SM 2

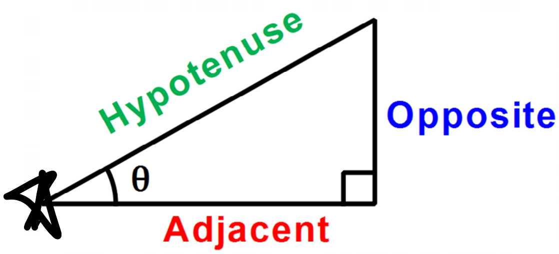
Section: 11.2

Objective: Find trigonometric ratios, use inverse functions to find missing angle measures

Trigonometry: "Triangle Measurement" – Using the ratios of the sides of right triangles to measure distances and angles. The study of relationships between angles and sides of a right triangle.

Trigonometric Ratio: A fraction made from two sides of a right triangle.

3 most common trigonometric ratios:



Sine: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ SOH

Cosine: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ CAH

Tangent: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ TOA

- ★ ***Adjacent*** means " next to θ " (θ is the angle you are focusing on)
- ★ ***Opposite*** means " across from θ " (θ is the angle you are focusing on)
- ★ ***Hypotenuse*** is the side across from the right angle longest side

How to remember:

SOH CAH TOA

Finding Sine, Cosine, and Tangent

1) Label the sides of the triangle as opposite, adjacent, and hypotenuse. (Opposite and adjacent depend on which angle you are focusing on, so it might help to circle that angle).

2) If one of the side lengths that you need for the ratio is missing, use the Pythagorean Theorem to find it.

$$a^2 + b^2 = c^2 \qquad \text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

3) Find $\sin\theta$, $\cos\theta$, or $\tan\theta$ by making a fraction using the definition of the function. (SOH-CAH-TOA)

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

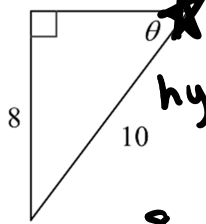
4) Simplify the fraction, if necessary.

★ If the question asks for an "exact value", that means no decimals. Write the answer with simplified square roots and simplified fractions.

SOH CAH TOA

Examples: Label the sides as opposite, adjacent, and hypotenuse. Find the lengths of any missing sides. Then find the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$. ← Find Ratio and reduce/simplify fraction

a)

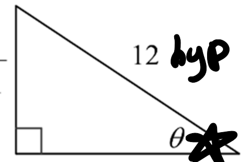


SOH $\sin\theta = \frac{8}{10} = \frac{4}{5}$

CAH $\cos\theta = \frac{6}{10} = \frac{3}{5}$

TOA $\tan\theta = \frac{8}{6} = \frac{4}{3}$

b)

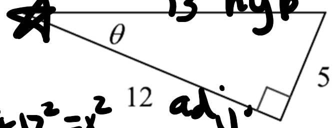


SOH $\sin\theta = \frac{\cancel{2}\sqrt{11}}{\cancel{12}_6} = \frac{\sqrt{11}}{6}$

CAH $\cos\theta = \frac{10}{12} = \frac{5}{6}$

TOA $\tan\theta = \frac{2\sqrt{11}}{10_5} = \frac{\sqrt{11}}{5}$

c)



$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$\sqrt{169} = \sqrt{x^2}$

$x = 13$

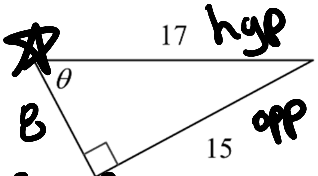
$\sin\theta = \frac{5}{13}$

384 ^

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

d)



$x^2 + 15^2 = 17^2$

$x^2 + 225 = 289$

$\sqrt{x^2} = \sqrt{64}$

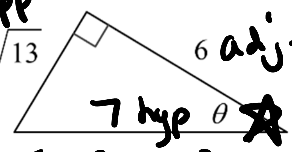
$x = 8$

$\sin\theta = \frac{15}{17}$

$\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{15}{8}$

e)



$(\sqrt{13})^2 + 6^2 = x^2$

$13 + 36 = x^2$

$\sqrt{49} = \sqrt{x^2}$

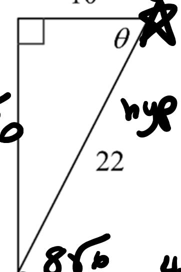
$x = 7$

$\sin\theta = \frac{\sqrt{13}}{7}$

$\cos\theta = \frac{6}{7}$

$\tan\theta = \frac{\sqrt{13}}{6}$

f)



$x^2 + 10^2 = 22^2$

$x^2 + 100 = 484$

$\sqrt{x^2} = \sqrt{384}$

$x = \sqrt{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot3}$

$x = 2\cdot2\cdot2\sqrt{6}$

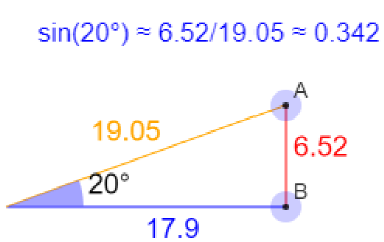
$x = 8\sqrt{6}$

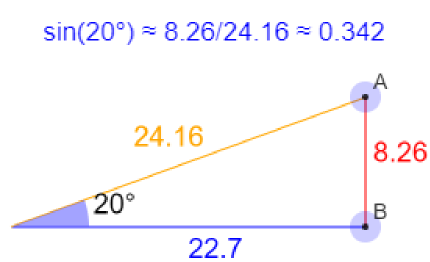
$\sin\theta = \frac{8\sqrt{6}}{22} = \frac{4\sqrt{6}}{11}$

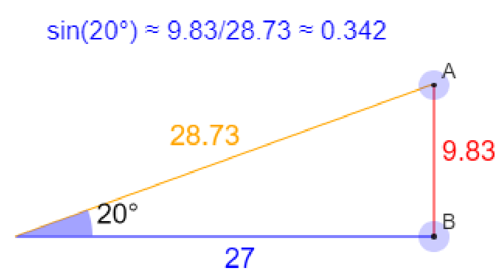
$\cos\theta = \frac{10}{22} = \frac{5}{11}$

$\tan\theta = \frac{8\sqrt{6}}{10} = \frac{4\sqrt{6}}{5}$

No matter how big the triangle is, the values of the trigonometric functions for a certain size angle will remain the same.







The triangles above all have a 20° angle. Even though they are all different sizes, when you divide the side opposite the 20° angle by the hypotenuse, you get the same thing for all three triangles. That means that $\sin(20°)$ is always the same, no matter how big the triangle is.

★ You need to make sure your calculator is in degree mode. Look for a button that says "mode" and set it to degree, or a button that says DRG and push it until you see degree on the screen.

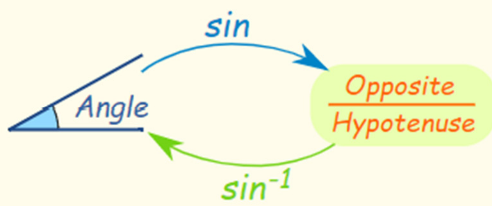
Type $\sin(20°)$ on your calculator and make sure you get 0.342…

* Make sure calc. is in Degrees.

This means that if you know how long two sides of a right triangle are, you can figure out how big the angle is. To do this, we use ***inverse trigonometric functions:*** $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

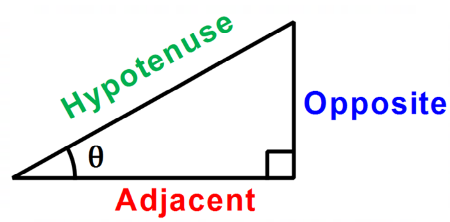
But what is the meaning of $\sin^{-1}$ ... ?

Well, the Sine function ***"sin"*** takes an angle and gives us the **ratio** "opposite/hypotenuse",



But $\sin^{-1}$ (called "inverse sine") goes the other way ...
... it takes the **ratio** "opposite/hypotenuse" and gives us an angle.

* use
$\sin^{-1}$ 2nd sin
$\cos^{-1}$ 2nd cos
$\tan^{-1}$ 2nd tan
when trying to find an angle



$$\sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)=\text{angle} \qquad \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)=\text{angle} \qquad \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)=\text{angle}$$

★ **Use inverse functions when you know how long at least two of the sides are and you want to know how big one of the angles is.**

Finding a Missing Angle Measure

1. Figure out which two sides you know out of Opposite, Adjacent, and Hypotenuse. Label them on the diagram. (If you know all three sides, pick which two you are going to use).

2. Use SOH-CAH-TOA to decide which one of sine, cosine, or tangent involves those two sides.

3. Figure out what the sine, cosine, or tangent equals (write the fraction).

$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta=\frac{\text{opposite}}{\text{adjacent}}$$

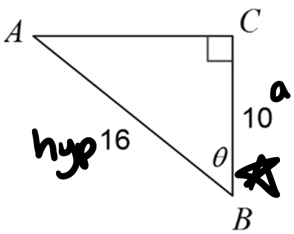
4. Use the inverse function ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$) of that fraction to find the angle measure.

SOH CAH TOA

Examples: Label the sides as opposite, adjacent, or hypotenuse. Write an equation involving sine, cosine or tangent. Then find the measure of θ to the nearest tenth of a degree. ← Find the angle θ

a)

CAH



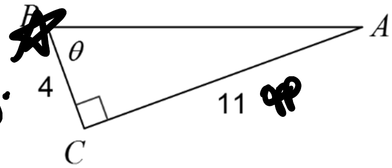
$\cos\theta = \frac{10}{16}$

what plug in calc.

$\theta = \cos^{-1}\left(\frac{10}{16}\right)$

$\boxed{\theta = 51.3°}$

b)



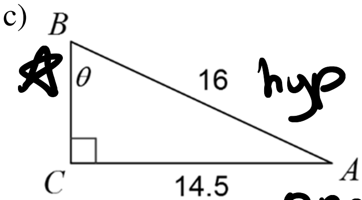
$\tan\theta = \frac{11}{4}$

what plug in calc.

$\theta = \tan^{-1}\left(\frac{11}{4}\right)$

$\boxed{\theta = 70.0°}$

c)



$\sin\theta = \frac{14.5}{16}$

what plug in calc.

$\theta = \sin^{-1}\left(\frac{14.5}{16}\right)$

64.99

$\boxed{\theta = 65°}$

SOH CAH TOA

d)

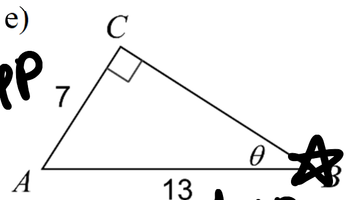
$\tan\theta = \frac{12.9}{4}$

what plug in calc.

$\theta = \tan^{-1}\left(\frac{12.9}{4}\right)$

$\boxed{\theta = 72.7°}$

e)



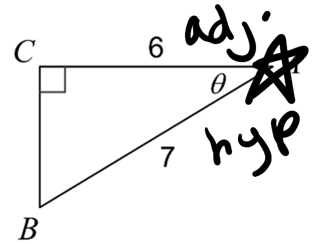
$\sin\theta = \frac{7}{13}$

what plug in calc.

$\theta = \sin^{-1}\left(\frac{7}{13}\right)$

$\boxed{\theta = 32.6°}$

f)



$\cos\theta = \frac{6}{7}$

what plug in calc.

$\theta = \cos^{-1}\left(\frac{6}{7}\right)$ 31.0027

$\boxed{\theta = 31°}$