8.2 Inverse Trigonometric Functions

SOH CAH TOA

★ **REMEMBER from 9.1 TrigFunction(angle/theta)=Ratio**

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Inverse Functions:

- The inverse sine of x ($\sin^{-1} x$) is the angle whose sine is x. If $\sin\theta = x$, then $\theta = \sin^{-1} x$.
- The inverse cosine of x ($\cos^{-1} x$) is the angle whose cosine is x. If $\cos\theta = x$, then $\theta = \cos^{-1} x$.
- The inverse tangent of x ($\tan^{-1} x$) is the angle whose tangent is x. If $\tan\theta = x$, then $\theta = \tan^{-1} x$.

★ **Use inverse functions when you know the sine, cosine, or tangent of an angle and <u>want to know how big the angle is.</u>**

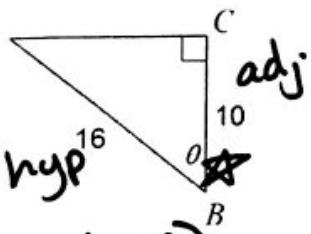
Use a calculator to find each angle measure to the nearest degree.

1. sin C = .2250 2. cos B= .1045 3. tan A = 1.2799

$C = \sin^{-1}(.2250) = 13°$ $B = \cos^{-1}(.1045) = 84°$ $A = \tan^{-1}(1.2799) = 52°$

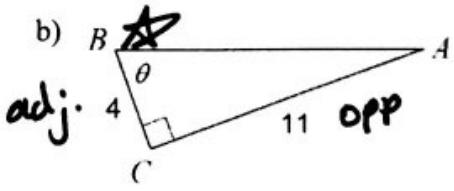
Examples: Find the measure of the indicated angle to the nearest tenth of a degree.

a)



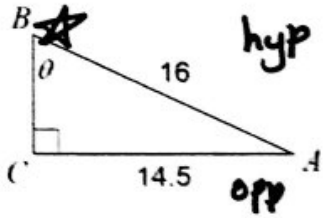
$\theta = \cos^{-1}\left(\frac{10}{16}\right) = 51.3°$

b)



$\theta = \tan^{-1}\left(\frac{11}{4}\right) = 70°$

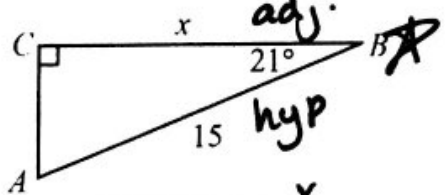
c)



$\theta = \sin^{-1}\left(\frac{14.5}{16}\right) = 65°$

Find the value of x in each figure. Round your answer to the nearest hundredth.

a)

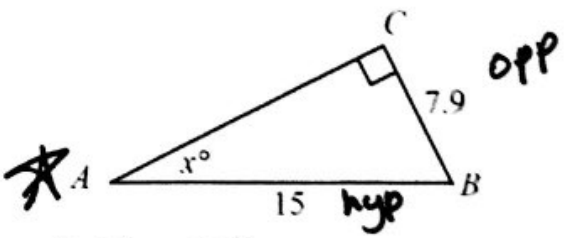


$\cos 21 = \frac{x}{15}$

$x = 15\cos 21$

$x = 14$

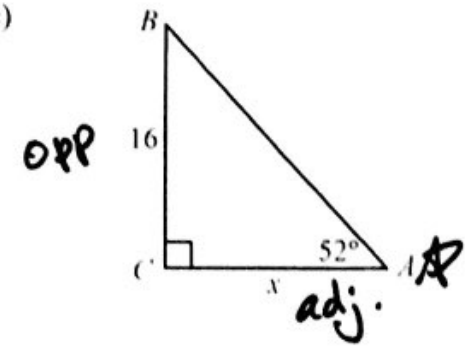
b)



$\sin x = \frac{7.9}{15}$

$x = \sin^{-1}(7.9/15) = 31.78°$

c)



opp

adj.

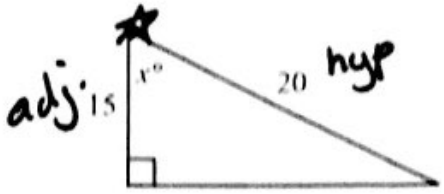
$\frac{\tan 52^\circ}{1} = \frac{16}{x}$

$x = 12.5$

$x \tan 52^\circ = 16$

$\frac{x \tan 52^\circ}{\tan 52^\circ} = \frac{16}{\tan 52^\circ}$

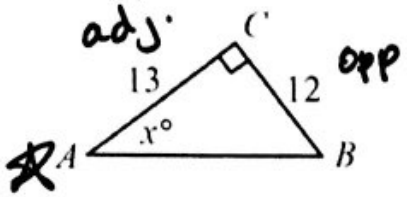
d)



$\cos x = \frac{15}{20}$

$x = \cos^{-1}\left(\frac{15}{20}\right) = 41.41^\circ$

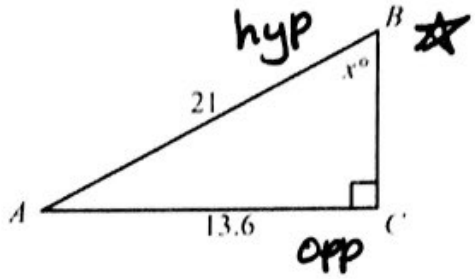
e)



$\tan x = \frac{12}{13}$

$x = \tan^{-1}(12/13) = 42.71^\circ$

f)

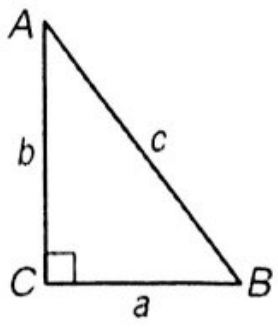


$\sin x = \frac{13.6}{21}$

$x = \sin^{-1}\left(\frac{13.6}{21}\right) = 40.36^\circ$

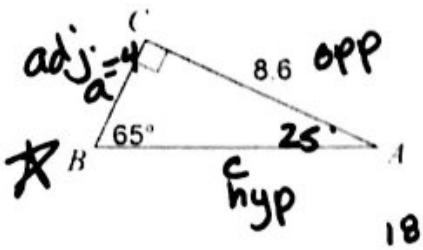
Solving a Triangle: Figuring out the lengths of all three sides and the measures of all three angles of a triangle.

- ★ If you know the lengths of two of the sides, use the Pythagorean Theorem to find the length of the third side.
- ★ If you know the measure of one of the acute angles, use the fact that the angles in a triangle add to 180° to find the measure of the other angle.
- ★ If you know the measure of one angle and the length of one side, use sin, cos, or tan to figure out the lengths of the other sides.
- ★ If you know the lengths of the sides and need to figure out the angle measures, use inverse functions ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).



Examples: ΔABC. Round answers to the nearest tenth. Show all your work.

a)



180-65-90

$m\angle A =$ 25°
$m\angle B =$ 65°
$m\angle C =$ 90°
$a =$ 4
$b =$ 8.6
$c =$ 9.5

$\tan 65 = \frac{8.6}{a}$

$\frac{a\tan 65}{\tan 65} = \frac{8.6}{\tan 65}$

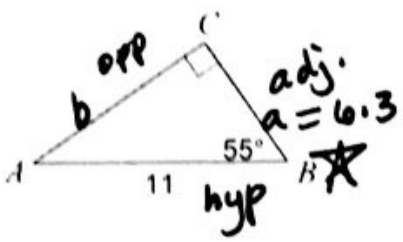
$a = 4$

$4^2 + 8.6^2 = c^2$

$89.96 = c^2$

$9.5 = c$

b)



180-55-90=35°

$m\angle A =$ 35°
$m\angle B =$ 55°
$m\angle C =$ 90°
$a =$ 6.3
$b =$ 9
$c =$ 11

$\cos 55 = \frac{a}{11}$

$a = 11\cos 55$

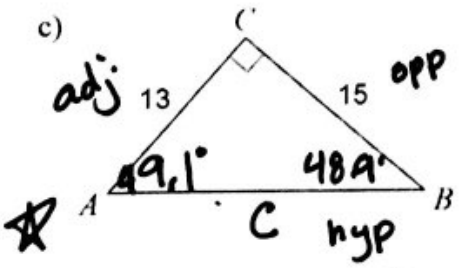
$a = 6.3$

$b^2 + 6.3^2 = 11^2$

$b^2 = 81.31$

$b = 9$

c)



$m\angle A =$ 49.1°
$m\angle B =$ 48.9
$m\angle C =$ 90°
$a =$ 15
$b =$ 13
$c =$ 19.8

$\tan A = \frac{15}{13}$

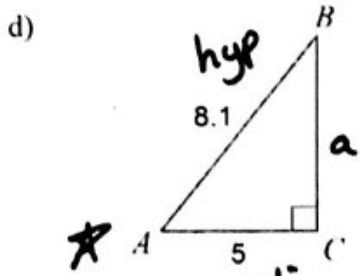
$A = \tan^{-1}\left(\frac{15}{13}\right) = 49.1°$

$13^2 + 15^2 = c^2$

$\sqrt{394} = \sqrt{c^2}$

$19.8 = c$

d)



$m\angle A =$ 51.9° adj.
$m\angle B =$ 38.1°
$m\angle C =$ 90°
$a =$ 6.4
$b =$ 5
$c =$ 8.1

$\cos A = \frac{5}{8.1}$

$A = \cos^{-1}\left(\frac{5}{8.1}\right)$

$A = 51.9$

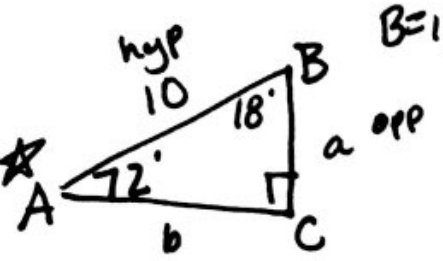
180-51.9-90

$5^2 + a^2 = 8.1^2$

$\sqrt{a^2} = \sqrt{40.61}$

$a = 6.4$

e) $m\angle A = 72°,\ c = 10$



$B = 180 - 90 - 72 = 18°$

$\sin 72 = \frac{a}{10}$

$a = 10\sin 72$

$a = 9.5$

$m\angle A = 72°$

$m\angle B = 18°$

$m\angle C = 90°$

$a = 9.5$

$b = 3.1$

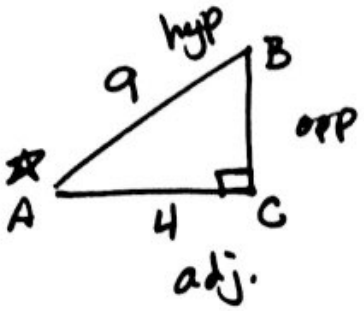
$c = 10$

$9.5^2 + b^2 = 10^2$

$b^2 = 9.75$

$b = 3.1$

f) $b = 4,\ c = 9$



$\cos A = \frac{4}{9}$

$A = \cos^{-1}(4/9)$

$A = 63.6°$

$B = 180 - 63.6 - 90 = 26.$

$m\angle A = 63.6°$

$m\angle B = 26.4$

$m\angle C = 90°$

$a = 8.1$

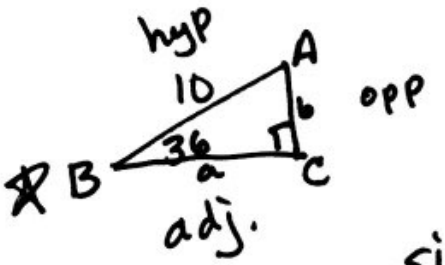
$b = 4$

$c = 9$

$4^2 + a^2 = 9^2$

$a = 8.1$

g) $m\angle B = 36°, c = 10$



$\sin 36 = \frac{b}{10}$

$10\sin 36 = b$

$b = 5.9$

$A = 180 - 90 - 36 = 54$

$m\angle A = 54°$

$m\angle B = 36°$

$m\angle C = 90°$

$a = 5.9$

$b = 8.1$

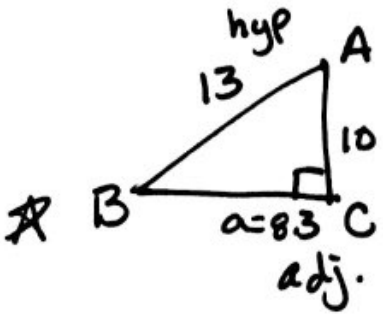
$c = 10$

$5.9^2 + b^2 = 10^2$

$b^2 = 65.19$

$b = 8.1$

h) $b = 10, c = 13$



$a^2 + 10^2 = 13^2$

$a = 8.3$

$\sin B = \frac{10}{13}$

$B = \sin^{-1}\left(\frac{10}{13}\right)$

$B = 50.3$

$A = 180 - 90 - 50.3 = 39.7°$

$m\angle A = 39.7°$

$m\angle B = 50.3°$

$m\angle C = 90$

$a = 8.3$

$b = 10$

$c = 13$