

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3.4 Rational Exponents

If n is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number then $a^{1/n} = \sqrt[n]{a}$.

★ **The denominator of the exponent tells you what type of root to take.**

Examples: Write an equivalent expression using radical notation and, if possible, simplify.

a) $25^{1/2}$

$\sqrt[2]{25}$

$\cancel{5\cdot5}$

$\boxed{5}$

b) $64^{1/3}$

$\sqrt[3]{64}$

$\sqrt[3]{\cancel{4\cdot4\cdot4}}$

$\boxed{4}$

c) $\left(xy^2z\right)^{1/6}$

$\sqrt[6]{xy^2z}$

d) $\left(36x^{10}\right)^{1/2}$

$\sqrt[2]{36x^{10}}$

$\boxed{6x^5}$

e) $2x^{1/4}$

$2\sqrt[4]{x}$

f) $(2x)^{1/4}$

$\sqrt[4]{2x}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[7]{2xy}$

$(2xy)^{1/7}$

b) $\sqrt[4]{\dfrac{ab^3}{7}}$

$\left(\dfrac{ab^3}{7}\right)^{1/4}$

c) $\sqrt{3z}$

$(3z)^{1/2}$

d) $3\sqrt{z}$

$3z^{1/2}$

e) $\sqrt[5]{xy^2z}$

$(xy^2z)^{1/5}$

Positive Rational Exponents

If m and n are positive integers (where $n \neq 1$) and $\sqrt[n]{a}$ exists, then $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

e.g.) $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ or $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Examples: Write an equivalent expression using radical notation and simplify.

a) $t^{5/6}$

$\sqrt[6]{t^5}$

b) $9^{3/2}$

$\left(\sqrt[2]{9}\right)^3$

$3^3 = \boxed{27}$

c) $64^{2/3}$

$\left(\sqrt[3]{64}\right)^2$

$4^2 = \boxed{16}$

d) $(2x)^{3/4}$

$\sqrt[4]{(2x)^3}$

or

$\sqrt[4]{2^3x^3} = \sqrt[4]{8x^3}$

e) $2x^{3/4}$

$2\sqrt[4]{x^3}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[3]{x^5}$

$x^{5/3}$

b) $\sqrt[7]{9^2}$

$9^{2/7}$

c) $\left(\sqrt[5]{6n}\right)^3$

$(6n)^{3/5}$

d) $6\sqrt[5]{n^3}$

$6n^{3/5}$

e) $\left(\sqrt[4]{2m}\right)^2$

$(2m)^{2/4} = (2m)^{1/2}$

Negative Rational Exponents

For any rational number m/n, and any nonzero real number $a^{m/n}$, $a^{-m/n} = \frac{1}{a^{m/n}}$.

★ **The sign of the base is not affected by the sign of the exponent.**

Examples: Write an equivalent expression using positive exponents and, if possible, simplify.

a) $49^{-1/2}$

$\frac{1}{49^{1/2}} = \frac{1}{\sqrt{49}} = \boxed{\frac{1}{7}}$

b) $(3mn)^{-2/5}$

$= \frac{1}{(3mn)^{2/5}} = \boxed{\frac{1}{\sqrt[5]{9m^2n^2}}}$

c) $7x^{-2/3}$

$= \frac{7}{x^{2/3}} = \boxed{\frac{7}{\sqrt[3]{x^2}}}$

Laws of Exponents: The laws of exponents apply to rational exponents as well as integer exponents.

Examples: Use the laws of exponents to simplify.

a) $2^{2/5} \cdot 2^{1/5}$

$= 2^{2/5+1/5} = \boxed{2^{3/5}}$

$= \sqrt[5]{2^3} = \boxed{\sqrt[5]{8}}$

b) $\frac{x^{7/3}}{x^{4/3}}$ $x^{7/3-4/3}$

$= x^{3/3} = x^{1} = \boxed{x}$

c) $(19^{2/5})^{5/3}$

$19^{2/5 \cdot 5/3} = \boxed{19^{2/3}}$

$= \sqrt[3]{19^2} = \boxed{\sqrt[3]{361}}$

d) $x^{1/2} \cdot x^{2/3}$

$\frac{1}{2} + \frac{2}{3}$

$= \frac{3}{6} + \frac{4}{6}$

$= 7/6$

$= x^{1/2+2/3}$

$= \boxed{x^{7/6}} = \sqrt[6]{x^7}$

$= \boxed{x\sqrt[6]{x}}$

e) $y^{-4/7} \cdot y^{6/7}$

$y^{-4/7+6/7} = \boxed{y^{2/7}}$

$\boxed{= \sqrt[7]{y^2}}$

f) $\frac{z^{3/4}}{z^{2/5}}$

$\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20}$

$= 7/20$

$y^{3/4-2/5}$

$= \boxed{y^{7/20}} = \boxed{\sqrt[20]{y^7}}$

g) $\frac{x^{3/4} \cdot x^{1/6} \cdot y}{y^{1/2}}$

$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12}$

$= \frac{11}{12}$

$x^{3/4+1/6} \cdot y^{1-1/2}$

$= x^{11/12} y^{1/2}$

h) $\frac{(2x^{2/5}y^{-1/3})^5}{x^2y}$

$= \frac{2^5 x^{10/5} y^{-5/3}}{x^2y}$

$= \frac{32\cancel{x^2}}{\cancel{x^2}y \cdot y^{5/3}} = \frac{32}{y^{1+5/3}}$

$= \boxed{\frac{32}{y^{8/3}}}$

To Simplify Radical Expressions:

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

Examples: Use rational exponents to simplify. Do not use exponents that are fractions in the final answer.

a) $\sqrt[8]{z^4}$

$z^{4/8} = z^{1/2} = \boxed{\sqrt{z}}$

b) $\left(\sqrt[3]{a^2bc^4}\right)^9$

$(a^2bc^4)^{9/3}$

$= \cancel{(a^2bc^4)^3}$

$= \boxed{a^6b^3c^{12}}$

c) $\sqrt{x} \cdot \sqrt[4]{x}$

$x^{1/2} \cdot x^{1/4}$

$\frac{1}{2} + \frac{1}{4} =$

$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$= x^{1/2+1/4}$

$= \boxed{x^{3/4}} = \boxed{\sqrt[4]{x^3}}$

d) $\sqrt[6]{y^2} \cdot \sqrt[9]{y}$

$= y^{2/6} \cdot y^{1/9}$

$\frac{1}{3} + \frac{1}{9}$

$= y^{1/3} \cdot y^{1/9}$

$= \frac{3}{9} + \frac{1}{9}$

$= y^{1/3+1/9}$

$= \frac{4}{9}$

$= \boxed{y^{4/9}} = \boxed{\sqrt[9]{y^4}}$

e) $\frac{\sqrt[3]{k}}{\sqrt[7]{k^2}} = \frac{k^{1/3}}{k^{2/7}}$

$= k^{1/3-2/7}$

$\frac{1}{3} - \frac{2}{7} = \frac{7}{21} - \frac{6}{21}$

$= \boxed{k^{1/21}}$

$= 1/21$

$= \boxed{\sqrt[21]{k}}$

f) $\frac{\sqrt[8]{m^4}}{\sqrt[6]{m}} = \frac{m^{4/8}}{m^{1/6}}$

$= \frac{m^{1/2}}{m^{1/6}} = m^{\frac{1}{2}-\frac{1}{6}}$

$\frac{1}{2} - \frac{1}{6} = \frac{3}{6} - \frac{1}{6}$

$= \frac{2}{6} = \frac{1}{3}$

$= \boxed{m^{1/3}}$

$= \boxed{\sqrt[3]{m}}$

g) $\sqrt[4]{\sqrt[5]{x}} = (x^{1/5})^{1/4}$

$= \boxed{x^{1/20}}$

$= \boxed{\sqrt[20]{x}}$

h) $\sqrt[3]{2} \cdot \sqrt[5]{3}$

can't simplify because the radicands are different