

Date:

Section: 1.1

# SM 2

## Objective: Functions

**Function:** When each domain element is paired with a unique range element.

- Only 1  $y$ -value for every  $x$ -value
- For every input there is one and only one output.

*x-values cannot repeat for the relation to be a function.*

**Domain:** The set of all inputs (the  $x$ -values) of a relation.

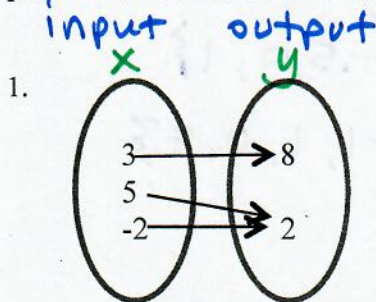
- If a relation is represented by a graph or ordered pairs, the domain is the set of all  $x$ -coordinates of points on the graph.

**Range:** The set of all outputs (the  $y$ -values) of a relation.

- If a relation is represented by a graph or ordered pairs, the range is the set of all  $y$ -coordinates of points on the graph.

**relation:** a relationship between two sets of values.

**Examples:** Decide whether each relation is a function. Then write the relation as a set of ordered pairs. Then find the domain and range.

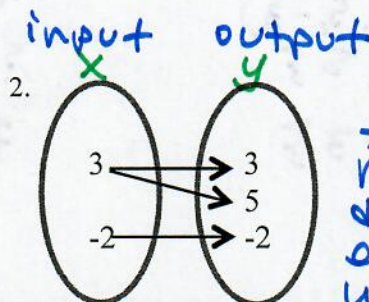


Function: *yes*

Ordered Pairs:  $(3, 8), (5, 2), (-2, 2)$

Domain:  $\{-2, 3, 5\}$

Range:  $\{2, 8\}$



Function: *no - the 3 repeats*

Ordered Pairs:  $(3, 3), (3, 5), (-2, -2)$

Domain:  $\{-2, 3\}$

Range:  $\{-2, 3, 5\}$

*\* ordering is not required. Do not duplicate a repeated element when writing the domain and range.*

3.

$x$	$y$
-2	5
-1	7
0	9
1	-2
2	-2

Function: *yes*

Domain:  $\{-2, -1, 0, 1, 2\}$

Range:  $\{-2, 5, 7, 9\}$

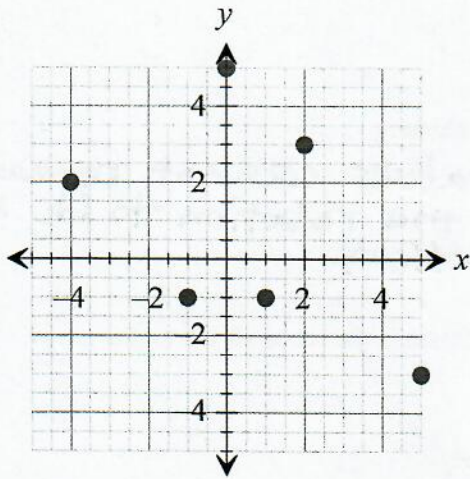
4.  $\{(4, -7), (-1, 5), (8, 2), (4, 5)\}$

Function: *no - the x-value of 4 repeats*

Domain:  $\{-1, 4, 8\}$

Range:  $\{-7, 2, 5\}$

5. \* Vertical line test



Function: *yes*

Ordered Pairs:

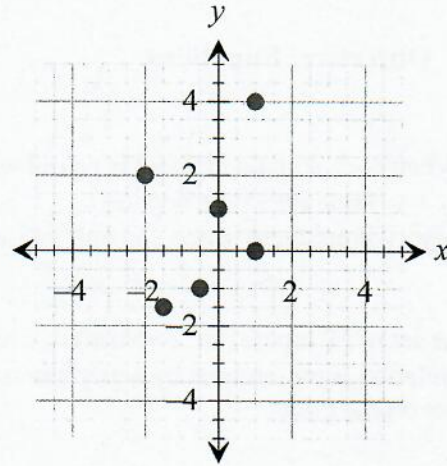
$(-4, 2), (-1, -1), (0, 5), (1, -1), (2, 3), (5, -3)$

Domain:

$\{-4, -1, 0, 1, 2, 5\}$

Range:

$\{2, -1, 5, 3, -3\}$



Function: *no - the x-value of 1 repeats*

Ordered Pairs:

$(-2, 2), (-1.5, -1.5), (-0.5, -1), (0, 1), (1, 0), (1, 4)$

Domain:

$\{-2, -1.5, -0.5, 0, 1\}$

Range:

$\{2, -1.5, -1, 1, 0, 4\}$

**Domain and Range in Real Life Situations:** In real life situations, it's important to think through what values make sense in the problem. You also need to think about what  $x$  stands for and what  $y$  stands for.

**Examples:** Determine if the situation is a function. Describe the domain and range for each real world situation.

- a) A teacher creates a list of ice cream flavors on the board. Then she has each student write their name next to the ice cream flavor they like best of the ones listed.

Is this a function? Why or why not? *no - the ice cream flavors*

Domain: *ice cream flavors*

Range: *names*

*Vanilla - bob, Jack  
chocolate - sue, Tyler, Ted  
strawberry - Beth*

- b) A teacher creates a table with the student's name in the first column. Then she has the students write down their favorite ice cream flavor next to their name.

Is this a function? Why or why not? *yes - the individuals in the class won't be repeated.*

Domain: *students*

Range: *flavors*

**Examples:** Describe the domain and range for each real world situation.

- c) Your cell phone plan charges a flat fee of \$10 for up to 1000 texts and \$0.10 per text over 1000. What are the domain and range?

Circle which unit represents the domain: Money or Texts

Domain: *number of texts [0, ∞)*

Range: *cost*

*[\$10, ∞)*

$$C = \$10 + .10(x - 1000) \\ \text{(over 1000 texts)}$$

- d) You are getting ready for the Homecoming dance. Your dad is going to let you borrow his new car, but you need to wash and fill it. The car wash costs \$5 and the gas costs \$3.89 per gallon. The car can hold 15 gallons of gas. What are the domain and range if the total cost is a function of the number of gallons and the car is completely empty when you pull into the gas station?

Circle which unit represents the domain: Money or Gallons

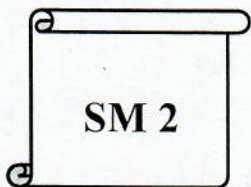
Domain: *number of gallons of gas [0, 15]*

Range: *total cost*

*[\$5, \$63.35]*

$$\text{Cost} = \$5 + 3.89x \\ C = \$5 + 3.89(15) \\ = 63.35$$

*x = gallons of gas*



Date:

Section: 1.2 Starter Notes

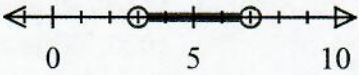

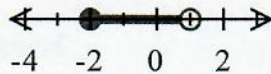
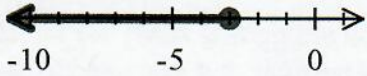
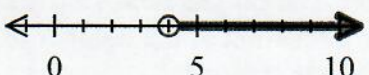
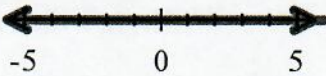
Objective: Interval Notation

### Interval Notation

Domain and range are often written in *interval notation*.

- The numbers are where the interval starts and stops.
- If an endpoint is *included*, put it in a bracket [ or ]
- If an endpoint is *not included*, put it in parentheses ( or )
- If the interval goes on forever, use  $-\infty$  or  $\infty$ . These always get put in parentheses ( or ).

### Examples:

- $(3, 7)$  means everything between 3 and 7, not including either 3 or 7.   
 $3 < x < 7$
- $[5, 8]$  means everything between 5 and 8, including both 5 and 8.   
 $5 \leq x \leq 8$
- $[-2, 1)$  means everything between -2 and 1, including -2, but not including 1.   
 $-2 \leq x < 1$
- $(-\infty, -3]$  means everything less than or equal to -3.   
 $x \leq -3$
- $(4, \infty)$  means everything greater than 4.   
 $x > 4$
- $(-\infty, \infty)$  means all real numbers (goes on forever in both directions).   
 $-\infty < x < \infty$

### Tips

- Use parentheses ( or ) around numbers that *are not included* in the domain or range.
  - This happens when there is an *open circle* at a point or an *asymptote* (a line that the graph gets really close to, but never actually touches).
- Use brackets [ or ] around endpoints that *are included* in the domain or range.
  - If there is a point on the graph with the given x- or y-coordinate, use a bracket.
- Always use parentheses around  $-\infty$  and  $\infty$ .
- **Read the domain from left to right and the range from down to up.**
  - Write the lower value or  $-\infty$  first and the higher value or  $\infty$  last.

## Objective: Domain and Range

**Domain:** The set of all inputs (the  $x$ -values) of a relation.

- If a relation is represented by a graph, the domain is the set of all  $x$ -coordinates of points on the graph.
- When modeling a real-life situation, the domain is the set of  $x$ 's that make sense in the problem.
- When given an equation, the domain is the set of  $x$ -values for which the equation is defined.

**Range:** The set of all outputs (the  $y$ -values) of a relation.

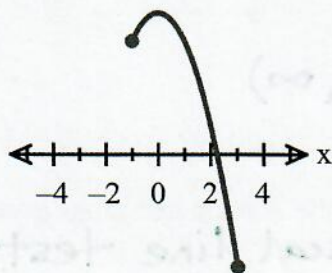
- If a relation is represented by a graph, the range is the set of all  $y$ -coordinates of points on the graph.
- When modeling a real-life situation, the range is the set of  $y$ 's that make sense in the problem.

**Domain - given the equation of a function.** When given an equation, think about what  $x$ -values will "work" as input into the equation. Are there any values of  $x$  for which the equation is not defined?

**Asymptote:** A line that the graph approaches but never reaches. It is represented as a dotted line. If the graph has an asymptote, use a parenthesis in your interval notation for domain and range.

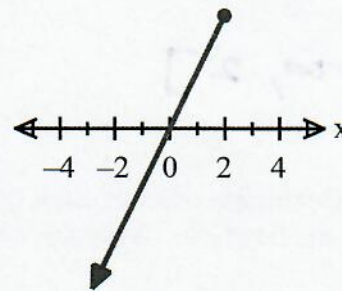
**Example:** State the domain in interval notation.

a)



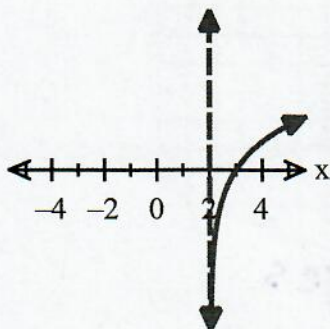
Domain:  $[-2, 3]$

b)



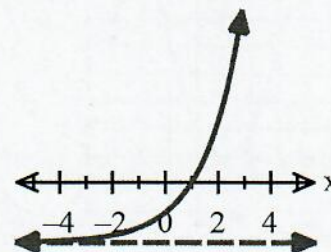
Domain:  $(-\infty, 2]$

c)



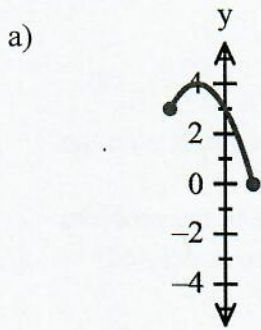
Domain:  $(2, \infty)$

d)

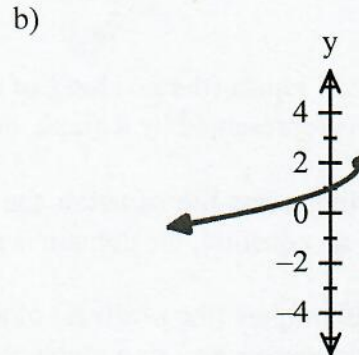


Domain:  $(-\infty, \infty)$

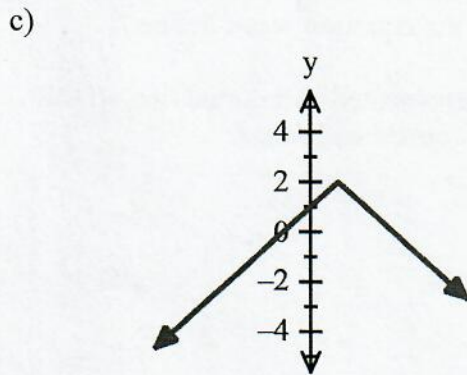
**Example:** State the range in interval notation.



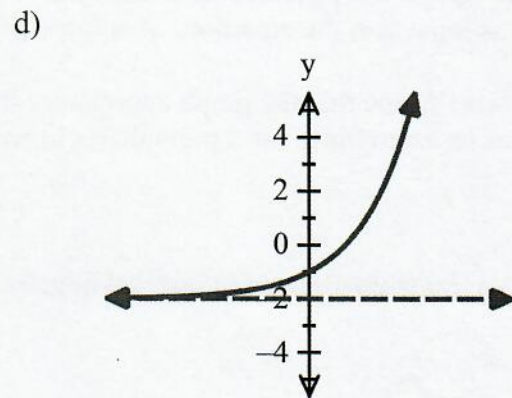
Range:  $[0, 4]$



Range:  $(-\infty, 2]$

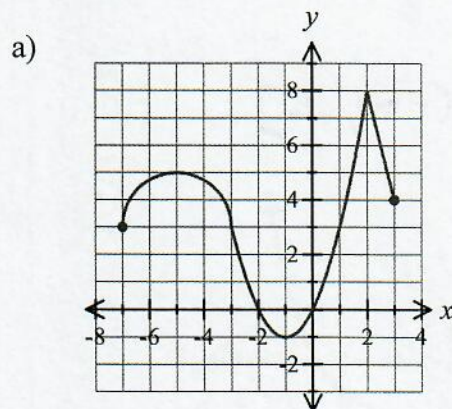


Range:  $(-\infty, 2]$



Range:  $(-2, \infty)$

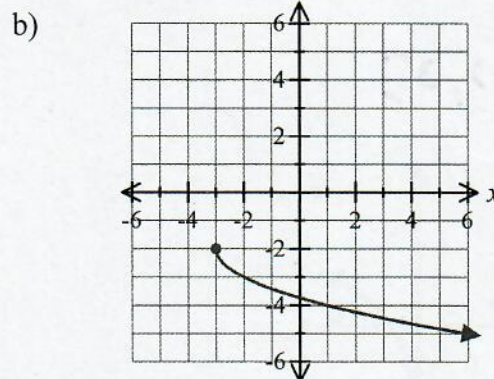
**Examples:** Determine whether each graph represents a function. Then state the domain and range given the graph of the function. Write answers in interval notation. *\*vertical line test*



Function? *yes*

Domain:  $[-7, 3]$

Range:  $[-1, 8]$

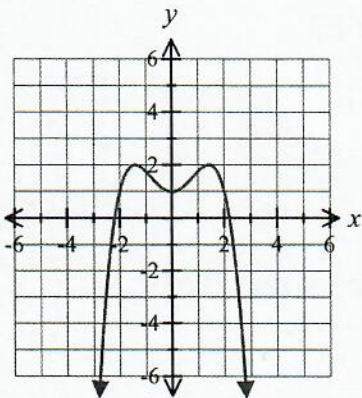


Function? *yes*

Domain:  $[-3, \infty)$

Range:  $(-\infty, -2]$

c)

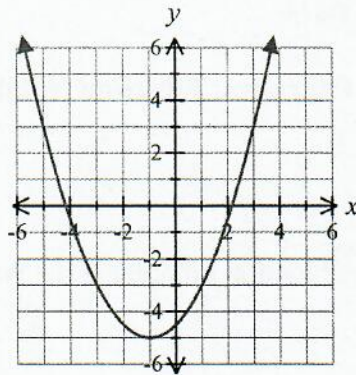


Function? *yes*

Domain:  $(-\infty, \infty)$

Range:  $(-\infty, 2]$

d)

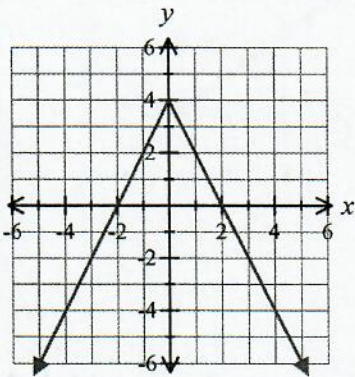


Function? *yes*

Domain:  $(-\infty, \infty)$

Range:  $[-5, \infty)$

e)

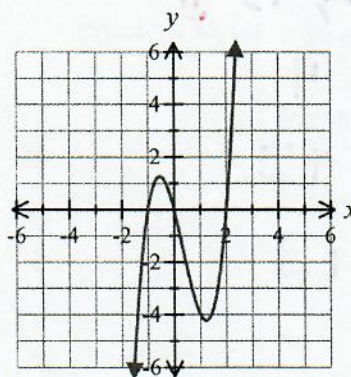


Function? *yes*

Domain:  $(-\infty, \infty)$

Range:  $(-\infty, 4]$

f)

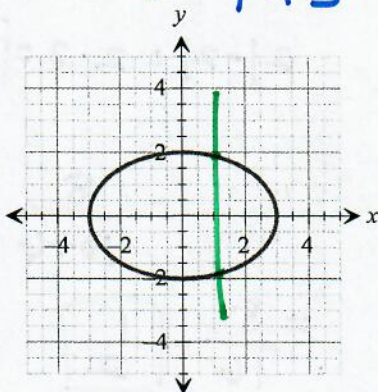


Function? *yes*

Domain:  $(-\infty, \infty)$

Range:  $(-\infty, \infty)$

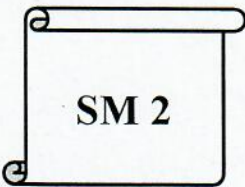
g)



Function? *no - fails the vertical line test.*

Domain:  $[-3, 3]$

Range:  $[-2, 2]$



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Objective: Function Notation

**Function notation:** Function notation is the way a function is written.  $f(x)$  is read "f of x".

$x$  = input

$f(x)$  = output

$f(x)$  is the same as  $y$ .

**Examples:** The graph of  $y = f(x)$  is shown below. Use it to answer the following questions.

a) Find  $f(-4)$ . 1

b) Find  $f(0)$ . 1

c) For what values of  $x$  is  $f(x) = 0$ ?

-3.75, -1/2, 1

d) For what values of  $x$  is  $f(x) = -3$ ?

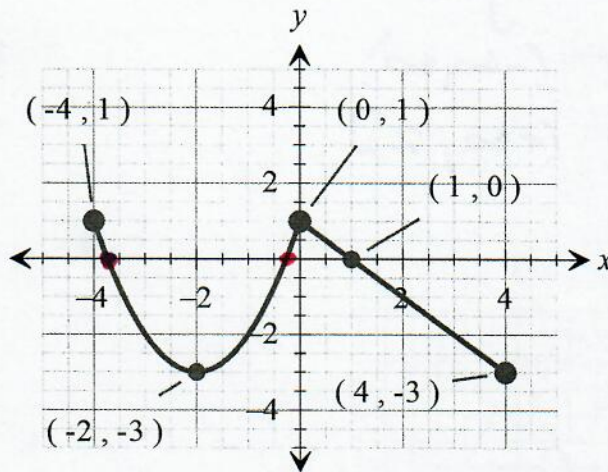
-2, 4

e) What is the domain?

[-4, 4]

f) What is the range?

[-3, 1]



Parenthesis  
exponents  
multiply/divide left to right  
add/subtract left to right

**REVIEW order of operations**

a)  $-4^2$

-16

b)  $(-4)^2$

16

c)  $-(1+3)^2$

$-(4)^2 = -16$

d)  $-2(3^2)$

$-2(9) = -18$

e)  $-3 \cdot (-2)^2 + 4$

$-3 \cdot 4 + 4$   
 $-12 + 4 = -8$

f)  $2|3-5|$

$2|-2| = 2 \cdot 2 = 4$

g)  $8 + |10-6|$

$8 + |4| = 12$

h)  $\frac{2+6}{-4}$

$= \frac{8}{-4} = -2$

i)  $\frac{8}{2(5)+6} = \frac{8}{10+6}$

$= \frac{8}{16} = \frac{1}{2}$



Evaluating a function or finding a value means substituting the given value for  $x$  in the equation. Evaluate the expression. You may use a calculator to evaluate the expression.

Examples: Find each value if  $f(x) = x^2 - 2x + 3$ ,  $g(x) = 3x - 5$ , and  $h(x) = \frac{x}{4-2x}$ .

Leave your answers as simplified fractions, if necessary. Show all your work.

a)  $f(2)$

$$\begin{aligned} & 2^2 - 2(2) + 3 \\ & = 4 - 4 + 3 = \boxed{3} \end{aligned}$$

b)  $g(-1)$

$$\begin{aligned} & 3(-1) - 5 \\ & = -3 - 5 = \boxed{-8} \end{aligned}$$

c)  $h(4)$

$$\frac{4}{4-2(4)} = \frac{4}{4-8} = \frac{4}{-4} = \boxed{-1}$$

d)  $g\left(\frac{2}{3}\right)$

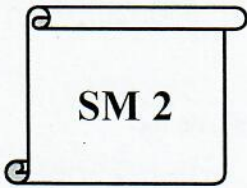
$$\begin{aligned} & 3\left(\frac{2}{3}\right) - 5 \\ & = 2 - 5 = \boxed{-3} \end{aligned}$$

e)  $f(-5)$

$$\begin{aligned} & (-5)^2 - 2(-5) + 3 \\ & = 25 + 10 + 3 \\ & = \boxed{38} \end{aligned}$$

f)  $h(-3)$

$$\frac{-3}{4-2(-3)} = \frac{-3}{4+6} = \boxed{\frac{-3}{10}}$$



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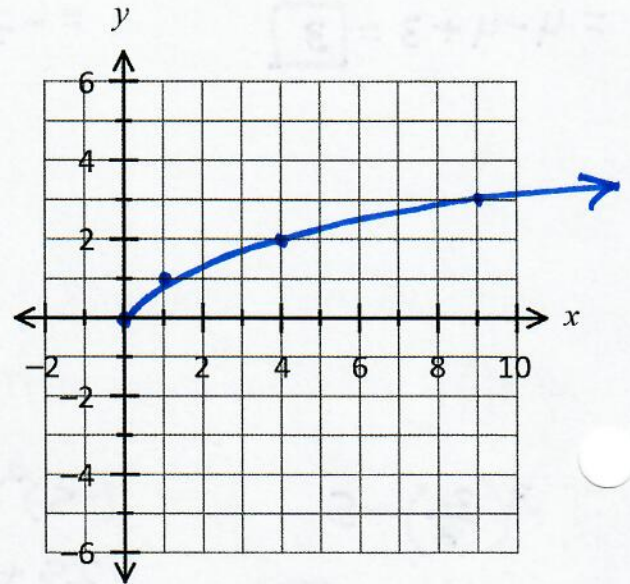
**Objective: Parent Graphs and Transformations**

**Parent Graphs** - Fill in the table to find some **key points** for some important graphs. Use the table to generate ordered pairs for points on the graph, then sketch the graph.

★ **Square Root Function:**  $f(x) = \sqrt{x}$

Domain:  $[0, \infty)$  Range:  $[0, \infty)$

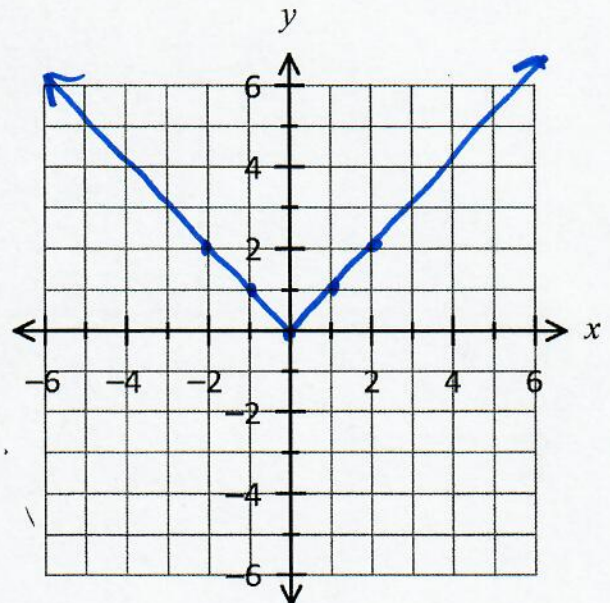
x	work	y	point
-1	$\sqrt{-1}$	undefined	
0	$\sqrt{0}$	0	(0,0)
1	$\sqrt{1}$	1	(1,1)
4	$\sqrt{4}$	2	(4,2)
9	$\sqrt{9}$	3	(9,3)



★ **Absolute Value Function:**  $f(x) = |x|$

Domain:  $(-\infty, \infty)$  Range:  $[0, \infty)$

x	work	y	point
-2	$ -2 $	2	(-2,2)
-1	$ -1 $	1	(-1,1)
0	$ 0 $	0	(0,0)
1	$ 1 $	1	(1,1)
2	$ 2 $	2	(2,2)

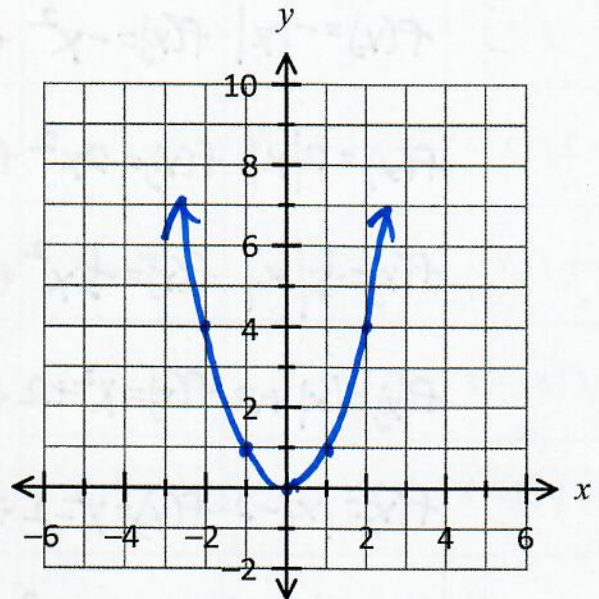


★ Quadratic Function:  $f(x) = x^2$

\* parabola

Domain:  $(-\infty, \infty)$  Range:  $[0, \infty)$

x	work	y	point
-2	$(-2)^2$	4	$(-2, 4)$
-1	$(-1)^2$	1	$(-1, 1)$
0	$0^2$	0	$(0, 0)$
1	$1^2$	1	$(1, 1)$
2	$2^2$	4	$(2, 4)$



### Types of transformations

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct "Order of Transformations."

#### 1. Reflection:

- Vertical Reflection - graph is reflected over the x-axis
- Horizontal Reflection - graph is reflected over the y-axis

#### 2. Stretch/Compression:

We will restrict our attention to Vertical stretches/compressions.

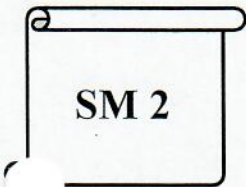
- Vertical Stretch - the y-coordinates are multiplied by a scalar that is greater than 1
- Vertical Compression - the y-coordinates are multiplied by a scalar that is between 0 and 1

#### 3. Translation (or Shift):

- Horizontal Translation - graph is shifted to the left or right
- Vertical Translation - graph is shifted up or down

**Transformations** of the parent graph:

	$f(x) =  x $	$f(x) = x^2$	$f(x) = \sqrt{x}$	Effect on Parent Graph
$y = -f(x)$	$f(x) = - x $	$f(x) = -x^2$	$f(x) = -\sqrt{x}$	reflects over the x-axis
$y = 2f(x)$	$f(x) = 2 x $	$f(x) = 2x^2$	$f(x) = 2\sqrt{x}$	vertical stretch of 2
$y = \frac{1}{2}f(x)$	$f(x) = \frac{1}{2} x $	$f(x) = \frac{1}{2}x^2$	$f(x) = \frac{1}{2}\sqrt{x}$	vertical shrink of $\frac{1}{2}$
$y = f(x) + 2$	$f(x) =  x  + 2$	$f(x) = x^2 + 2$	$f(x) = \sqrt{x} + 2$	shift up 2
$y = f(x) - 2$	$f(x) =  x  - 2$	$f(x) = x^2 - 2$	$f(x) = \sqrt{x} - 2$	shift down 2
$y = f(x + 2)$	$f(x) =  x + 2 $	$f(x) = (x + 2)^2$	$f(x) = \sqrt{x + 2}$	shift left 2
$y = f(x - 2)$	$f(x) =  x - 2 $	$f(x) = (x - 2)^2$	$f(x) = \sqrt{x - 2}$	shift right 2



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Objective: One-Step Transformations

A. Types of transformations

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct "Order of Transformations."

1. Reflection:

- a. Vertical Reflection – graph is reflected over the x-axis
- b. Horizontal Reflection – graph is reflected over the y-axis

2. Stretch/Compression (Dilation):

We will restrict our attention to Vertical stretches/compressions.

- a. Vertical Stretch (dilate by a factor) – the y-coordinates are multiplied by a scalar that is greater than 1
- b. Vertical Compression (dilate by a factor) – the y-coordinates are multiplied by a scalar that is between 0 and 1

3. Translation (or Shift):

- a. Horizontal Translation – graph is shifted to the left or right
- b. Vertical Translation – graph is shifted up or down

B. Applying Transformations to the Function:

For each graph, do the following:

1. Identify the parent graph ( $y = |x|$ ,  $y = x^2$ , or  $y = \sqrt{x}$ ).
2. Fill in the x, y table for the parent graph.
3. Draw the graph of the parent graph with a dashed line.
4. Identify the transformation.
5. Fill in the x, y table for the transformation.
6. Draw the final graph with a solid line.

\* Think about whether the transformation affects the x-values or y-values.

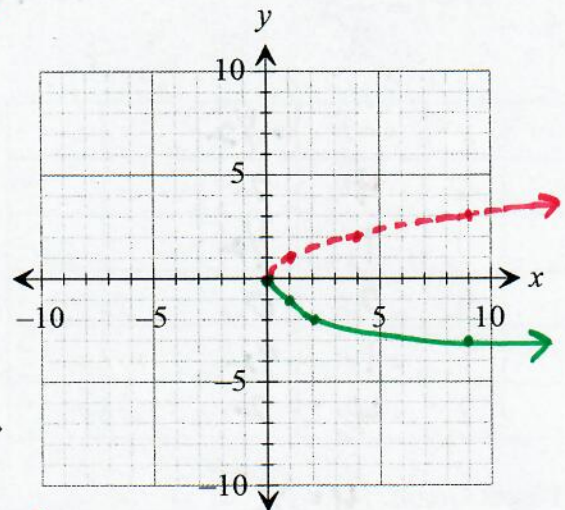
Vertical reflection/Reflection over the x-axis:

1.  $f(x) = -\sqrt{x}$  *affects y-values*

x	$y = \sqrt{x}$	$\cdot -1$		
0	0	0		
1	1	-1		
4	2	-2		
9	3	-3		

Parent Graph:  $y = \sqrt{x}$

Transformation: *reflect over the x-axis*

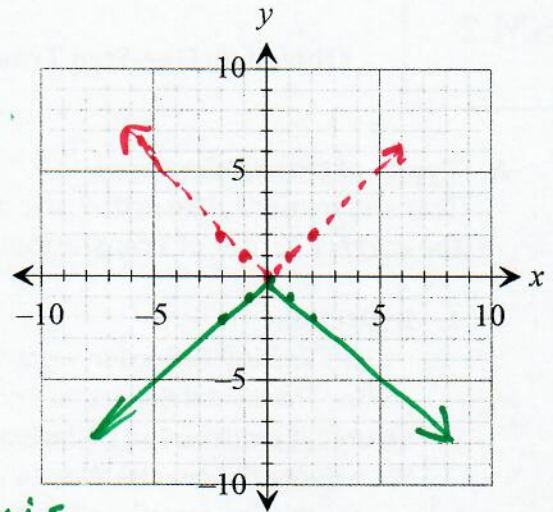


2.  $f(x) = -|x|$  \* affects y values

x	y =  x	• -1		
-2	2	-2		
-1	1	-1		
0	0	0		
1	1	-1		
2	2	-2		

Parent Graph:  $y = |x|$

Transformation: reflects over the x axis



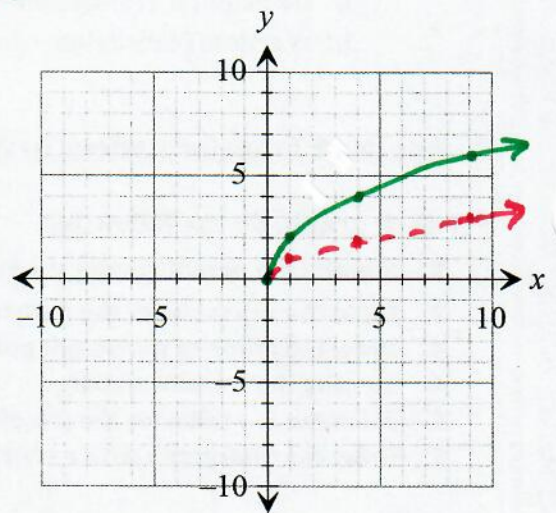
Vertical stretch/compression (shrink)

3.  $f(x) = 2\sqrt{x}$  \* affects y values

x	y = $\sqrt{x}$	• 2		
0	0	0		
1	1	2		
4	2	4		
9	3	6		

Parent Graph:  $y = \sqrt{x}$

Transformation: vertical stretch of 2

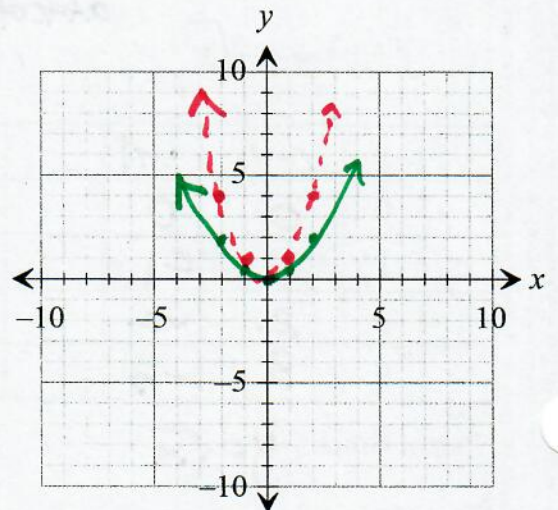


4.  $f(x) = \frac{1}{2}x^2$  \* affects y values

x	y = $x^2$	• 1/2		
-2	4	2		
-1	1	1/2		
0	0	0		
1	1	1/2		
2	4	2		

Parent Graph:  $y = x^2$

Transformation: vertical shrink by a factor of 1/2



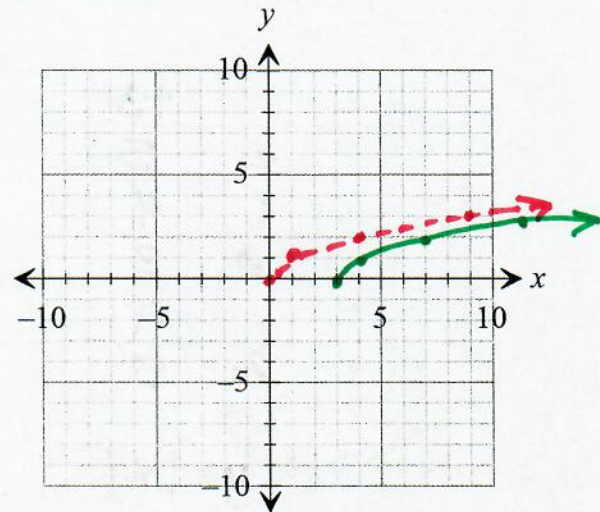
**Horizontal Translation:**

★ **ALWAYS** do the opposite of the horizontal number.

\* affects x-values

5.  $f(x) = \sqrt{x-3}$

+3	x	y = $\sqrt{x}$			
3	0	0			
4	1	1			
7	4	2			
12	9	3			



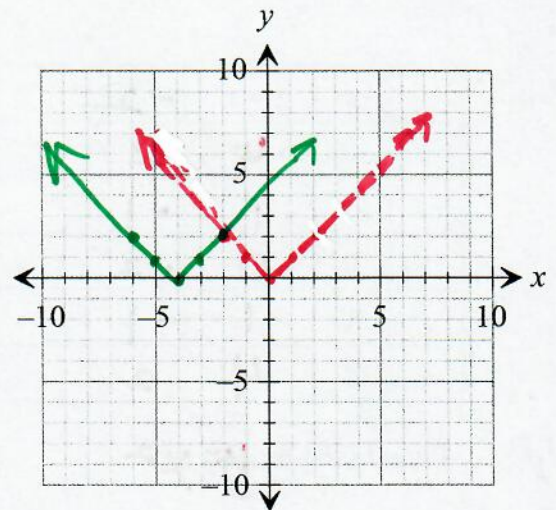
Parent Graph:  $y = \sqrt{x}$

Transformation: shifts right 3 units

\* affects x-values

6.  $f(x) = |x+4|$

-4	x	y =  x			
-6	-2	2			
-5	-1	1			
-4	0	0			
-3	1	1			
-2	2	2			



Parent Graph:  $y = |x|$

Transformation: shift left 4 units

This is what a horizontal translation looks in a quadratic function.

$f(x) = (x+4)^2$

Shift left 4

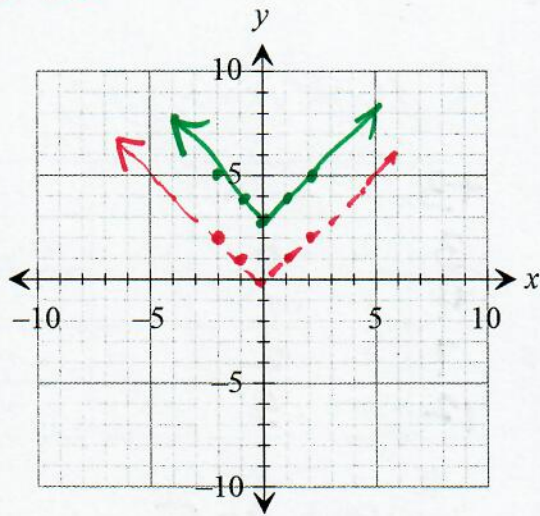
\* Shift up & down in the same direction as the sign.

**Vertical Translation:**

\* affects the y-values

7.  $f(x) = |x| + 3$

x	$y =  x $	+3		
-2	2	5		
-1	1	4		
0	0	3		
1	1	4		
2	2	5		

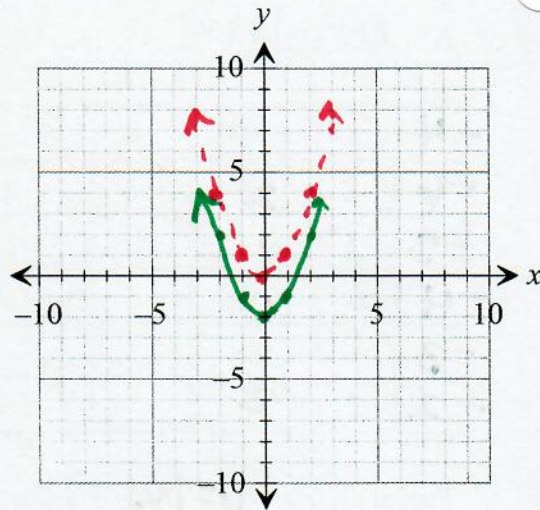


Parent Graph:  $y = |x|$

Transformation: Shift up 3 units

8.  $f(x) = x^2 - 2$

x	$y = x^2$	-2		
-2	4	2		
-1	1	-1		
0	0	-2		
1	1	-1		
2	4	2		



Parent Graph:  $y = x^2$

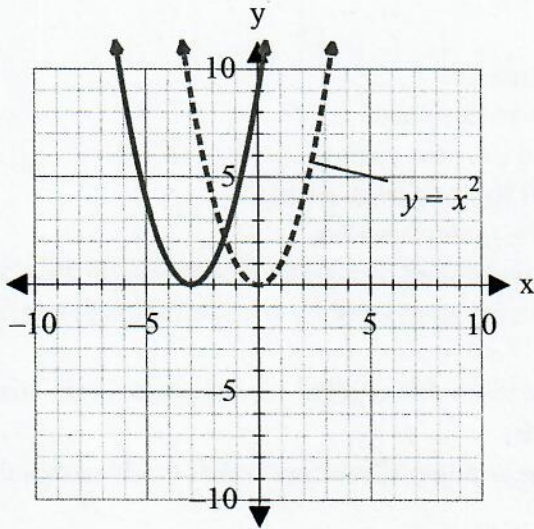
Transformation: Shift down 2 units



C. Given the graph, write the equation of the function.

On each graph, the parent graph is shown as a dashed line, and a transformed graph is shown as a solid line. Determine what transformation of the parent graph was performed and write an equation of the final graph.

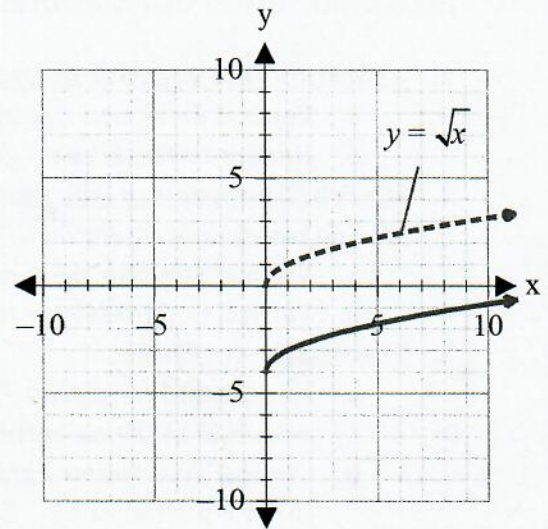
9.



Transformation: shift left 3

Equation:  $y = (x+3)^2$

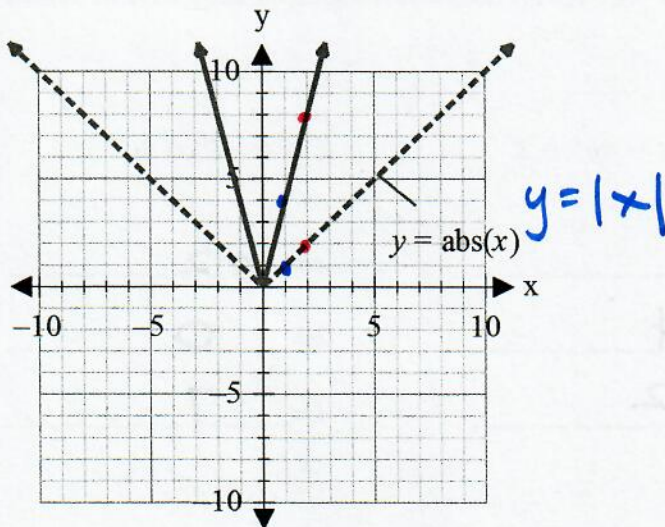
10.



Transformation: shift down 4

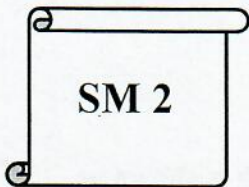
Equation:  $y = \sqrt{x} - 4$

11.



Transformation: Vertical stretch of 4

Equation:  $y = 4|x|$



Date:

Section: 1.6

**Objective: Multiple-Step Transformations**

**A. Types of transformations**

Transformations, like arithmetic operations, need to be applied in a particular order. Here is the correct "Order of Transformations."

**1. Reflection (the negative in front of the equation):**

- a. Vertical Reflection – graph is reflected over the  $x$ -axis
- b. Horizontal Reflection – graph is reflected over the  $y$ -axis

**2. Stretch/Compression (the number in front of the equation or  $a$ ):**

We will restrict our attention to Vertical stretches/compressions.

- a. Vertical Stretch – the  $y$ -coordinates are multiplied by a scalar that is greater than 1
- b. Vertical Compression – the  $y$ -coordinates are multiplied by a scalar that is between 0 and 1

**3. Translation (or Shift):**

- a. Horizontal Translation – graph is shifted to the left or right (the opposite of the number with  $x$ , or in the parentheses with  $x$ , or  $h$ )
- b. Vertical Translation – graph is shifted up or down (the number that is added or subtracted to the equation or  $k$ )

**B. Finding the number that tells you the transformation**

Answer the following questions using the equations:  $y = a\sqrt{(x - h)} + k$ ,

$y = a((x - h))^2 + k$ ,  $y = a|(x - h)| + k$ . Given the following equations find  $a$ ,  $h$ , and  $k$ .

A.  $y = 3\sqrt{x - 6} + 8$

$a =$  3

$h =$  6

$k =$  8

B.  $y = (x - 4)^2 - 2$

$a =$  1

$h =$  4

$k =$  -2

C.  $y = -2|x| + 7$

$a =$  -2

$h =$  0

$k =$  7

C. Writing transformations in the correct order

For each function, identify the parent graph ( $y = \sqrt{x}$ ,  $y = x^2$ , or  $y = |x|$ ), then list the transformations needed to get from the parent graph to the final graph. Make sure to list the transformations in the order in which they should be applied.

A.  $y = -4(x+2)^2$

Parent:  $y = x^2$

Transformations:

1. reflect over x-axis
2. vertical stretch of 4
3. shift left 2

B.  $y = \frac{1}{4}|x+3|-6$

Parent:  $y = |x|$

Transformations:

1. vertical shrink of  $\frac{1}{2}$
2. shift left 3
3. shift down 6

C.  $y = -2\sqrt{x}-1$

Parent:  $y = \sqrt{x}$

Transformations:

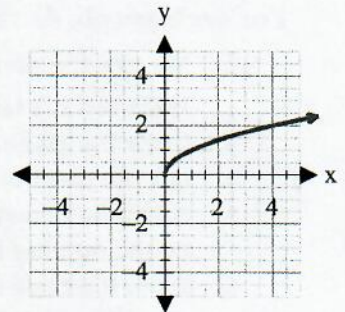
1. reflect over x-axis
2. vertical stretch of 2
3. shift down 1

D. Vocabulary

a. Endpoint:

The point that marks the end of the square root graph.

**\*\*** The point where the square root function begins.

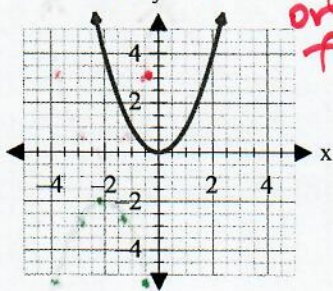


Endpoint: (0,0)

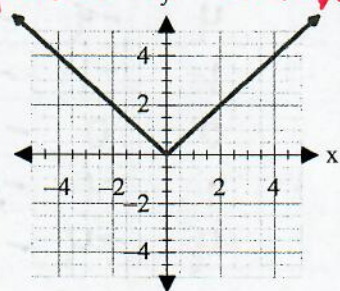
b. Vertex:

The point where the parabola or square root graph crosses its axis of symmetry.

**\*\*** The highest (or lowest) point of a square root or absolute value function.



Vertex: (0,0)



Vertex: (0,0)

E. How to put a, h, and k in the table

A.  $y = 2(x - 3)^2 + 1$

(opposite of the sign in parenthesis)

$a = 2$        $h = 3$        $k = 1$

Parent Columns:

Column for stretch and reflection:  
Multiply the y-value of the Parent columns by a

Column for translation or shift:

Add h to x-value of the previous column  
Add k to y-value of the previous column

h	x	$y = x^2$	$\cdot a$	k
+3			$\cdot 2$	+1
1	-2	4	8	9
2	-1	1	2	3
3	0	0	0	1
4	1	1	2	3
5	2	4	8	9

F. Graphing multiple-step transformations:

For each graph, do the following:

1. Identify the parent graph ( $y = |x|$ ,  $y = x^2$ , or  $y = \sqrt{x}$ ).
2. Fill in the x, y table for the parent graph.
3. Draw the graph of the parent graph with a dashed line.
4. List the transformations in the correct order.
5. Fill in the table to apply the reflections and stretches/compressions (by multiplying the y-coordinates by the number in front or multiply by a).
6. Fill in the table to apply the translations. (Add or subtract h and k from the x's and y's to move the graph in the correct directions.)
7. Draw the final graph with a solid line.
8. State the vertex or endpoint, domain, and range of the final graph.

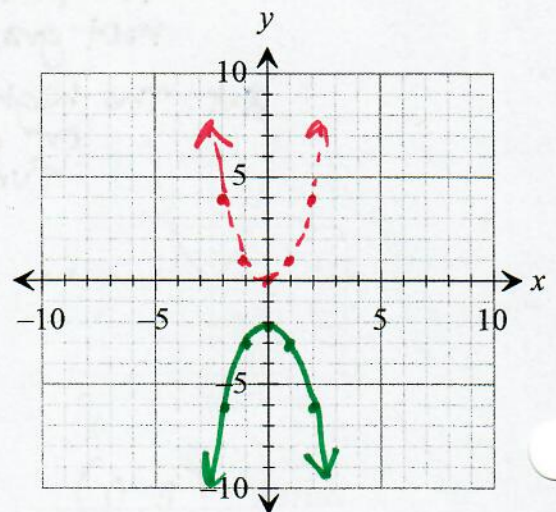
A.  $f(x) = -x^2 - 2$        $a = -1$        $h = 0$        $k = -2$

x	$y = x^2$	$\cdot -1$	$-2$
-2	4	-4	-6
-1	1	-1	-3
0	0	0	-2
1	1	-1	-3
2	4	-4	-6

Parent Graph:  $y = x^2$

Transformations: reflect over x-axis  
shift down 2

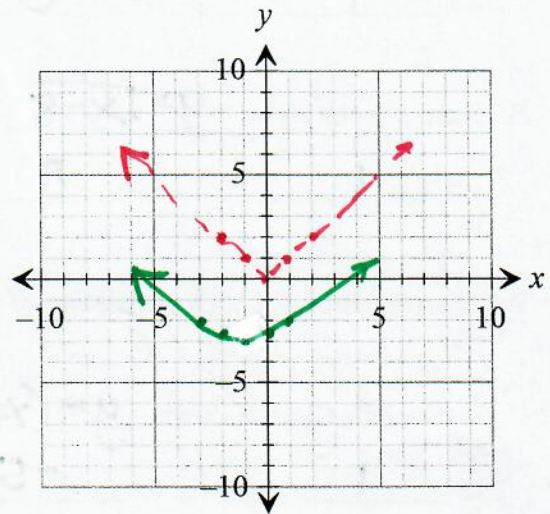
Vertex:  $(0, -2)$       Domain:  $(-\infty, \infty)$



Range:  $(-\infty, -2]$

B.  $f(x) = \frac{1}{2}|x + 1| - 3$   $a = \frac{1}{2}$   $h = -1$   $k = -3$

-1	x	y =  x	• 1/2	-3	
-3	-2	2	1	-2	
-2	-1	1	1/2	-2.5	
-1	0	0	0	-3	
0	1	1	1/2	-2.5	
1	2	2	1	-2	



Parent Graph:  $y = |x|$

Transformations: vertical shrink of 1/2  
shift left 1  
shift down 3

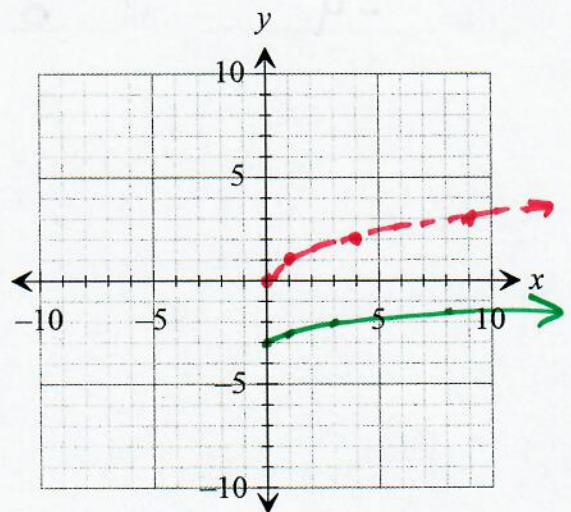
Vertex:  $(-1, -3)$

Domain:  $(-\infty, \infty)$

Range:  $[-3, \infty)$

C.  $f(x) = \frac{1}{2}\sqrt{x+1} - 3$   $a = \frac{1}{2}$   $h = -1$   $k = -3$

-1	x	y = $\sqrt{x}$	• 1/2	-3	
-1	0	0	0	-3	
0	1	1	1/2	-2.5	
3	4	2	1	-2	
8	9	3	1.5	-1.5	



Parent Graph:  $y = \sqrt{x}$

Transformations: vertical shrink of 1/2  
shift left 1  
shift down 3

Vertex:  $(0, -3)$

Domain:  $[0, \infty)$

Range:  $[-3, \infty)$

Write an equation for each translation of the **parent function**  $y = \sqrt{x}$ .

A: 5 unit up:  $y = \sqrt{x} + 5$

$a = 1$        $h = 0$        $k = 5$

B: 8 units to the right:  $y = \sqrt{x-8}$

$a = 1$        $h = 8$        $k = 0$

Write an equation for each translation of the **parent function**  $y = x^2$ .

C: 5 units left, 12 units down:  $y = (x+5)^2 - 12$

$a = 1$        $h = -5$        $k = -12$

D: Vertical stretch by a factor of  $\frac{1}{2}$ , 9 units right:  $y = \frac{1}{2}(x-9)^2$

$a = \frac{1}{2}$        $h = 9$        $k = 0$

Write an equation for each translation of the **parent function**  $y = |x|$

E: Vertical stretch by a factor of 4, reflect over the  $x$ -axis, 10 units up:  $y = -4|x| + 10$

$a = -4$        $h = 0$        $k = 10$

# Analyzing Functions Study Guide

## Domain and Range:

- **Domain:** all  $x$ -coordinates on the graph from *left to right*.
- **Range:** all  $y$ -coordinates on the graph from *bottom to top*.
  - Graphs with unconnected dots (no solid line): List  $x$ 's and  $y$ 's in { and }.
    - Don't list repeated numbers more than once.
  - Graphs with solid lines (even if there are labeled dots on it):
    - Use interval notation:  $(\_, \_)$ ,  $(\_, \_]$ ,  $[\_, \_)$ , or  $[\_, \_]$ .
    - If there's an arrow on the end of a graph, the domain and range will involve  $-\infty$  or  $\infty$ .
    - Use [ or ] for endpoints and vertices (places where the graph changes direction).
    - Use ( or ) for  $-\infty$ ,  $\infty$ , asymptotes, or open circles.

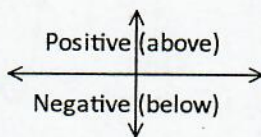
## Increasing, Decreasing or Constant: (Write $x$ 's)

- Write  $x$ -coordinates where graph *starts* and *stops* going each direction from *left to right*.
- Always use ( and ).
- **Increasing:** Uphill from left to right.
- **Decreasing:** Downhill from left to right.
- **Constant:** Flat.
- **Hint:** Look for places where the graph changes direction (relative maxima or relative minima) to help you break the graph into intervals.
- Use the  $\cup$  sign to connect multiple intervals:  $(\_, \_) \cup (\_, \_)$



## Positive or Negative: (Write $x$ 's)

- **Positive:** Above  $x$ -axis.
- **Negative:** Below  $x$ -axis.
- Divide the graph into the parts that are above the  $x$ -axis and the parts that are below the  $x$ -axis using the  $x$ -intercepts. Write  $x$ -coordinates for the *start* and *end* of each interval from *left to right*.
- Use ( and ) at  $x$ -intercepts.
- Use [ or ] only when there is an endpoint above or below the  $x$ -axis.
- Use the  $\cup$  sign to connect multiple intervals:  $(\_, \_) \cup (\_, \_)$



## Intercepts: The points where the graph crosses the $x$ - or $y$ -axis.

- Write intercepts as ordered pairs.
  - $x$ -intercepts are written as  $(x, 0)$ .
  - $y$ -intercepts are written as  $(0, y)$ .
- To find  $x$ -intercepts algebraically, set  $y = 0$  and solve for  $x$ .
- To find  $y$ -intercepts algebraically, set  $x = 0$  and solve for  $y$ .

## Relative Maximum or Relative Minimum:

- **Relative maximum:** a point on the graph that is **higher** than all the points around it.
- **Relative minimum:** a point on the graph that is **lower** than all the points around it.
- **Maximum or minimum point:** Write ordered pair:  $(x, y)$ .
- **Maximum or minimum value:** Write  $y$ -coordinate of the point.

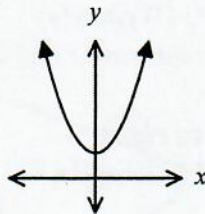
**End Behavior:** End behavior describes what is happening to the **y-coordinates** of the graph as you move left ( $x \rightarrow -\infty$ ) or as you move right ( $x \rightarrow \infty$ ).

- **Left end behavior** looks like this:  $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ .
- **Right end behavior** looks like this:  $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ .
- **Arrow pointing up:** Write  $\infty$
- **Arrow pointing down:** Write  $-\infty$
- **Endpoint (no arrow):** Write D.N.E. (does not exist)
- **Asymptote or flat end with arrow:** Write y-coordinate of asymptote or flat part

**Symmetry:**

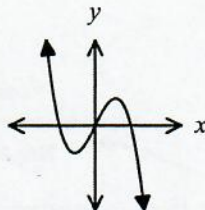
- **Even symmetry (y-axis):**
  - The left and right sides are mirror images around the y-axis. (Left and right sides would overlap if you fold the graph along the y-axis).

Even:



- **Odd symmetry (origin):**
  - If you fold the graph along the x-axis and then along the y-axis, the two halves will overlap.
  - If you spin the graph around  $180^\circ$ , you will end up with what you started with.

Odd:



$$y = a\sqrt{x-h} + k$$

$$y = a|x-h| + k$$

$$y = a(x-h)^2 + k$$

### Order of Transformations

- ① reflect over x-axis
- ② vertical stretch or shrink
- ③ shift left or right
- ④ shift up or down

		h		a		k	
		x	y =				
		-2					
		-1					
		0					
		1					
		2					