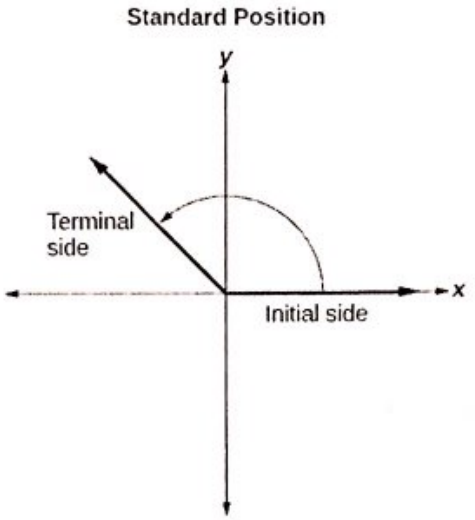
8.3 Trigonometry on the Cartesian Plane

Cartesian Plane- is a **plane** with a rectangular coordinate system that associates each point in the **plane**with a pair of numbers. We know this as the x and y axis.

Standard Position- the vertex of the angle is on the origin of the x and y axis and the angle is measured counterclockwise from the positive x-axis.

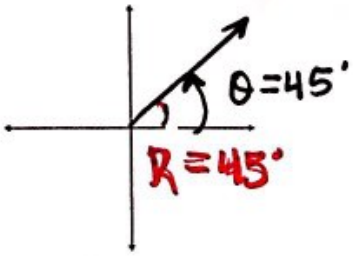
Terminal Side- the ray that makes the angle when its initial side is in standard position

Reference angle-is the smallest **angle** that you can make from the terminal side of an **angle** with the x-axis. This angle measure will always be less than 90°.

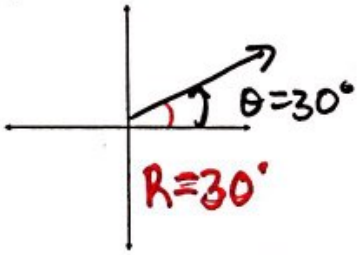


Example: Draw the angle measurement in standard position. Identify the location of the reference angle and its measure.

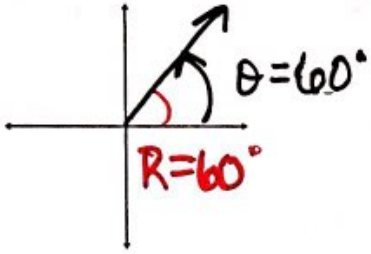
a. 45°



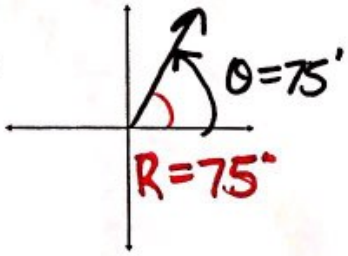
b. 30°



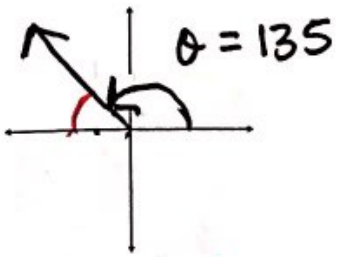
c. 60°



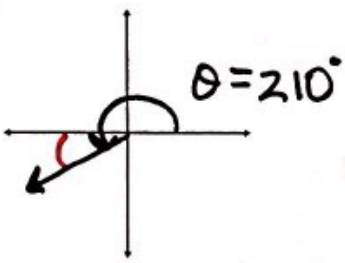
d. 75°



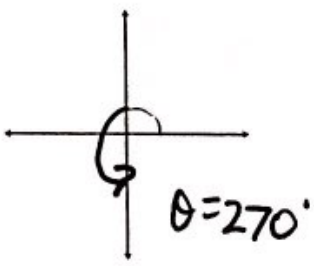
e. 135°



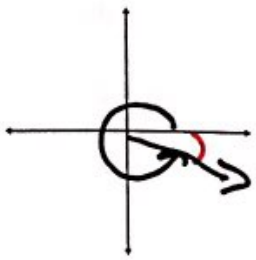
f. 210°



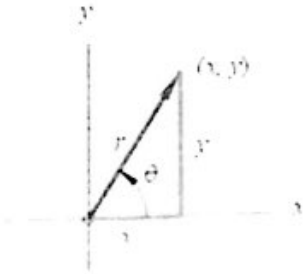
g. 270°



h. 325°



When we first defined the trigonometric functions the angle θ was between 0° and 90° and we used the terms *adjacent*, *opposite* and *hypotenuse* to refer to the sides of a triangle.



But we now want to allow angle θ to have values outside this range. These triangles can have an angle that is bigger than 90°.

To allow for angles bigger than 90° we now imagine an arrow pointing out from the origin with length r and orientated at angle θ, and with its terminal side ending at (x, y).

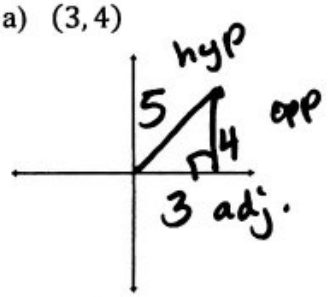
We construct a triangle by drawing a line vertically from the arrowhead to the x axis and another line horizontally across to the y axis.

We now redefine the six trigonometric functions like this: $\boldsymbol{\sin\theta = \frac{y}{r}}$ $\boldsymbol{\cos\theta = \frac{x}{r}}$ $\boldsymbol{\tan\theta = \frac{y}{x}}$

If we are given a coordinate, we will know the value of x and y, but how could you find the value of r?

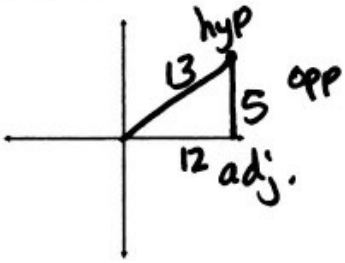
<u>Example</u>: Find the sine, cosine, and tangent of the following angles made by coordinate points. Keep answers in simplified radical form (NO DECIMALS!)

a) (3, 4)



$\sin\theta = \frac{4}{5}$
$\cos\theta = \frac{3}{5}$
$\tan\theta = \frac{4}{3}$

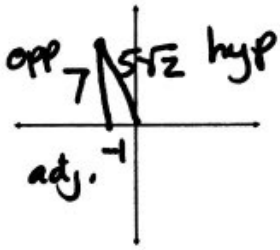
b) (12, 5)



$\sin\theta = 5/13$
$\cos\theta = 12/13$
$\tan\theta = 5/12$

c) (−1, 7)

$(-1)^2 + 7^2 = c^2$
$50 = c^2$ $c = 5\sqrt{2}$

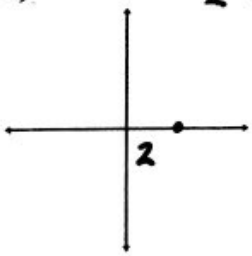


$\sin\theta = \frac{7}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{10}$
$\cos\theta = \frac{-1}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{10}$
$\tan\theta = \frac{7}{-1} = -7$

x y

d) (2, 0)

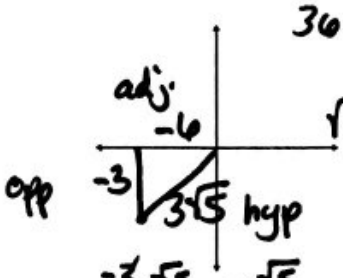
$2^2 + 0^2 = r^2$
$2^2 = r^2$
$2 = r$



$\sin\theta = \frac{y}{r} = \frac{0}{2} = 0$
$\cos\theta = \frac{x}{r} = \frac{2}{2} = 1$
$\tan\theta = \frac{y}{x} = \frac{0}{2} = 0$

e) (−6, −3)

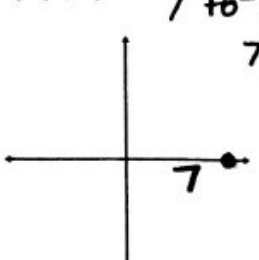
$(-6)^2 + (-3)^2 = c^2$
$36 + 9 = c^2$
$45 = c^2$
$\sqrt{3 \cdot 3 \cdot 5} = \sqrt{c^2}$
$3\sqrt{5} = c$



$\sin\theta = \frac{-3}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$
$\cos\theta = \frac{-6}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$
$\tan\theta = \frac{-3}{-6} = \frac{1}{2}$

f) (7, 0)

$7^2 + 0^2 = r^2$
$7^2 = r^2$
$7 = r$



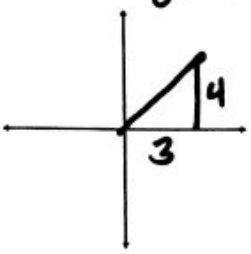
$\sin\theta = \frac{y}{r} = \frac{0}{7} = 0$
$\cos\theta = \frac{x}{r} = \frac{7}{7} = 1$
$\tan\theta = \frac{y}{x} = \frac{0}{7} = 0$

We can also use inverse trigonometric functions to find the angle created by points on the coordinate plane. Remember: <u>TrigFunction(angle/theta) = Ratio</u> so <u>Inverse function(ratio) = theta.</u>

Find the measurement of the STANDARD ANGLE (you will need to first find the reference angle!) that is created by the coordinate point. Draw a picture. Round to the ten-thousandths place.

a) (3, 4)

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

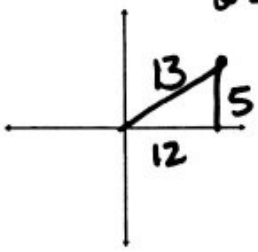


$\theta = 53.1301$

$\theta = 53.1301°$

b) (12, 5)

$\theta = \tan^{-1}\left(\frac{5}{12}\right)$

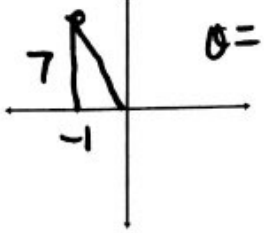


$\theta = 22.6199$

$\theta = 22.6199°$

c) (−1, 7)

$\theta = \tan^{-1}\left(\frac{7}{-1}\right)$



$\theta = -81.869$

$\theta = 98.1301°$

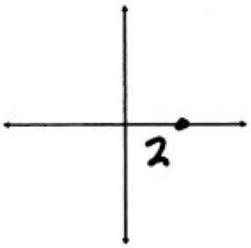
$180 - 81.8699$

d) (2, 0)

$2^2 + 0^2 = r^2$

$2^2 = r^2$

$2 = r$

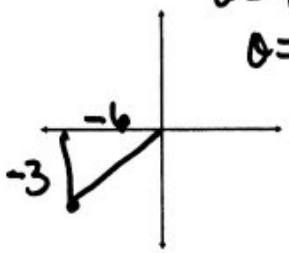


$\theta =$

e) (−6, −3)

$\theta = \tan^{-1}\left(\frac{-3}{-6}\right)$

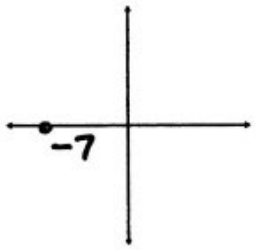
$\theta = 26.5651$



$\theta = 206.5651$

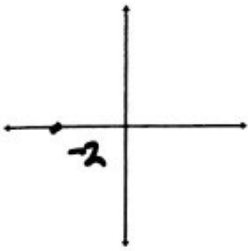
$180 + 26.5651$

f) (−7, 0)



$\theta =$

g) (−2, 0)

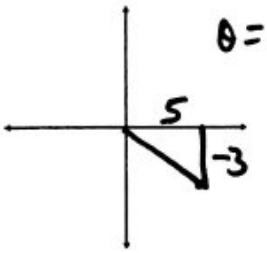


$\theta =$

h) (5, −3)

$\theta = \tan^{-1}\left(\frac{-3}{5}\right)$

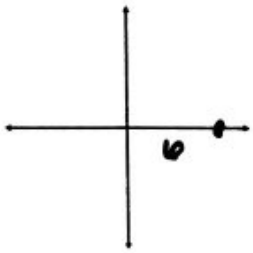
$\theta = -30.9638$



$\theta = 329.0362°$

$360 - 30.9638$

i) (6, 0)



$\theta =$