SOLUTIONS FOR THE ACT PRACTICE MATHEMATICS TEST

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Solutions to the ACT practice test which is available for free at http://www.act.org/aap/pdf/preparing.pdf. These solutions were written by TJ Leone, who is solely responsible for their content.

1.

$$\begin{aligned} |7-3|-|3-7| &= |4|-|-4| \\ &= 4-4 \\ &= 0 \end{aligned}$$

Answer: D

2.

$$\begin{aligned} 45h+30 &= 210 \\ 45h &= 180 \\ h &= 4 \end{aligned}$$

Answer: G

3.

$$\begin{aligned} \text{Vehicle A needs } \frac{1,008\text{mi}}{14\text{mi/gal}} &= 72 \text{ gal} \\ \text{Vehicle B needs } \frac{1,008\text{mi}}{36\text{mi/gal}} &= 28 \text{ gal} \\ \text{Vehicle A needs } 72 \text{ gal} - 28 \text{ gal} &= 44 \text{ gal more than Vehicle B} \end{aligned}$$

Answer: C

4.

$$\begin{aligned} t^2 - 59t + 54 - 82t^2 + 60t &= t^2 - 82t^2 - 59t + 60t + 54 \\ &= -81t^2 + t + 54 \end{aligned}$$

Answer: J

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5. Since BCDE is a square,
$BC = DE = EB = CD = 6$ inches.
Since $\triangle ABE$ is equilateral,
$AB = EA = EB = 6$ inches.
So the perimeter of the figure is

$$\begin{aligned} AB + BC + CD + DE + EA &= 6 + 6 + 6 + 6 + 6 \\ &= 5(6) \\ &= 30 \text{ inches} \end{aligned}$$

Answer: C

6.

$$\begin{aligned} (4z+3)(z-2) &= (4z)(z) + (4z)(-2) + (3)(z) + (3)(-2) \\ &= 4z^2 - 8z + 3z - 6 \\ &= 4z^2 - 5z - 6 \end{aligned}$$

Answer: J

7.

$$\begin{aligned} 40\% \text{ of } x &= 8 \\ 0.4x &= 8 \\ x &= \frac{8}{0.4} \\ &= \frac{80}{4} \\ &= 20 \\ 15\% \text{ of } 20 &= (0.15)(20) \\ &= 3.0 \end{aligned}$$

Answer: C

8.

We are given that $(x-2)+(x-1)+(x)+(x+1)+(x+2)+(x+3)=447$

Notice that

$$\begin{aligned}(x-1)+(x+1) &= x+x-1+1 = 2x+0 = 2x\\ (x-2)+(x+2) &= x+x-2+2 = 2x+0 = 2x\\ x+(x+3) &= 2x+3\end{aligned}$$

so

$$\begin{aligned}(x-1)+(x)+(x+1)+(x+2)+(x+3) &= 2x+2x+2x+3\\ &= 6x+3\\ 6x+3 &= 447\\ 6x &= 444\\ x &= 72\end{aligned}$$

Answer: H

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9. Point M has coordinates $(5, 4)$.
Point B has coordinates $(7, 3)$.
Let A have coordinates (x, y).
Then

$$\begin{aligned} \frac{x+7}{2} &= 5 \\ x+7 &= 10 \\ x &= 3 \\ \frac{y+3}{2} &= 4 \\ y+3 &= 8 \\ y &= 5 \end{aligned}$$

So the coordinates of A are $(3, 5)$
Answer: D

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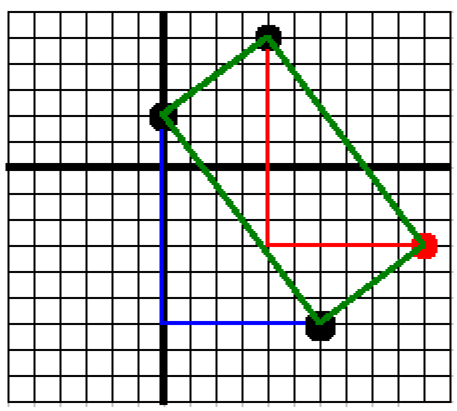
10. To get from B to C, we go down $2 - (-6) = 8$ and right $6 - 0 = 6$. The line from A to D is the same length in the same direction because $ABCD$ is a rectangle. So, to get from A to D, we start at A and go down 8 to get to (4,3) and right to get 6 to get to (10,3)



Answer: F

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11. Total value of inventory for store X:

$$\begin{aligned} 100(\$5) + 200(\$10) + 150(\$15) &= \$500 + \$2000 + \$2250 \\ &= \$4750 \end{aligned}$$

Total value of inventory for store Y:

$$\begin{aligned} 120(\$5) + 50(\$10) + 100(\$15) &= \$600 + \$500 + \$1500 \\ &= \$2600 \end{aligned}$$

Total value of inventory for both stores:

$$\$4750 + \$2600 = \$7350$$

Answer: E

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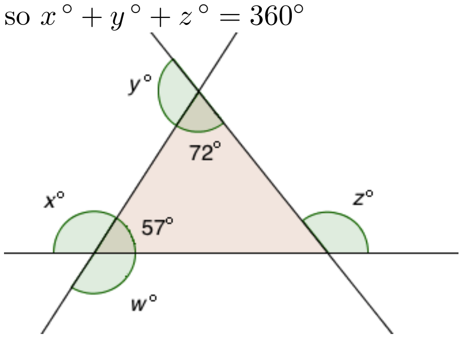
12. $w\,^\circ + y\,^\circ + z\,^\circ = 360^\circ$ (sum of exterior angles is 360 degrees)
$w\,^\circ = x\,^\circ$ (vertical angles are equal)
so $x\,^\circ + y\,^\circ + z\,^\circ = 360^\circ$



Answer: J

13.

$$\begin{aligned} \% \text{ that chose Whitney} &= \frac{\text{number of polled voters that chose Whitney}}{\text{total number of voters polled}} \\ &= \frac{30}{200} \\ &= \frac{15}{100} \\ &= 15\% \end{aligned}$$

Answer: A

14.

$$
\begin{aligned}
\text{\% that chose Lue} &= \frac{\text{number of polled voters that chose Lue}}{\text{total number of voters polled}}\\
&= \frac{80}{200}\\
&= \frac{40}{100}\\
&= 40\%\\
\text{40\% of 10,000} &= 4,000
\end{aligned}
$$

Answer: H

15.

$$
\begin{aligned}
\text{\% that chose Gomez} &= \frac{\text{number of polled voters that chose Gomez}}{\text{total number of voters polled}}\\
&= \frac{40}{200}\\
&= \frac{20}{100}\\
&= 20\%\\
20\% \text{ of } 360^\circ &= (0.2)(360^\circ)\\
&= 72^\circ
\end{aligned}
$$

Answer: B

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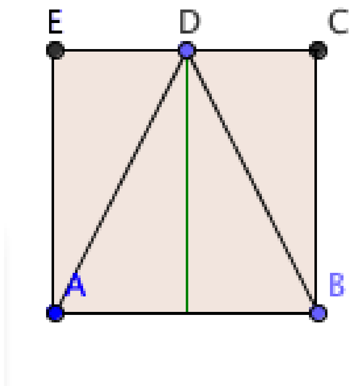
16. If we draw a line straight down from point D on $\triangle ABD$ we can see that it divides $\triangle ABD$ into two triangles the same size as $\triangle ADE$.
So the ratio of the area of $\triangle ADE$ to the area of $\triangle ABD$ is $1 : 2$



Answer: G

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17. Two lines are parallel if they have the same slope. For equations in slope-intercept form (the form $y = mx + b$), the slop is given by the coefficient of x (i.e., the "m" in $y = mx + b$).
So the slope of a line parallel to the line $y = \frac{2}{3}x - 4$ is $\frac{2}{3}$.
Answer: E

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18. In the figure below, all rectangular portions are the same length, so the ratio of the red section to the blue section is $2 : 3$. If the total length of the board is 30, then each of the five equal sections has length $\frac{30}{5} = 6$, and the short piece is $2 \times 6 = 12$ feet long.

Answer: H

19.

$$\begin{aligned} 7^2 &= 49 < 58 \\ 8^2 &= 64 > 58 \\ \sqrt{49} < \sqrt{58} &< \sqrt{64} \\ 7 < \sqrt{58} &< 8 \end{aligned}$$

So the smallest integer greater than $\sqrt{58}$ is 8.
Answer: C

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20. Each wall is 10 feet $\times$ 15 feet $= 150$ square feet, so the 4 walls are $4 \times 150 = 600$ square feet. Subtract out the area of the window $(3 \times 5 = 15)$ and the door $(3.5 \times 7 = 24.5)$ to get $600 - 15 - 24.5 = 560.5$. One gallon of paint is not enough $(560.5 > 350)$. Two gallons are more than enough $(560.5 < 600)$.
Answer: G

21.

$$\begin{aligned} x^2 + 2x &= 8 \\ x^2 + 2x - 8 &= 0 \\ (x+4)(x-2) &= 0 \\ x = -4 \text{ or } x &= 2 \end{aligned}$$

Answer: A

22.

$$\begin{aligned}
\frac{3a^4}{3a^6} &= \frac{3}{3} \times \frac{a^4}{a^6} \\
&= 1 \times \frac{a^4}{a^6} \\
&= \frac{a^4}{a^6} \\
&= \frac{a \times a \times a \times a}{a \times a \times a \times a \times a \times a} \\
&= \frac{a \times a \times a \times a \times 1}{a \times a \times a \times a \times a \times a} \\
&= \frac{a}{a} \times \frac{a}{a} \times \frac{a}{a} \times \frac{a}{a} \times \frac{1}{a \times a} \\
&= \frac{1}{a^2}
\end{aligned}$$

Answer: K

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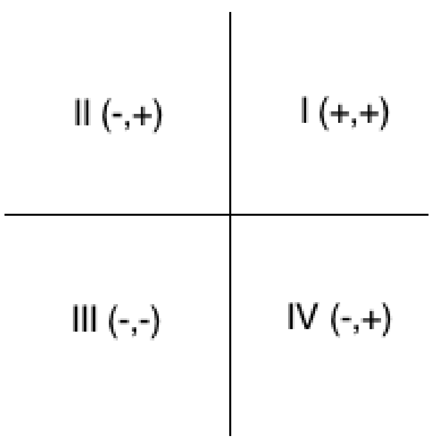
23. Coordinates have opposite signs in quadrants II and IV.



Answer: E

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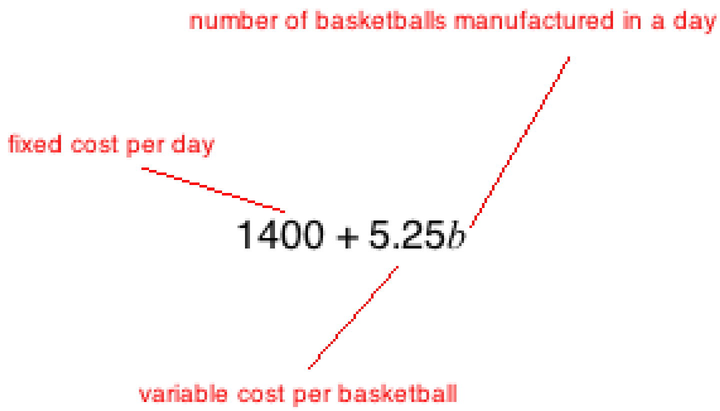
24. Fixed cost per day: \$1400
Variable cost per basketball: \$5.25
Number of basketballs manufactured in a day: b.



Answer: K

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25. We need to find the perimeter of $\triangle ABC = AB + BC + CA$.
We already know that:
$CA = 3$
$\frac{AB}{AC} = \frac{AB}{3} = \frac{12.5}{7.5} = \frac{KL}{MK}$
$\frac{BC}{AC} = \frac{BC}{3} = \frac{15}{7.5} = \frac{LM}{MK}$
From $\frac{AB}{3} = \frac{12.5}{7.5}$ we get

$$\begin{aligned} \frac{AB}{3} &= \frac{12.5}{7.5} = \frac{5 \times 2.5}{3 \times 2.5} \\ &= \frac{5}{3} \\ AB &= 5 \end{aligned}$$

From $\frac{BC}{3} = \frac{15}{7.5}$ we get

$$\begin{aligned} \frac{BC}{3} &= 2 \\ BC &= 6 \end{aligned}$$

So $AB + BC + CA = 5 + 6 + 3 = 14$
Answer: B

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26. If $\frac{3\sqrt{7}}{a\sqrt{7}} = \frac{3\sqrt{7}}{7}$, then
$a\sqrt{7} = 7$.
Since $a \times \sqrt{7} = 7$ and $\sqrt{7} \times \sqrt{7} = 7$,
$a = \sqrt{7}$.
Answer: G

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27. Let s be the number of seconds it takes for the two balloons to be at the same height.
Height of the falling balloon after s seconds: $70 - 6s$
Height of the rising balloon after s seconds: $10 + 15s$
Both heights are the same when

$$\begin{aligned} 70 - 6s &= 10 + 15s \\ 70 - 10 &= 15s + 6s \\ 60 &= 21s \\ s &= \frac{60}{21} \\ &= \frac{20}{7} \\ &= 2\frac{6}{7} \approx 2.9 \end{aligned}$$

Answer: C

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28. For each of the 4 paths from the town to the village 2 paths can be taken to the waterfall for a total of $4 \times 2 = 8$ paths from the town to the waterfall.
For each of the 8 paths from the town to the waterfall, 6 paths can be taken to the camp site, for a total of $8 \times 6 = 48$ paths.
Answer: J

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29. The side lengths and volumes of the two cubes are shown in the table below.

	side length	volume
Cube A	2	$2^3 = 8$
Cube B	4	$4^3 = 64$

The volume of Cube B is 64.
Answer: E

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30. We are given the formula $A = P(1 + r)^n$ along with the following values:

A	current value	unknown
P	amount deposited	\$10,000
r	rate of interest for one compounding period	$4\% = 0.04$
n	number of compounding periods	5

So $A = \$10,000(1 + 0.04)^5 \approx \$12,167$
Answer: G

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31. The diameter of the base is 20, so the radius of the base is 10. The height of the cylinder is 20. So $r = 10$, $h = 20$.

$$\begin{aligned} 2\pi r^2 + 2\pi rh &= 2\pi r(r + h) \\ &= 2\pi(10)(10 + 20) \\ &= 2\pi(10)(30) \\ &= 2\pi(300) \\ &= 600\pi \end{aligned}$$

Answer: D

32.

$$\begin{aligned} f(g(x)) &= f(x^2 - 2) \\ &= 4(x^2 - 2) + 1 \\ &= 4x^2 - 8 + 1 \\ &= 4x^2 - 7 \end{aligned}$$

Answer: H

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33. The average number of goals scored per match is $\frac{\text{Total number of goals}}{\text{Total number of matches}}$

4 matches had 0 goals, for a total of $4 \times 0 = 0$ goals.
10 matches had 1 goal, for a total of $10 \times 1 = 10$ goals.
5 matches had 2 goals, for a total of $5 \times 2 = 10$ goals.
9 matches had 3 goals, for a total of $9 \times 3 = 27$ goals.
7 matches had 4 goals, for a total of $7 \times 4 = 28$ goals.
5 matches had 5 goals, for a total of $5 \times 5 = 25$ goals.
1 match had 6 goals, for a total of $1 \times 6 = 6$ goals.
2 matches had 7 goals, for a total of $2 \times 7 = 14$ goals.

So the total number of goals is $0 + 10 + 10 + 27 + 28 + 25 + 6 + 14 = 120$.
$127 \div 43 \approx 2.8$
Answer: B

34.

These angles are supplementary with $\angle x$	because
$\angle 1$	adjacent angles with on-adjacent sides lying along line c.
$\angle 2$	adjacent angles with non-adjacent sides lying along line a.
$\angle 9$	corresponds to $\angle 1$ where line c cuts parallel lines a and b, and $\angle 1$ is supplementary to $\angle x$.
$\angle 10$	$\angle 10$ and $\angle 9$ are vertical angles, and $\angle 9$ is supplementary to $\angle x$.

Answer: H

35.

$$\begin{aligned}(3x^3)^3 &= (3x^3)(3x^3)(3x^3)\\ &= (3)(3)(3)(x^3)(x^3)(x^3)\\ &= 27x^9\end{aligned}$$

Answer: E

36.

$$\begin{aligned}4x - 8 &> 8x + 16\\ -4x &> 24\\ x &< -6\end{aligned}$$

Answer: F

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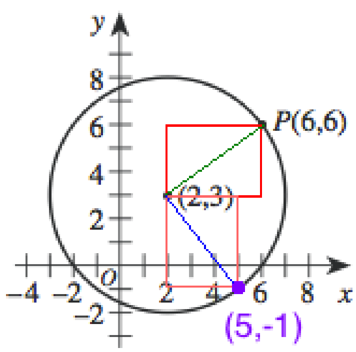
37. Imagine a rectangle with lower left corner at $(2, 3)$ and upper right corner at $(6, 6)$. This rectangle has width 4 and height 3. The line segment from $(2, 3)$ to $(6, 6)$ is a diagonal of this rectangle with length 5. We know this because the radius of the circle is 5. Also, the diagonal cuts the rectangle into two right triangles with legs 3 and 4, and a right triangle with legs 3 and 4 always has a hypotenuse of 5.

If we rotate this rectangle 90 degrees clockwise, we have a rectangle with width 3 and height 4. The right bottom corner of this rectangle is on a point on the circle rotated 90 degrees clockwise from the point $(6, 6)$. Because the width of the rectangle is 3 and the height of the rectangle is 4, the rotated point on the circle must be 3 units to the right of $(2, 3)$ and 4 units below $(2, 3)$, or $(2 + 3, 3 - 4) = (5, -1)$.



Answer: C

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38. We know that $\sin\angle M = \frac{KL}{KM}$
We are given that $LM = 10$ and $KM = 12$, but we also need to know KL to find $\sin\angle M$.
Since $\triangle KLM$ is a right triangle,

$$\begin{aligned} KL^2 + LM^2 &= KM^2 \\ KL^2 + 10^2 &= 12^2 \\ KL^2 + 100 &= 144 \\ KL^2 &= 44 \\ KL &= \sqrt{44} \end{aligned}$$

So $\sin\angle M = \frac{KL}{KM} = \frac{\sqrt{44}}{12}$
Answer: K

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39. Since $\overline{BD}$ bisects $\angle ABE$ and $\overline{BE}$ bisects $\angle CBD$, we have

$$\begin{aligned} \angle ABD &= \angle DBE \\ \angle CDE &= \angle DBE \end{aligned}$$

So $\angle ABD = \angle CDE = \angle DBE$.
Also, since A, B, and C lie on a straight line,
$\angle ABD + \angle CDE + \angle DBE = 180$,
which means that $\angle ABD = \angle CDE = \angle DBE = 60^\circ$
Answer: B

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40. average number of hydrogen molecules per cubic centimeter $= \frac{\text{number of hydrogen molecules in given volume}}{\text{given volume}}$

$$\begin{aligned} &= \frac{8 \times 10^{12}}{4 \times 10^4} \\ &= \frac{8}{4} \times \frac{10^{12}}{10^4} \\ &= 2 \times 10^8 \end{aligned}$$

Answer: H

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41. The figure below shows how we will apply the law of cosines. Referring to the original figure, we see that

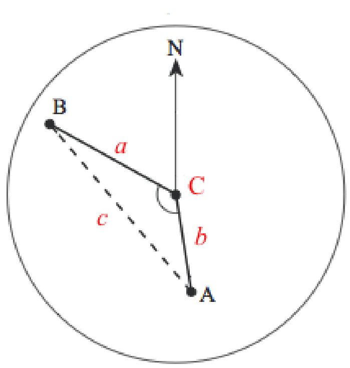
$$\begin{aligned} a &= 30 \\ b &= 20 \\ \angle C &= 300 - 170 \\ &= 130 \end{aligned}$$

Using the law of cosines, we have

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos\angle C \\ c^2 &= 30^2 + 20^2 - 2(30)(20)\cos 130^\circ \\ c &= \sqrt{30^2 + 20^2 - 2(30)(20)\cos 130^\circ} \end{aligned}$$

None of the answers are in this form, but we can rearrange terms to get

$$c = \sqrt{20^2 + 30^2 - 2(20)(30)\cos 130^\circ}$$



Answer: B

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42. A general strategy for comparing fractions is to give them a common denominator. $\frac{1}{5} = \frac{3}{15}$ and $\frac{1}{3} = \frac{5}{15}$. The rational number halfway between them is $\frac{4}{15}$.
Answer: J

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43. For this problem, it helps to label a point E as shown in the figure below. Since $\overline{BD}$ cuts the parallel lines $\overline{AB}$ and $\overline{CD}$, we know that $m\angle BDC = m\angle ABD$ (alternate interior angles), so $m\angle ABD = 25^\circ$. Since the trapezoid $ABCD$ is isosceles, we know that $\triangle ABE$ is isosceles, so $m\angle BAE = m\angle ABD = 25^\circ$. Since the interior angles of a triangle add up to 180°, we have, for $\triangle ABE$

$$\begin{aligned} m\angle BAE + m\angle ABE + m\angle AEB &= 180^\circ \\ 25^\circ + 25^\circ + m\angle AEB &= 180^\circ \\ 50^\circ + m\angle AEB &= 180^\circ \\ m\angle AEB &= 130^\circ \end{aligned}$$

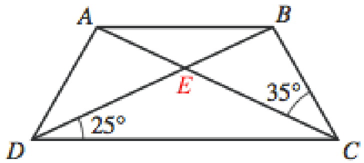
Since $\angle AEB$ and $\angle BEC$ are supplementary, we have

$$\begin{aligned} m\angle AEB + m\angle BEC &= 180^\circ \\ 130^\circ + m\angle BEC &= 180^\circ \\ m\angle BEC &= 50^\circ \end{aligned}$$

Since the interior angles of a triangle add up to 180°, we have, for $\triangle BEC$

$$\begin{aligned} m\angle BCE + m\angle BEC + m\angle EBC &= 180^\circ \\ 35^\circ + 50^\circ + m\angle EBC &= 180^\circ \\ 85^\circ + m\angle EBC &= 180^\circ \\ m\angle EBC &= 95^\circ \end{aligned}$$

Notice that $\angle EBC$ is another name for our required angle $\angle DBC$, so $m\angle DBC = 95\,^\circ$



Answer: B

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44. Each side of the larger square is $\sqrt{50}$. Each side of the smaller square is $\sqrt{18}$ So

$$\begin{aligned} x &= \sqrt{50} - \sqrt{18} \\ &= \sqrt{25(2)} - \sqrt{9(2)} \\ &= 5\sqrt{2} - 3\sqrt{18} \\ &= 2\sqrt{2} \end{aligned}$$

Answer: G

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45. A rational number is a number that can be expressed in the form $\frac{x}{y}$, where x and y are integers. We can write $\sqrt{\frac{64}{49}}$ as $\frac{8}{7}$, so it is a rational number.
Answer: E

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46. Since $a < b$, $a - b < 0$. In general, if $x < 0$, then $|x| = -x$, so $|a - b| = -(a - b)$.
Answer: K

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47. We know that Tom scored a total of $5(78.0) = 390.0$ points on the first 5 tests to get an average of 78.0 points. Let x be the number of points he scores on the 6th test. To get an average of 80.0 points, he would need

$$\begin{aligned} \frac{390.0 + x}{6} &= 80.0 \\ 390.0 + x &= 480.0 \\ x &= 90.0 \end{aligned}$$

Answer: A

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48. We are told that the point (a, b) in the complex plane is comparable to the point (a, b) graphed in the standard coordinate plane. We know that in the standard coordinate plane, $\sqrt{a^2 + b^2}$ represents the distance from the origin to the point (a, b). Since $\sqrt{a^2 + b^2}$ is also the *modulus*, we find the complex number with the greatest modulus by finding the point that is farthest from the origin. This is point z_1.
Answer: F

49.

$$\begin{aligned}
8^{2x+1} &= 4^{1-x} \\
(2^3)^{2x+1} &= (2^2)^{1-x} \\
2^{3(2x+1)} &= 2^{2(1-x)} \\
2^{6x+3} &= 2^{2-2x} \\
6x + 3 &= 2 - 2x \\
6x + 2x &= 2 - 3 \\
8x &= -1 \\
x &= -\frac{1}{8}
\end{aligned}$$

Answer: C

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50. The graph is symmetric about the y-axis, so it is even.
Answer: F

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51. The probability to be found is

$$\frac{\text{number from 100 to 999 with 1 or 2 zeros}}{\text{number of integers from 100 to 999, inclusive}}$$

There are 900 integers from 100 to 999, inclusive. In this range, there are 90 integers with

a zero in the units place (100, 110, 120, etc.) and 90 integers with a zero in the tens place (100, 101, 102, 200, 201, 202, 900, 901, 909). There are also 9 integers with zeros in *both* the units place and the tens place (100, 200, 900). So the total number of integers from 100 to 999 with at least one zero is $90 + 90 - 9 = 171$. So

$\frac{\text{number from 100 to 999 with 1 or 2 zeros}}{\text{number of integers from 100 to 999, inclusive}} = \frac{171}{900}$

Answer: D

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52. The equation for q in y-intercept form is

$$y = 2x + 1$$

So the slope of q is 2. We are told that $\angle b = \angle a$, so where q moves up two units each time we go one unit to the right, r moves down two units each time we go one unit to the right. The slope of r is -2.
Answer: F

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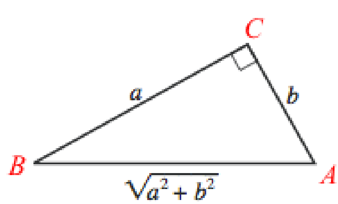
53. In the figure below, letters have been added to the original figure to refer to angles of the triangle.

$$\begin{aligned} \tan^{-1}\frac{a}{b} &= \angle A \\ \cos\left(\tan^{-1}\frac{a}{b}\right) &= \cos\angle A \\ &= \frac{b}{\sqrt{a^2+b^2}} \end{aligned}$$



Answer: D

54.

$$\begin{aligned}\pi r^2 &= \pi(52^2)\\ &\approx 8495\end{aligned}$$

The closest answer is 8500.

Answer: J

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55. The equation of a circle of radius r is $x^2 + y^2 = r^2$, so the equation of the circle shown on the map is $x^2 + y^2 = 52^2$.
Answer: E

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56. The transmitter of WGWB is 100 miles away from the transmitter of WGGW. Since it has a range of 60 miles, it can be heard once you travel 40 miles along the highway from WGGW. After 52 miles, you will be out of range of WGGW. So you are within range of both towers between 40 and 52 miles of WGGW, for a total of 12 miles.
Answer: G

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57. From the graph, it looks like the curve $y = (x-1)^4$ is below the line $y = x-1$ between $x = 1$ and $x = 2$. Let's look at values of $x < 1$. When $x < 1$, $x - 1 < 0$ but $(x-1)^4 > 0$, so $(x-1)^4 > (x-1)$. When $x > 2$, $(x-1) > 1$, so $(x-1)^4 > (x-1)$.
In the range $1 < x < 2$, we have $0 < (x-1) < 1$, so $(x-1)^4 < (x-1)$. So $1 < x < 2$ is the required range.
Answer: E

58.

$$\begin{aligned}x &= 10t + u\\ y &= 10u + t\\ x - y &= 10t + u - (10u + t)\\ &= 10t + u - 10u - t\\ &= 9t - 9u\\ &= 9(t - u)\end{aligned}$$

Answer: F

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59. The area of a triangle is $\frac{1}{2}bh$.
For the triangle in the figure

$$\begin{aligned} b &= \text{the horizontal distance between point } C \text{ and point } B \\ &= 5 - 1 \\ &= 4 \\ h &= \text{the vertical distance between point } A \text{ and } \overline{BC} \\ &= 5 - 3 \\ &= 2 \\ \frac{1}{2}bh &= \frac{1}{2}(4)(2) \\ &= 4 \end{aligned}$$

Answer: A

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60. We are given that $\frac{a}{1-r} = 200$ and $r = 0.15$. So

$$\begin{aligned} \frac{a}{1-0.15} &= 200 \\ \frac{a}{0.85} &= 200 \\ a &= 200(0.85) \\ &= 170 \end{aligned}$$

So far, we have only found the first term of the series, $a = 170$.
The second term of the series is $ar = (170)(0.15) = 25.5$.
Answer: F