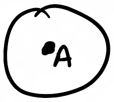
SM 2

Date: **Section: 12.1**

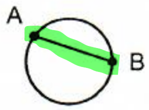
Objective: Circle Vocabulary, Arc, and Angle Measures

Circle: All points in a plane that are the same distance from a given point, called the ***center*** of the circle.



circle A

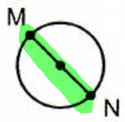
Chord: A segment with both endpoints on a circle.



$\overline{AB}$ is a chord.

$\overline{BA}$

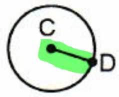
Diameter: A chord that passes through the center of a circle.



$\overline{MN}$ is a diameter.

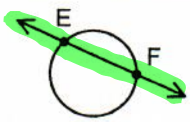
$\overline{NM}$

Radius: A segment with one endpoint on the circle and one endpoint at the center of the circle.



$\overline{CD}$ is a radius.

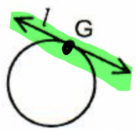
Secant: A line that intersects a circle at two points.



$\overleftrightarrow{EF}$ is a secant.

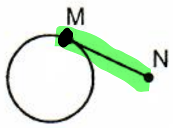
Tangent: A line in the plane of the circle that intersects a circle at exactly one point.

Point of Tangency: The point where a tangent intersects a circle.



Line l is a tangent. G is the point of tangency.

Tangent Segment: A segment that touches a circle at one of its endpoints and lies in the line that is tangent to the circle at that point.



$\overline{MN}$ is a tangent segment.

Example: In circle P, name the term that best describes the given line, segment, or point.

$\overline{AF}$ diameter

$\overline{EG}$ chord

$\overline{PF}$ radius

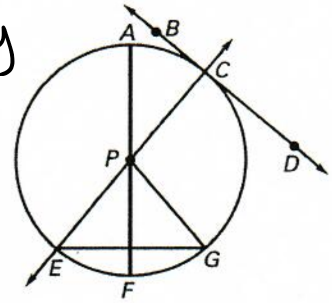
$\overline{PG}$ radius

C point of tangency

$\overleftrightarrow{CE}$ secant

$\overleftrightarrow{BD}$ tangent

P center



Example: In $\odot Q$, identify a chord, a diameter, two radii, a secant, two tangents, and two points of tangency.

Chord: $\overline{KM}$, $\overline{PM}$, $\overline{PK}$

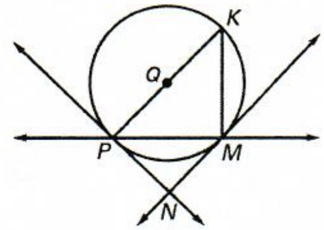
Diameter: $\overline{PK}$

Radii: $\overline{QK}$, $\overline{QP}$

Secant: (line) $\overleftrightarrow{PM}$

Tangents: (line) $\overleftrightarrow{PN}$, $\overleftrightarrow{MN}$

Points of tangency: P, M



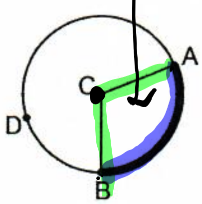
Central Angle: An angle in a circle whose vertex is the center of the circle and whose sides are radii of the circle

$\angle ACB$

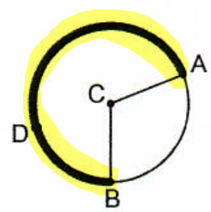
Minor Arc: All the points on a circle that lie in the interior of a central angle whose measure is less than 180°.

Major Arc: All the points on a circle that do not lie on the corresponding minor arc.

$\overset{\frown}{AB}$ is a minor arc.



$\overset{\frown}{ADB}$ is a major arc.

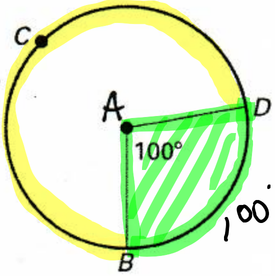


Measure of a Central Angle: is the measure of the angle with its vertex at the center of a circle.

Measure of a Minor Arc: is the measure of its central angle.

Measure of a Major Arc: 360° minus the measure of the minor arc.

Example:



Measure of central angle: 100°

Measure of the minor arc: 100°

Measure of the major arc: 260°

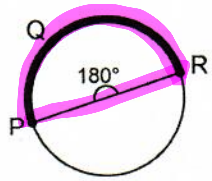
$360° - 100° = 260°$

Name the central angle: $\angle DAB$ or $\angle BAD$

Name the minor arc: $\overset{\frown}{DB}$ or $\overset{\frown}{BD}$

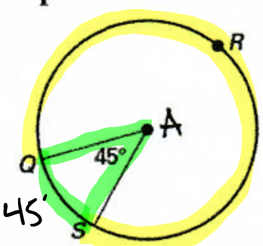
Name the major arc: $\overset{\frown}{DCB}$ or $\overset{\frown}{BCD}$

Semicircle: An arc whose central angle measures 180°.



Examples: Name the major and minor arcs and the central angle. Find the measure of each.

a)



45°

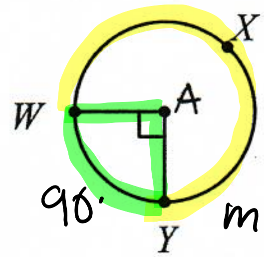
$360° - 45° = 315°$

major arc: $\overset{\frown}{QRS}$ 315°

minor arc: $\overset{\frown}{QS}$ 45°

central angle: $\angle QAS$

b)



90°

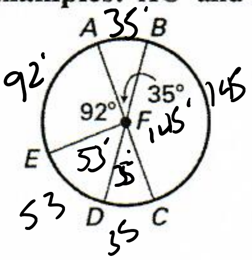
$360 - 90 = 270°$

major arc: $\overset{\frown}{WXY}$ 270°

minor arc: $\overset{\frown}{WY}$ 90°

central angle: $\angle WAY$

Examples: $\overline{AC}$ and $\overline{BD}$ are diameters. Find the indicated measures.



$180 - 92 - 35 = 53°$

a) $m\overset{\frown}{DC} = 35°$

d) $m\overset{\frown}{DE}$ 53°

b) $m\overset{\frown}{BC} = 145°$

$180 - 35$

e) $m\overset{\frown}{ABE} = 268°$

$360 - 92 = 268°$

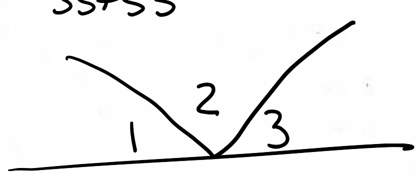
c) $m\overset{\frown}{CDE} = 88°$

$35 + 53$

f) $m\overset{\frown}{ABD} = 215°$

$35 + 145 + 35$

vertical ∠s always ≅

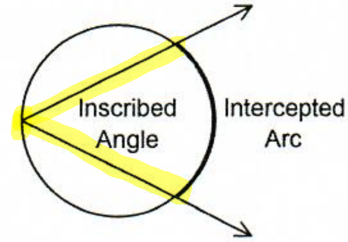


180°

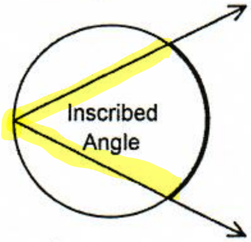
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

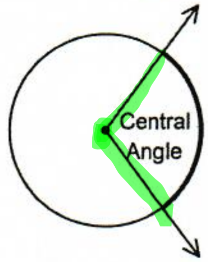
Inscribed Angle: An angle whose vertex is on a circle and whose sides contain chords of the circle.

Intercepted Arc: An arc that lies in the interior of an inscribed angle and has endpoints on the sides of the angle.



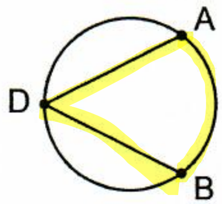
WARNING: Don't get ***inscribed*** angles and ***central*** angles mixed up!





ange $\cong$ arc

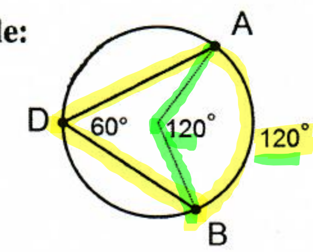
Theorem: If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



$$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$$

$$m\overset{\frown}{AB} = 2m\angle ADB$$

Example:

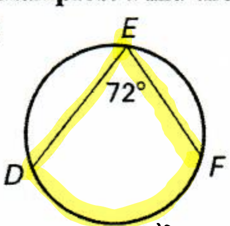


$$m\angle ADB = 60°$$

$$m\overset{\frown}{AB} = 120°$$

Examples: Find the measure of the inscribed angle or the intercepted arc.

a)

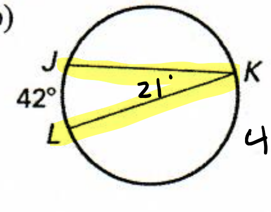


72·2

144°

$m\ \overset{\frown}{DF} = 144°$

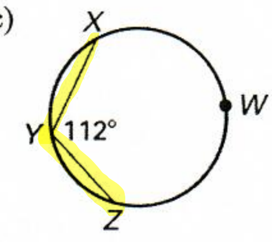
b)



42÷2

$m\angle JKL = 21°$

c)



112°·2

224°

$m\overset{\frown}{XWZ} = 224°$

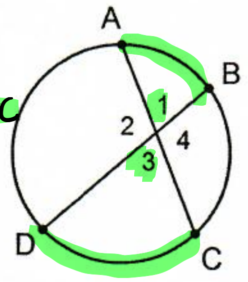
SM 2 **Date:** **Section: 12.2**

Objective: Tangent and Chord Theorems

Theorem:

- If two chords intersect inside a circle, then the measure of each angle formed is the average of the measures of the arcs intercepted by the angle and its vertical angle.

$\text{Angle} = \frac{\text{Arc}+\text{Arc}}{2}$

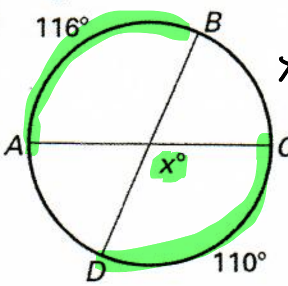


$m\angle 1 = m\angle 3 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$ $\quad \frac{m\overset{\frown}{AB} + m\overset{\frown}{CD}}{2}$

$m\angle 2 = m\angle 4 = \frac{1}{2}\left(m\overset{\frown}{BC} + m\overset{\frown}{AD}\right)$ $\quad \frac{m\overset{\frown}{BC} + m\overset{\frown}{AD}}{2}$

Examples: Find the value of *x*.

a)



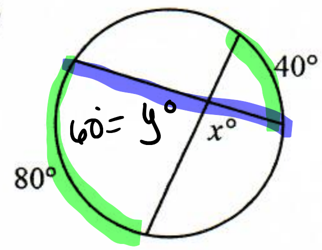
$x = \frac{116+110}{2}$

$x = \frac{226}{2}$

$\boxed{x = 113°}$

b)

$\frac{1/2}{} \ 180°$

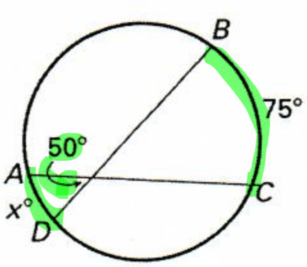


$y = \frac{40+80}{2}$

$y = \frac{120}{2} = 60°$

$x = 180 - 60 = \boxed{120°}$

c)



$\text{Angle} = \frac{\text{Arc}+\text{Arc}}{2}$

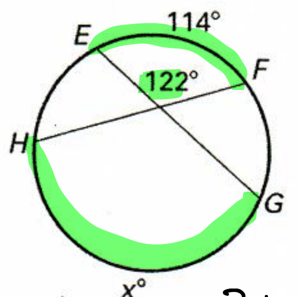
$2 \cdot 50 = \frac{75 + x°}{2} \cdot 2$

$100 = 75 + x$

$-75 \quad -75$

$\boxed{x = 25°}$

d)



$\text{Angle} = \frac{\text{Arc}+\text{Arc}}{2}$

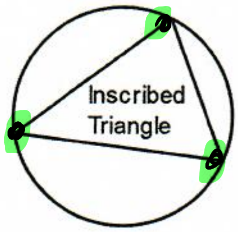
$2 \cdot 122° = \frac{114 + x}{2} \cdot 2$

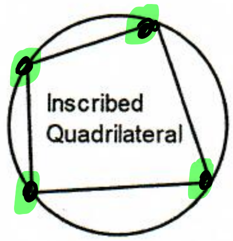
$244 = 114 + x$

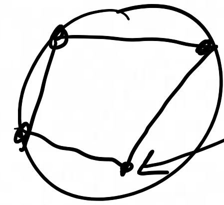
$-114 \quad -114$

$\boxed{130° = x}$

Inscribed Polygon: A polygon whose vertices all lie on a circle.





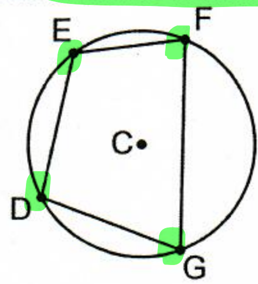


vertice not on circle

NOT INSCRIBED

Theorem:

- If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary. 180°

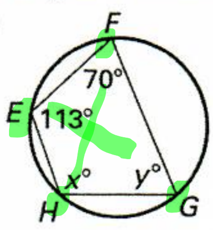


$m\angle D + m\angle F = 180°$

$m\angle E + m\angle G = 180°$

Examples: Find the values of x and y.

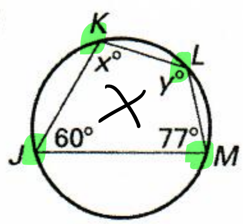
a)



$x + 70 = 180$
$-70 \quad -70$
$\boxed{x = 110°}$

$y + 113 = 180$
$-113 \quad -113$
$\boxed{y = 67°}$

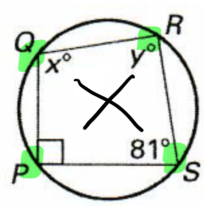
b)



$x + 77 = 180$
$-77 \quad -77$
$\boxed{x = 103°}$

$y + 60 = 180$
$-60 \quad -60$
$\boxed{y = 120°}$

c)

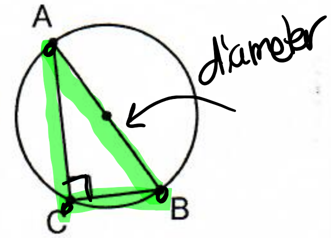


$x + 81 = 180$
$-81 \quad -81$
$\boxed{x = 99°}$

$y + 90 = 180$
$-90 \quad -90$
$\boxed{y = 90°}$

Theorems:

- If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.



If $\triangle ABC$ is a right triangle with hypotenuse $\overline{AB}$, then $\overline{AB}$ is a diameter of the circle.

Converse

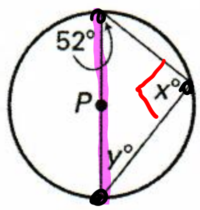
- If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.

If $\overline{AB}$ is a diameter of the circle, then $\triangle ABC$ is a right triangle with $\overline{AB}$ as hypotenuse.

$\boxed{\text{All } \angle\text{'s of } \Delta \text{ add up to } 180°}$

Examples: Find the values of x and y in $\odot$P. ← P is center

a)



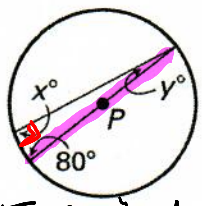
Inscribed Δ

$\boxed{x = 90°}$

$y + 90 + 52 = 180$
$\quad -90 - 52 \quad -90 - 52$

$\boxed{y = 38°}$

b)



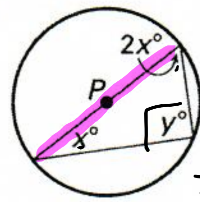
Inscribed Δ

$\boxed{x = 90°}$

$y + 80 + 90 = 180$
$\quad -80 - 90 \quad -80 - 90$

$\boxed{y = 10°}$

c)



Inscribed Δ

$\boxed{y = 90°}$

$2x + x + 90 = 180$

$3x + 90 = 180$
$\quad -90 \quad -90$

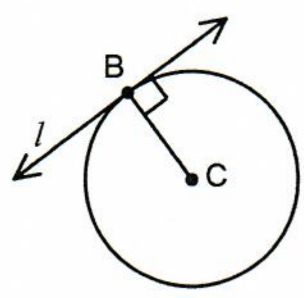
$\frac{3x}{3} = \frac{90}{3}$

$\boxed{x = 30°}$

Theorems About Tangents:

- If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency.

If line l is tangent to $\odot C$ at B, then $l \perp \overline{CB}$.



- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

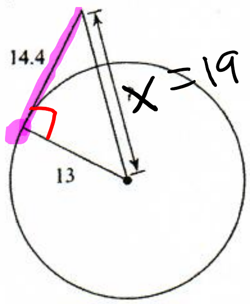
If $l \perp \overline{CB}$, then line l is tangent to $\odot C$ at B.

*use Pythagorean Thm to find missing side of Δ

Examples: Find the length of the missing segment. Assume that segments which appear to be tangent to the circle are tangent to the circle.

$\underbrace{a^2}_{\text{leg}} + \underbrace{b^2}_{\text{leg}} = \underbrace{c^2}_{\text{hyp}}$

a)



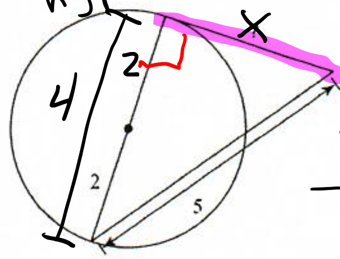
$13^2 + 14.4^2 = x^2$

$169 + 207.36 = x^2$

$\sqrt{376.36} = \sqrt{x^2}$

$\boxed{x = 19.4}$

b)



$4^2 + x^2 = 5^2$

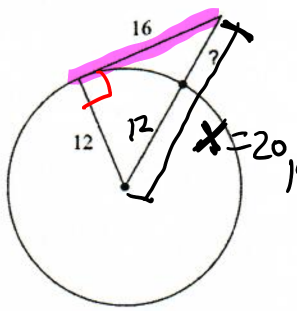
$16 + x^2 = 25$

$-16 \qquad -16$

$\sqrt{x^2} = \sqrt{9}$

$\boxed{x = 3}$

c)



$12^2 + 16^2 = x^2$

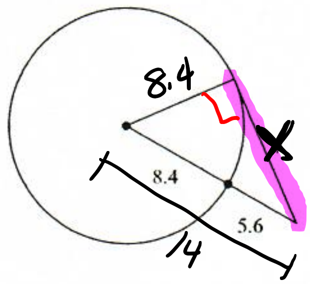
$144 + 256 = x^2$

$\sqrt{400} = \sqrt{x^2}$

$20 = x$

$? = 20 - 12 = \boxed{8}$

d)



$x^2 + 8.4^2 = 14^2$

$x^2 + 70.56 = 196$

$-70.56 \qquad -70.56$

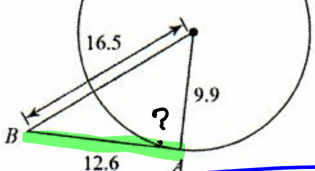
$\sqrt{x^2} = \sqrt{125.44}$

$\boxed{x = 11.2}$

Examples: Determine whether $\overline{AB}$ is tangent to the circle. Explain your reasoning.

Does $a^2 + b^2 = c^2$?

a)



must show work for credit

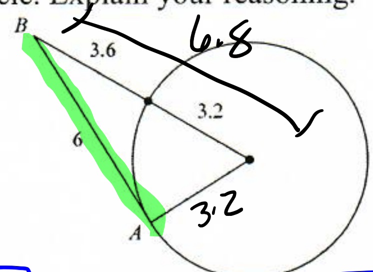
$12.6^2 + 9.9^2 \ ? \ 16.5^2$

$158.76 + 98.01 \ ? \ 272.25$

$256.77 \neq 272.25$

$\overline{AB}$ is not tangent because $a^2 + b^2 \neq c^2$

b)



$3.2^2 + 6^2 \ ? \ 6.8^2$

$10.24 + 36 \ ? \ 46.24$

$46.24 = 46.24$

$\overline{AB}$ is tangent because $a^2 + b^2 = c^2$

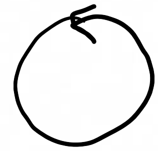
* If Pythag. Thm works the Δ is a right Δ and the side is tangent.

SM 2

Date: **Section: 12.3**

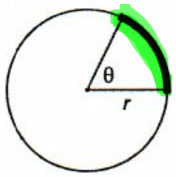
Objective: Arc Length and Sector Area

circumference of circle



$2\pi r$

Arc Length: Arc Length $= \dfrac{\theta}{360°} \cdot$ circumference of circle $= \dfrac{\theta}{360°} \cdot 2\pi r$

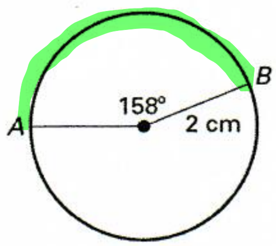


Exact

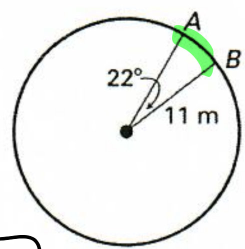
Examples: Find the length of $\overset{\frown}{AB}$. Write your answers in terms of π and as decimals rounded to the nearest hundredth. Arc length

46+90

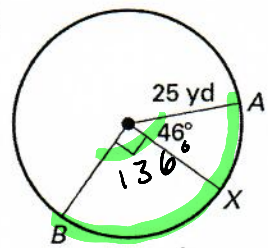
a)



b)



c)



$\dfrac{158}{360}\cdot\dfrac{2\pi\cdot 2}{1}=\dfrac{632\pi}{360}=\boxed{\dfrac{79\pi}{45}\text{ cm}}$ Exact

or $\boxed{5.52\text{cm}}$

$\dfrac{22}{360}\cdot\dfrac{2\pi 11}{1}$ Exact

$\dfrac{484\pi}{360}=\boxed{\dfrac{121\pi}{90}\text{m}}$

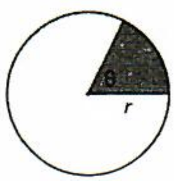
or $\boxed{4.22\text{m}}$

$\dfrac{136}{360}\cdot\dfrac{2\pi\cdot 25}{1}=\dfrac{6800\pi}{360}$

Exact $\boxed{\dfrac{170\pi}{9}\text{yd}}$

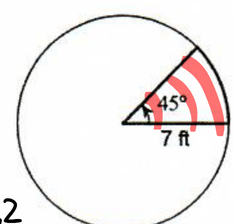
$\boxed{59.34\text{ yd}}$

Sector Area: Sector Area $= \dfrac{\theta}{360°} \cdot$ area of circle $= \dfrac{\theta}{360°} \cdot \pi r^2$

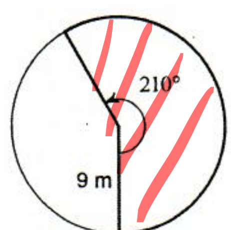


Examples: Find the area of each sector. Write your answers in terms of π and as decimals rounded to the nearest tenth.

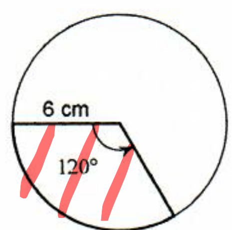
a)



b)



c)



$\dfrac{45}{360}\cdot\dfrac{\pi\cdot 7^2}{1}=\dfrac{2205\pi}{360}$

Exact $\boxed{\dfrac{49\pi}{8}\text{ or } 19.2\text{ft}^2}$

$\dfrac{210}{360}\cdot\dfrac{\pi\cdot 9^2}{1}=\dfrac{17010\pi}{360}$

$\boxed{\dfrac{189\pi}{4}\text{ or } 148.4\text{ m}^2}$

$\dfrac{120}{360}\cdot\dfrac{\pi\cdot 6^2}{1}=\dfrac{4320\pi}{360}$ 12

$\boxed{12\pi\text{ or } 37.7\text{cm}^2}$

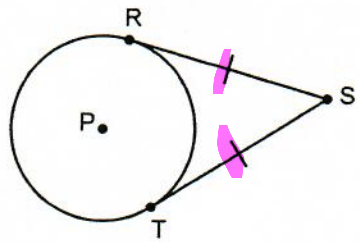
SM 2

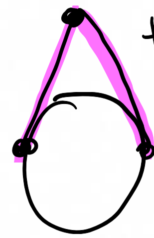
Date: **Section: 12.4**

Objective: More Tangent and Chord Theorems

Theorem: If two segments from the same point outside a circle are both tangent to the circle, then they are congruent.

If $\overline{SR}$ and $\overline{ST}$ are tangent to circle P at points R and T then $\overline{SR} \cong \overline{ST}$.

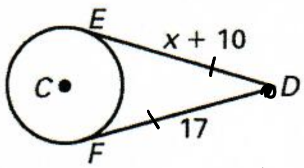




Hat Thm.

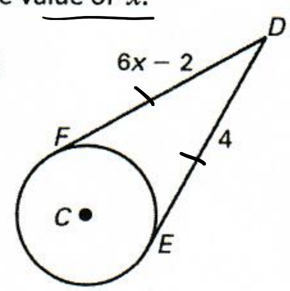
Examples: $\overline{DE}$ and $\overline{DF}$ are both tangent to ⊙C. Find the value of x.

a)



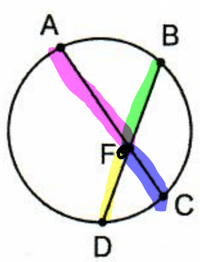
$x+10=17$
$-10 \quad -10$
$\boxed{x=7}$

b)



$6x-2=4$
$+2 \quad +2$
$\frac{6x}{6}=\frac{6}{6}$
$\boxed{x=1}$

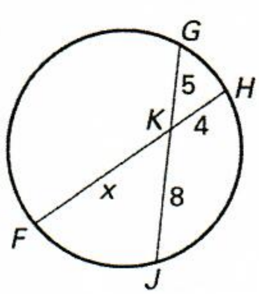
Theorem: If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$\overline{AF} \cdot \overline{FC} = \overline{BF} \cdot \overline{FD}$

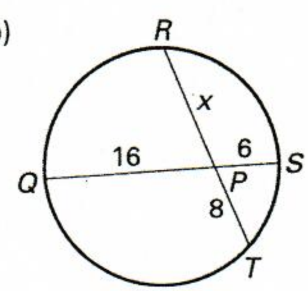
Examples: Find the value of x.

a)



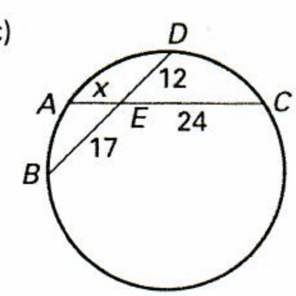
$4 \cdot x = 5 \cdot 8$
$\frac{4x}{4} = \frac{40}{4}$
$\boxed{x=10}$

b)



$8x = 16 \cdot 6$
$\frac{8x}{8} = \frac{128}{8}$
$\boxed{x=16}$

c)



$24x = 12 \cdot 17$
$\frac{24x}{24} = \frac{204}{24}$
$\boxed{x=8.5}$

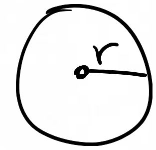
SM 2

Date: **Section: 12.5**

Objective: Graphing Circles

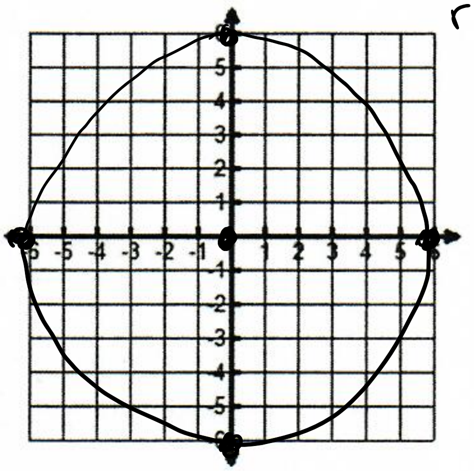
(0,0)

Equation of a Circle with Center at the Origin and Radius r: $x^2+y^2=r^2$ ← radius



Examples: Determine the center and radius of each circle, then graph the circle.

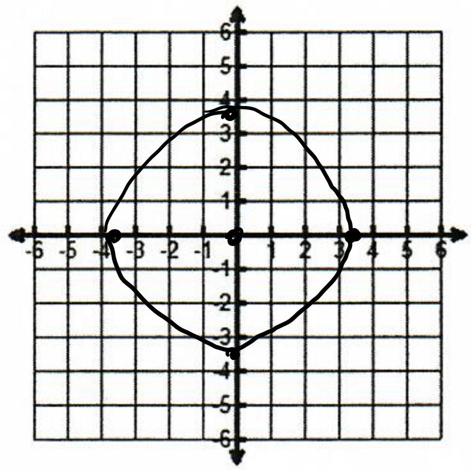
a) $x^2+y^2=36$ ← r^2 $\sqrt{36}=\sqrt{r^2}$ $r=6$



Radius: $\sqrt{36}=6$

Center: (0,0)

b) $x^2+y^2=13$ $\sqrt{r^2}=\sqrt{13}$ $r=\sqrt{13}\approx 3.6$



Radius: $\sqrt{13}\approx 3.6$

Center: (0,0)

Example: Write the equation of a circle with center at $(0,0)$ and radius 11. ← $r=11$ $r^2=121$

$x^2+y^2=r^2$

$\boxed{x^2+y^2=121}$

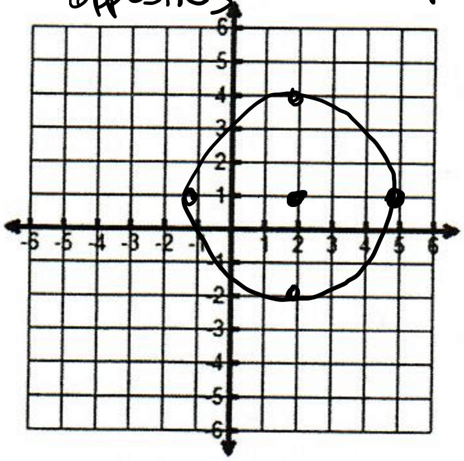
Equation of a Circle with Center at (h,k) and Radius r: $(x-h)^2+(y-k)^2=r^2$

opposites

Examples: Determine the center and radius of each circle, then graph the circle.

a) $(x-2)^2+(y-1)^2=9$ $\sqrt{r^2}=\sqrt{9}$

opposites $r=3$

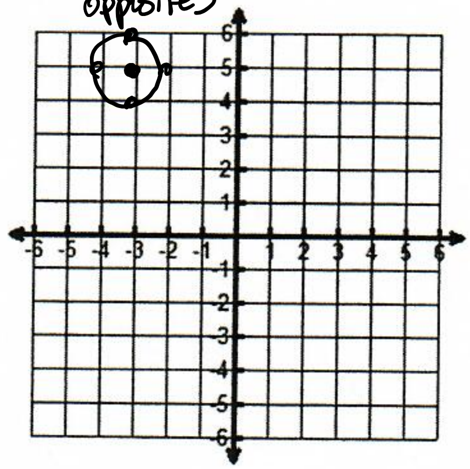


Radius: 3

Center: (2, 1)

b) $(x+3)^2+(y-5)^2=1$ $\sqrt{r^2}=\sqrt{1}$

opposites $r=1$

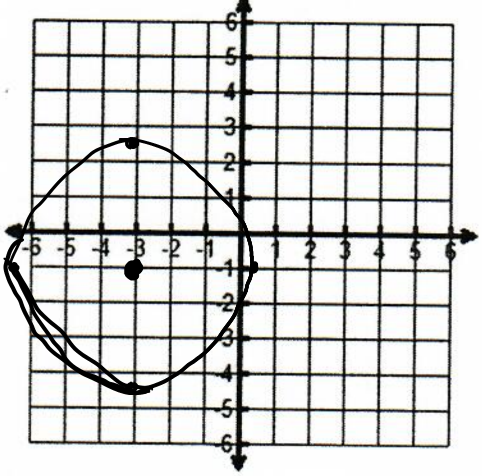


Radius: 1

Center: (-3, 5)

c) $(x+3)^2+(y+1)^2=12$

center (opposite) $\sqrt{r^2}=\sqrt{12}$



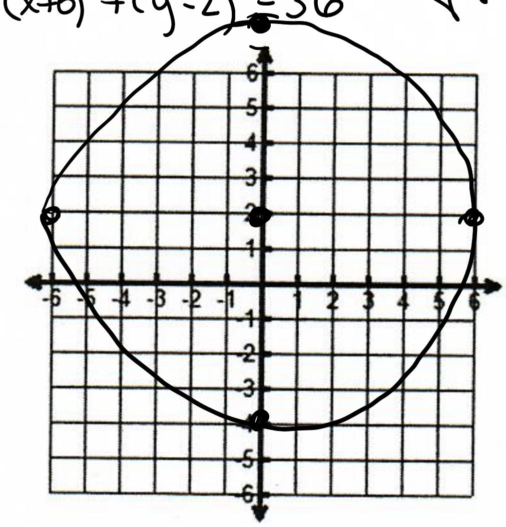
Radius: $\sqrt{12} \approx 3.46$

Center: (-3, -1)

d) $x^2+(y-2)^2=36$

$(x+0)^2+(y-2)^2=36$ $\sqrt{r^2}=\sqrt{36}$

$r=6$



Radius: 6

Center: (0, 2)

Examples: Write the equation of the circle with the given center and radius. $(x-h)^2+(y-k)^2=r^2$

(h,k) center

a) $(2,5)$; $r=7$

Equation: $(x-2)^2+(y-5)^2=49$

b) $(3,-1)$; $r=\sqrt{13}$ $(\sqrt{13})^2$

Equation: $(x-3)^2+(y+1)^2=13$

c) $(-2,12)$; $r=15$ $15^2=225$

Equation: $(x+2)^2+(y-12)^2=225$

d) $(-5,0)$; $r=2\sqrt{3}$ $r^2=(2\sqrt{3})^2$

$2\sqrt{3}\cdot2\sqrt{3}=4\sqrt{9}$

$4\cdot3$

Equation: $(x+5)^2+(y+0)^2=12$

$(x+5)^2+y^2=12$

e) $(-6,-9)$; $r=1$

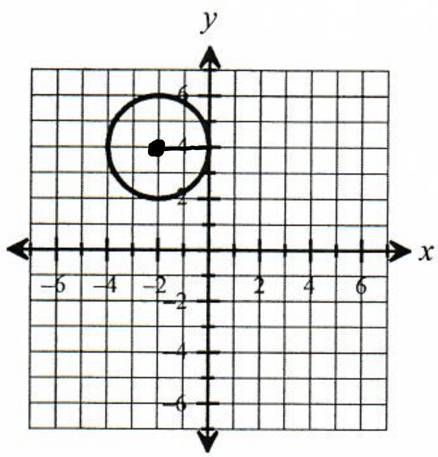
Equation: $(x+6)^2+(y+9)^2=1$ opp.

f) $(0,4)$; $r=\frac{1}{2}$ $r^2=\left(\frac{1}{2}\right)^2=\frac{1^2}{2^2}=\frac{1}{4}$

Equation: $(x-0)^2+(y-4)^2=\frac{1}{4}$

$x^2+(y-4)^2=\frac{1}{4}$

g)

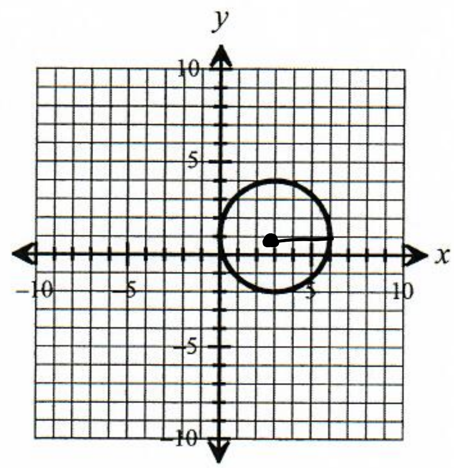


Radius: 2

Center: $(-2,4)$

Equation: $(x+2)^2+(y-4)^2=4$

h)



Radius: 3

Center: $(3,1)$

Equation: $(x-3)^2+(y-1)^2=9$