UNIT 2 NOTES

Date: **Section: 2.2**

Objective: Analyzing Functions: Maxima, Minima, Increasing, Decreasing, Constant

Relative Maxima and Minima

- When a point is _____ than all the points near it, it is called a ***relative*** _____.

- When a point is _____ than all the points near it, it is called a ***relative*** _____.

- If you are asked for a ***maximum or a minimum <u>point</u>,*** write the answer as an _____.

- If you are asked for a ***maximum or a minimum <u>value</u>,*** the answer is the _____.

- Infinity (positive or negative) is NOT a maximum or a minimum.

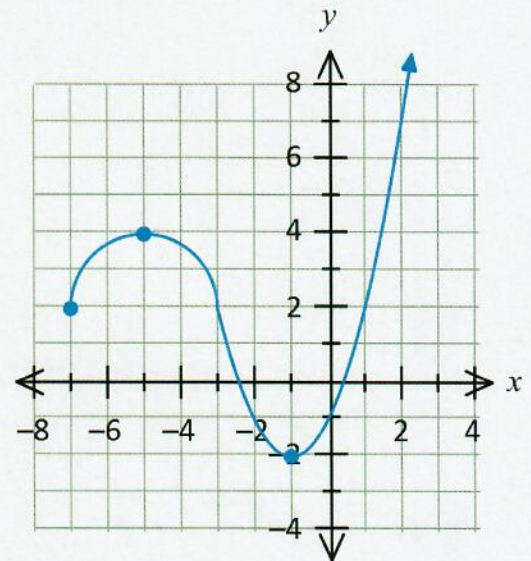- Maximum or minimum points are usually the endpoints or vertices.

Example:
a) Find the relative maximum point.

b) Find the relative maximum value.

c) Find the relative minimum points.

d) Find the relative minimum values.

Increasing, Decreasing, and Constant

If you look from left to right along the graph of the function, you will notice parts are *rising*, parts are *falling* and parts are *flat*. The different parts of the graph are described as intervals on which the function is *increasing*, *decreasing*, or *constant*, respectively.

- _____: ***Uphill*** from ***left*** to ***right***.

- _____: ***Downhill*** from ***left*** to ***right***.

- _____: *Flat.*

Writing Intervals Where the Graph is Increasing, Decreasing or Constant:

- Write the intervals of _____-coordinates showing where the graph *starts* and *stops* going each direction from _____ to _____.

- **Always use (and). Never use [and].**

- **Hint:** Look for places where the graph changes direction (relative maxima or relative minima) to help you break the graph into intervals.

- Use the ∪ sign to connect multiple intervals: $(\_,\_)\cup(\_,\_)$

- **REMEMBER:** Only write down *x*-coordinates! **You might want to cross out the numbers on the y-axis to help you remember not to write down the y's.**

Example: Highlight the increasing, decreasing, and constant sections of the graph each a different color. Then write the intervals where the graph is increasing, decreasing, and constant in interval notation.
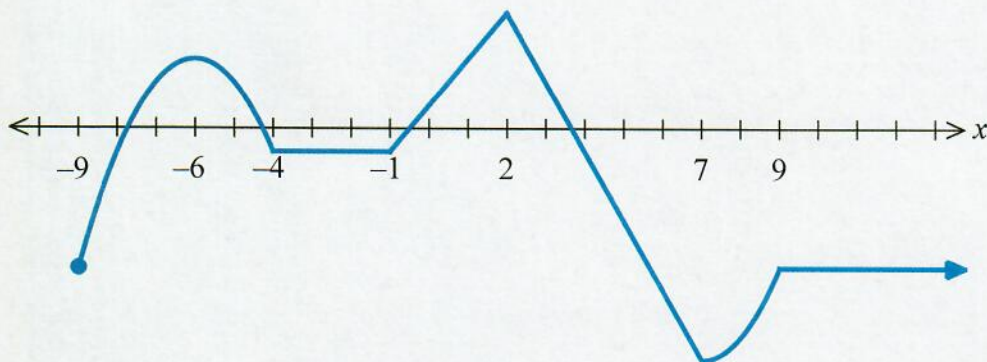
a)

The increasing section(s) are _____ color.   Increasing interval(s): _____
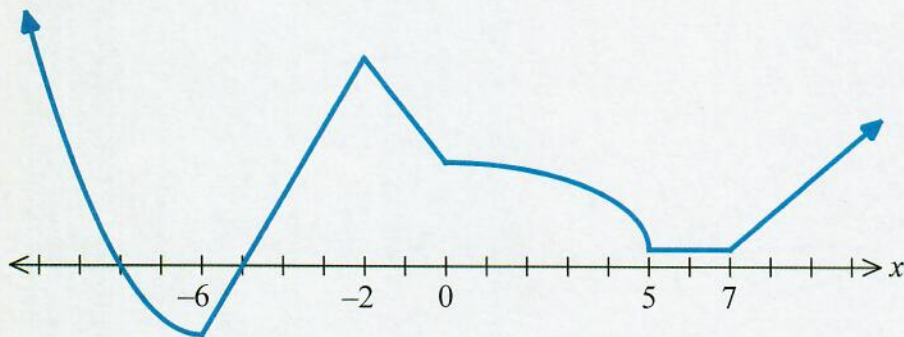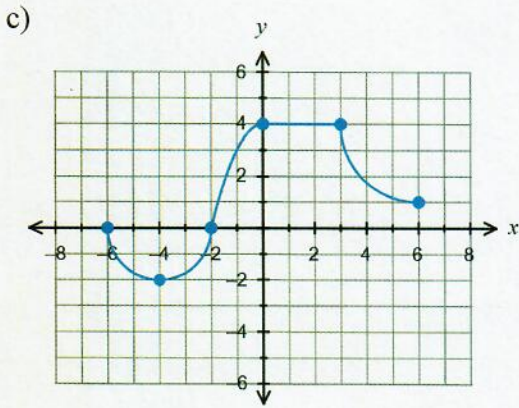
The decreasing section(s) are _____ color.   Decreasing interval(s): _____

The constant section(s) are _____ color.   Constant interval(s): _____

b)

The increasing section(s) are _____ color.   Increasing interval(s): _____

The decreasing section(s) are _____ color.   Decreasing interval(s): _____

The constant section(s) are _____ color.   Constant interval(s): _____

c)

Increasing:
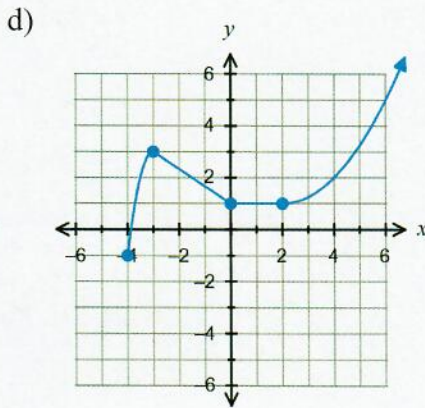 Color:

  Interval(s):

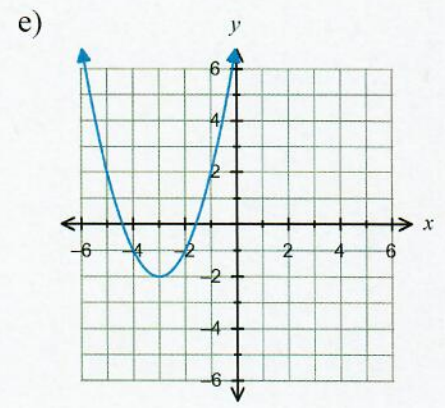**Decreasing:**
  Color:

  Interval(s):

Constant:
  Color:

  Interval(s):

d)

Increasing:
  Color:

  Interval(s):

Decreasing:
  Color:

  Interval(s):

Constant:
  Color:

  Interval(s):

e)

Increasing:
  Color:

  Interval(s):

Decreasing:
  Color:

  Interval(s):

Constant:
  Color:

  Interval(s):

Intercepts

    x-Intercepts: The points where a graph crosses the _____. They have the form $(x, 0)$.

       • To find the *x*-intercept(s), _____.

    y-Intercepts: The points where a graph crosses the _____. They have the form $(0, y)$.

       • To find the *y*-intercept(s), _____.

Examples: Find the intercepts of each graph. Write the intercepts as ordered pairs.

a) $f(x) = 2x + 6$ b) $f(x) = -3x + 2$

x-intercept _____   *x*-intercept _____

y-intercept _____   *y*-intercept _____

c) $3x + 2y = 12$ d) $x - 2y = 5$

x-intercept _____   *x*-intercept _____
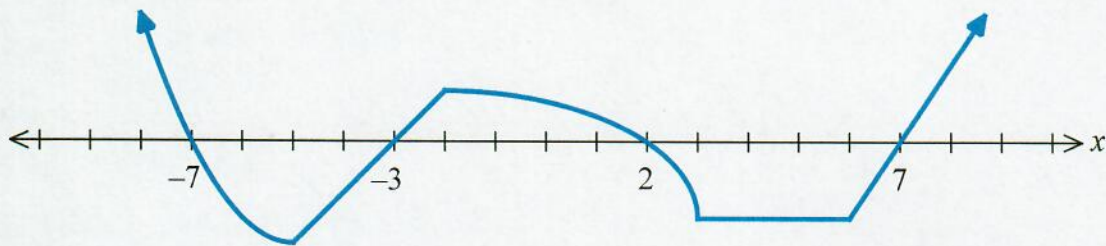
y-intercept _____   *y*-intercept _____

Positive and Negative

- A function is *positive* where the y-coordinates are positive. The graph is _____ *the x-axis.*

- A function is *negative* where the y-coordinates are negative. The graph is _____ *the x-axis.*

★ When you are asked to state where the graph is positive and negative, write the intervals of the

    of _____ - coordinates from _____ to _____.

★ Use _____ at the x-intercepts, where the graph crosses over from positive to negative. The y-coordinate is zero at the intercepts, so the graph is neither positive or negative there. That means those points are not included in the interval.

★ Use _____ if the graph has an *endpoint* somewhere above or below the x-axis.

Example: Put a large dot on the x-intercepts. Highlight the positive and negative sections of the graph each a different color. Then write the intervals where the graph is positive or negative in interval notation.
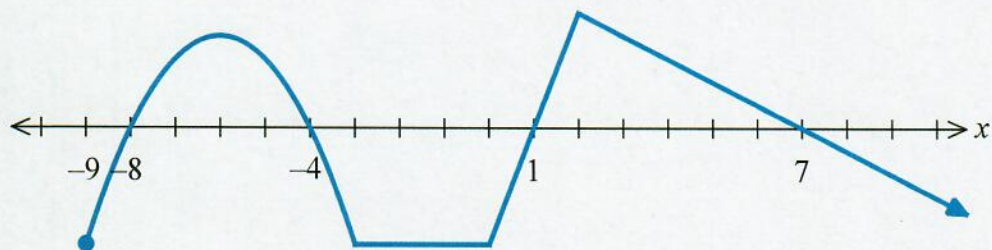
a)

Draw a dotted line through the x-intercepts.

The positive section(s) are _____ color.   Positive interval(s):_____

The negative section(s) are _____ color.   Negative interval(s):_____

b)

Draw a dotted line through the x-intercepts.

The positive section(s) are _____ color.   Positive interval(s):_____
The negative section(s) are _____ color.   Negative interval(s):_____

Example: Give the coordinates of the intercepts as ordered pairs. Then, highlight the parts of the graph where the function is positive and the parts where the function is negative in different colors. Write the intervals where the function is positive and negative in interval notation.
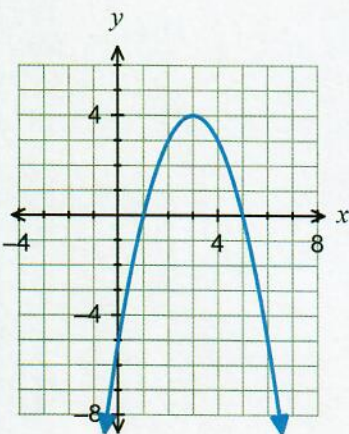
a)

x-intercept(s):_____

y-intercept:_____
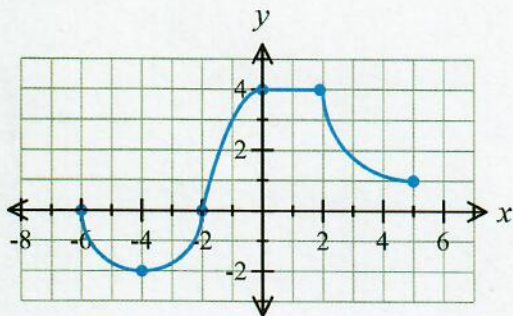
Positive color:_____

Positive interval(s):_____

Negative color:_____

Negative interval(s):_____
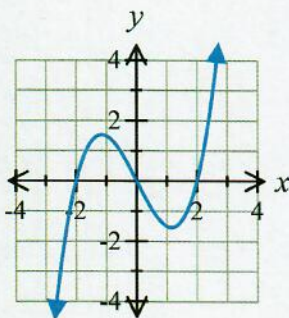
b)

x-intercept(s):_____

y-intercept:_____

Positive color:_____

Positive interval(s):_____

Negative color:_____

Negative interval(s):_____

c)

x-intercept(s):_____

y-intercept:_____

Positive color:_____

Positive interval(s):_____

Negative color:_____

Negative interval(s):_____

SM 2

Date:

Section: 2.4

**Objective: Analyzing Functions: Symmetry and End Behavior**

Symmetry

y-axis or _____ *symmetry:*
- The left and right sides are mirror images around the ____-axis.
- The left and right sides would overlap if you fold the graph along the ____-axis.
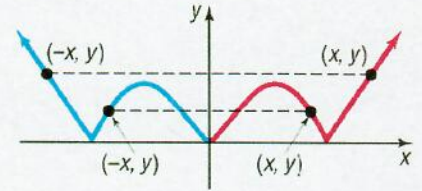
Origin or _____ *symmetry*
- When you rotate the graph around 180°, you end up with the same graph you started with.
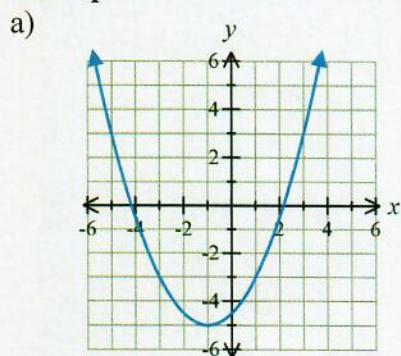- If you fold the graph along the x-axis and then again along the y-axis, the two halves would overlap.
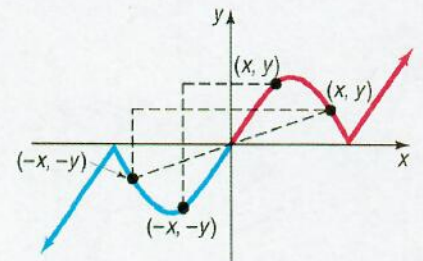
Examples: Determine what type of symmetry each function has (even, odd, or neither).
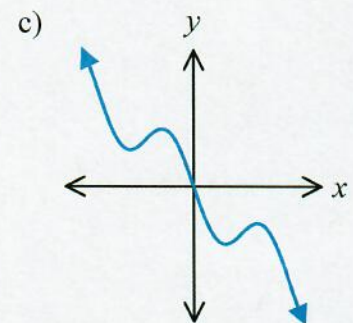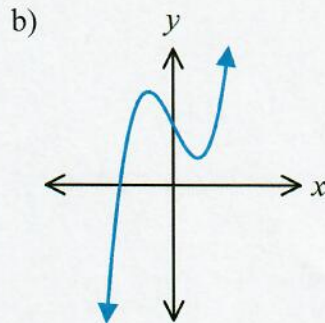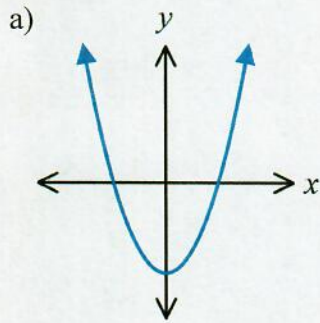
a)

b)

c)

End Behavior

End behavior describes what is happening to the **y-coordinates** of the graph as you move left $(x \to -\infty)$ or as you move right $(x \to \infty)$.
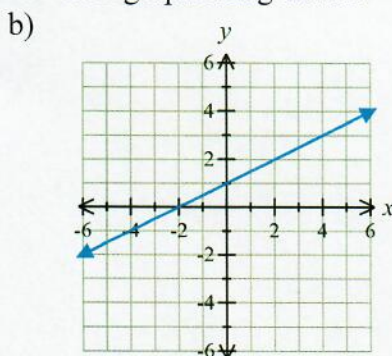
- **Left end behavior** looks like this: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$.
- **Right end behavior** looks like this: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$.
- **Arrow pointing up:** Write ∞
- **Arrow pointing down:** Write $-\infty$
- **Endpoint (no arrow):** Write D.N.E. (does not exist)
- **Asymptote or flat end with arrow:** Write y-coordinate of asymptote or flat part

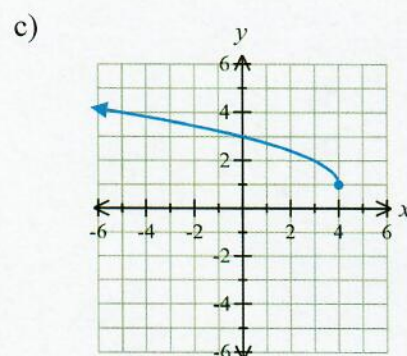**Examples:** Describe the end behavior of each graph using limits.

a)

Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

b)

Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

c)

Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

d)

Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

e)

Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

f)
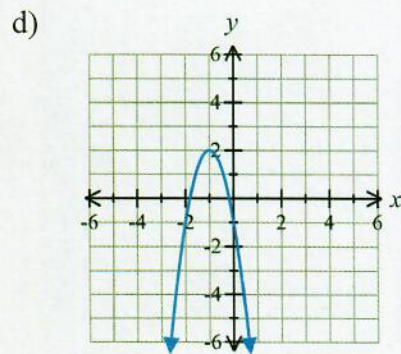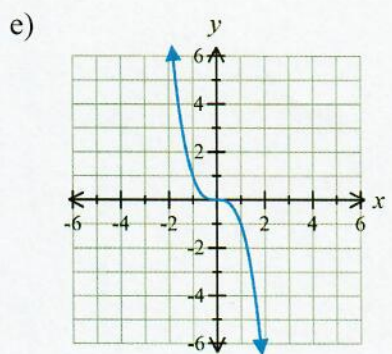
Left: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$

Right: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$

Analyzing Functions Study Guide

Domain and Range:
- **Domain:** all *x*-coordinates on the graph from *left* to *right*.
- **Range:** all *y*-coordinates on the graph from *bottom* to *top*.
  - Graphs with unconnected dots (no solid line): List *x*'s and *y*'s in { and }.
    - Don't list repeated numbers more than once.
  - Graphs with solid lines (even if there are labeled dots on it):
    - Use interval notation: (__,__), (__,__], [__,__), or [__,__].
    - If there's an arrow on the end of a graph, the domain and range will involve −∞ or ∞.
    - **Use [or] for endpoints and vertices** (places where the graph changes direction).
   - **Use (or) for −∞, ∞, asymptotes, or open circles.**

Increasing, Decreasing or Constant: (Write *x*'s)
- Write **_x-coordinates_** where graph *starts* and *stops* going each direction **from *left* to *right*.**
- **Always use (and).**
- **Increasing:** *Uphill* from *left* to *right*.
- **Decreasing:** *Downhill* from *left* to *right*.
- **Constant:** *Flat*.
- **Hint:** Look for places where the graph changes direction (relative maxima or relative minima) to help you break the graph into intervals.
- Use the ∪ sign to connect multiple intervals: (__,__)∪(__,__)
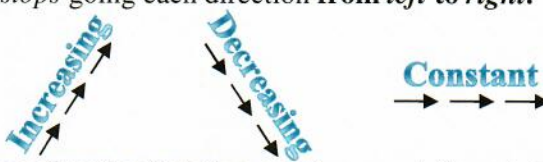
Positive or Negative: (Write *x*'s)
- **Positive:** *Above x*-axis.
- **Negative:** *Below x*-axis.
- Divide the graph into the parts that are above the *x*-axis and the parts that are below the *x*-axis using the *x*-intercepts. Write **_x-coordinates_** for the *start* and *end* of each interval **from *left* to *right*.**
- Use (and) at *x*-intercepts.
- Use [or] only when there is an *endpoint above or below the x-axis*.
- Use the ∪ sign to connect multiple intervals: (__,__)∪(__,__)
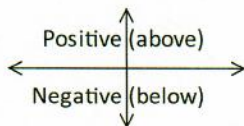
Intercepts: The points where the graph crosses the *x*- or *y*-axis.
- Write intercepts as ordered pairs.
  - *x*-intercepts are written as $(x, 0)$.
  - *y*-intercepts are written as $(0, y)$.
- To find *x*-intercepts algebraically, **set $y = 0$ and solve for *x*.**
- To find *y*-intercepts algebraically, **set $x = 0$ and solve for *y*.**

Relative Maximum or Relative Minimum:
- **Relative maximum:** a point on the graph that is **higher** than all the points around it.
- **Relative minimum:** a point on the graph that is **lower** than all the points around it.
- **Maximum or minimum _point_:** Write ordered pair: (x, y).
- **Maximum or minimum _value_:** Write *y*-coordinate of the point.

End Behavior: End behavior describes what is happening to the **y-coordinates** of the graph as you move left $(x \to -\infty)$ or as you move right $(x \to \infty)$.

- **Left end behavior** looks like this: $\lim\limits_{x \to -\infty} f(x) = \underline{\quad}$.

- **Right end behavior** looks like this: $\lim\limits_{x \to \infty} f(x) = \underline{\quad}$.

- **Arrow pointing up:** Write ∞
- **Arrow pointing down:** Write $-\infty$
- **Endpoint (no arrow):** Write D.N.E. (does not exist)
- **Asymptote or flat end with arrow:** Write y-coordinate of asymptote or flat part

Symmetry:
- **Even symmetry (y-axis):**
  - The left and right sides are mirror images around the y-axis. (Left and right sides would overlap if you fold the graph along the y-axis).

    Even:
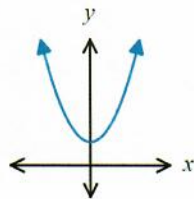
- **Odd symmetry (origin):**
  - If you fold the graph along the x-axis and then along the y-axis, the two halves will overlap.
  - If you spin the graph around $180°$, you will end up with what you started with.

    Odd: