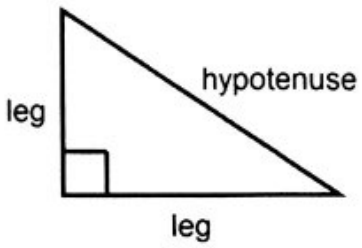
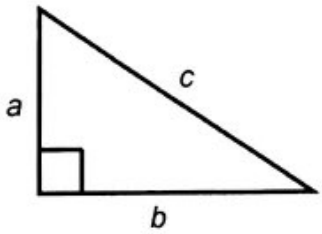
# 8.1 The Pythagorean Theorem/Trigonometric Ratios

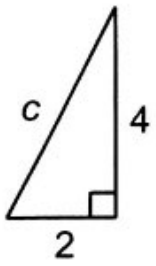
In a right triangle, $a^2 + b^2 = c^2$, or **leg² + leg² = hypotenuse².**




★ **The hypotenuse (the longest side – the one across from the right angle) should always be by itself on one side of the equation.**

**To find the length of the hypotenuse:**




$2^2 + 4^2 = c^2$

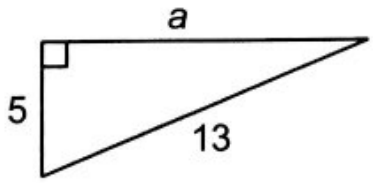
$4 + 16 = c^2$

$c^2 = 20$

$c = \sqrt{20} = \sqrt{2 \cdot 2 \cdot 5}$

$c = 2\sqrt{5} \approx 4.47$

**To find the length of a leg:**




$a^2 + 5^2 = 13^2$

$a^2 + 25 = 169$

$a^2 = 169 - 25$

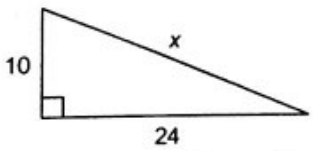
$a^2 = 144$

$a = \sqrt{144}$

$a = 12$

**Examples:** Find the length of the missing side of each triangle.

a)



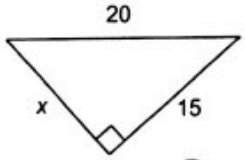

$10^2 + 24^2 = x^2$

$100 + 576 = x^2$

$\sqrt{676} = \sqrt{x^2}$

$26 = x$

b)




$x^2 + 15^2 = 20^2$

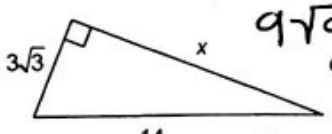
$x^2 + 225 = 400$

$\sqrt{x^2} = \sqrt{175}$

$x = \sqrt{5 \cdot 5 \cdot 7}$

$x = 5\sqrt{7} \approx 13.2$

c)




$3\sqrt{3} \cdot 3\sqrt{3}$

$9\sqrt{9}$ $3 \cdot 3$

$9 \cdot 3$

$x^2 + (3\sqrt{3})^2 = 14^2$

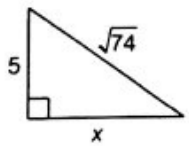
$x^2 + 9 \cdot 3 = 196$

$x^2 + 27 = 196$

$x^2 = 169$

$x = 13$

d)



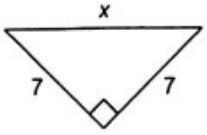

$x^2 + 5^2 = (\sqrt{74})^2$

$x^2 + 25 = 74$

$\sqrt{x^2} = \sqrt{49}$

$x = 7$

e)




$7^2 + 7^2 = x^2$

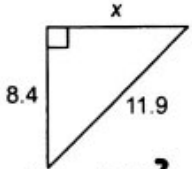
$49 + 49 = x^2$

$\sqrt{98} = \sqrt{x^2}$

$\sqrt{2 \cdot 7 \cdot 7} = x$

$9.9 \approx 7\sqrt{2} = x$

f)




$x^2 + 8.4^2 = 11.9^2$

$x^2 + 70.56 = 141.61$

$\sqrt{x^2} = \sqrt{71.05}$

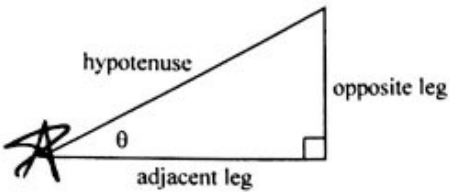
$x \approx 8.4$

**Right Triangle Trigonometric Ratios**

***Trigonometry:*** The study of the relationships among the sides and angles of right triangles.

***Trigonometric Ratio:*** A ratio of the lengths of two sides of a right triangle. The three main trigonometric rations are sine (sin), cosine (cos), and tangent (tan).

If $\theta$ is an acute angle of a right triangle, "adj" is the length of the leg adjacent (next to) $\theta$, "opp" is the length of the leg opposite $\theta$, and "hyp" is the length of the hypotenuse, then:




SOH CAH TOA

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

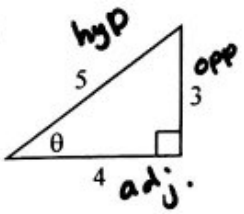
**A common way to remember this is SOH-CAH-TOA**

**Three other trig ratios:** cosecant $\csc\theta = \frac{\text{hyp}}{\text{opp}}$ secant $\sec\theta = \frac{\text{hyp}}{\text{adj}}$ cotangent $\cot\theta = \frac{\text{adj}}{\text{opp}}$

reciprocal of sine — reciprocal of cosine — reciprocal of tangent

**Examples:** Find the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$ and $\cot\theta$.

a)




opp= 3

adj= 4

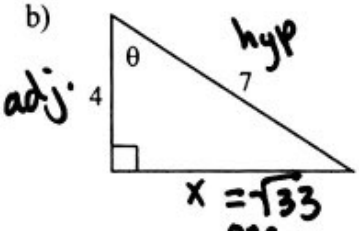
hyp= 5

SOH $\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$

CAH $\cos\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{4}$

TOA $\tan\theta = \frac{3}{4}$ $\cot\theta = \frac{4}{3}$

b)




opp= $\sqrt{33}$

adj= 4

hyp= 7

opp

$\sin\theta = \frac{\sqrt{33}}{7}$ $\csc\theta = \frac{7}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$

$\cos\theta = \frac{4}{7}$ $\sec\theta = \frac{7}{4}$

$\tan\theta = \frac{\sqrt{33}}{4}$ $\cot\theta = \frac{4}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

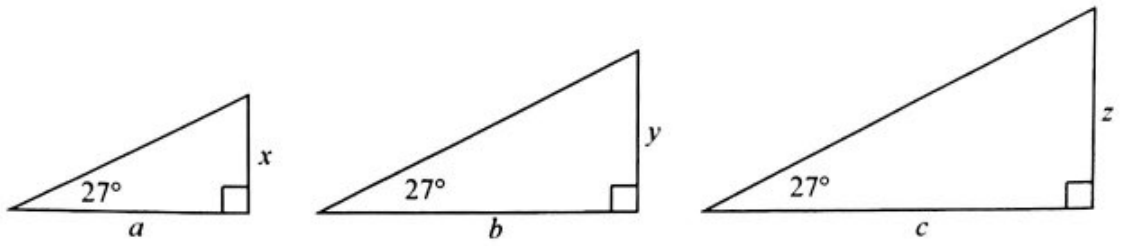
$x^2 + 4^2 = 7^2$

$x^2 + 16 = 49$

$\sqrt{x^2} = \sqrt{33}$

$x = \sqrt{33}$

No matter how big the triangle is, the values of the trigonometric functions for a certain size angle will remain the same. For example, in the diagram below, $\tan 27° = \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$. The value of the tangent is the same in all three triangles even though they are different sizes. The same is true for the sine and cosine.




**Examples:** Draw and label a triangle, find the length of the missing side, and find the requested values.

Find $sin\theta$, $cos\theta$, $tan\theta$, $csc\theta$, $sec\theta$, and $cot\theta$ if $\sin\theta = \frac{5}{13}$


$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$x^2 = 144$

$x = 12$

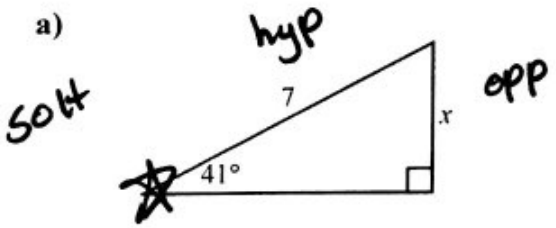
$\sin\theta = \frac{5}{13}$ $\quad \csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\quad \sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad \cot\theta = \frac{12}{5}$

**Identify which trigonometric ratio is needed to solve for the missing side. Write the correct equation, then solve. Round to the nearest hundredths.**

**a)**

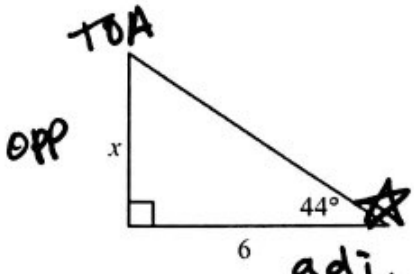



$\sin 41 = \frac{x}{7}$

$x = 7\sin 41$

$x = 4.59$

**b)**

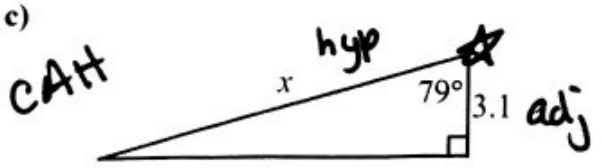



$\tan 44 = \frac{x}{6}$

$x = 6\tan 44$

$x = 5.79$

**c)**

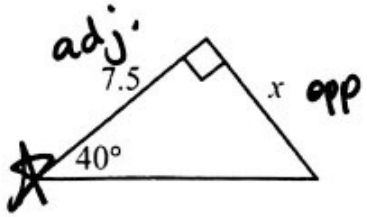



$\frac{\cos 79}{1} = \frac{3.1}{x}$

$\frac{x\cos 79}{\cos 79} = \frac{3.1}{\cos 79}$

$x = 16.25$

**d)**

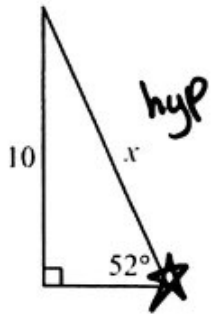



$\frac{\tan 40}{1} = \frac{x}{7.5}$

$x = 7.5\tan 40$

$x = 6.29$

e) SOH



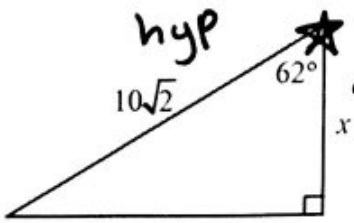

opp, hyp

$\frac{\sin 52}{1} = \frac{10}{x}$

$\frac{x \sin 52}{\sin 52} = \frac{10}{\sin 52}$

$x = 12.69$

f) CAH

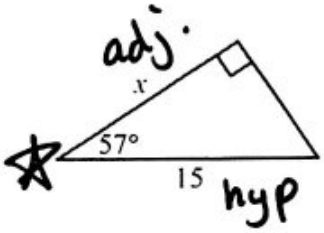



$\cos 62 = \frac{x}{10\sqrt{2}}$

$x = 10\sqrt{2} \cos 62$

$x = 6.64$

g) CAH

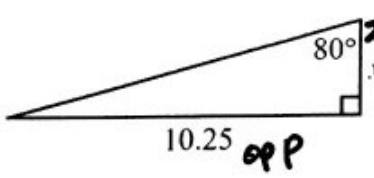



$\cos 57 = \frac{x}{15}$

$x = 15 \cos 57$

$x = 8.17$

h) TOA




$\tan 80 = \frac{10.25}{x}$

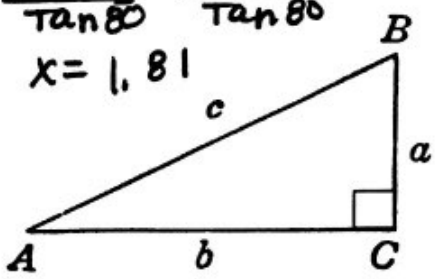
$\frac{x \tan 80}{\tan 80} = \frac{10.25}{\tan 80}$

$x = 1.81$

**Triangle Notation:**

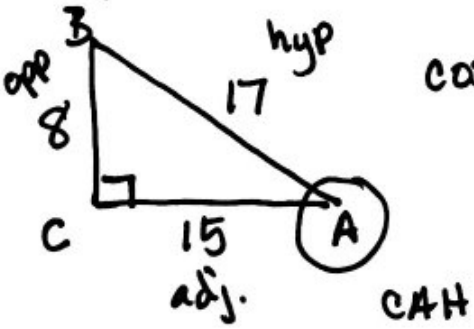
**When labeling a triangle, there are a few things to remember.**

1. **Capital letters refer to angles**
2. **Lower case letters are the side opposite their capital letter angle.**
3. **In a right triangle, "C" is 90°and "c" is the hypotenuese**




**In each triangle ABC, angle C is a right angle. Find the value of the trig function indicated.**

a) Find Find cosA if $a = 8$, $c = 17$, $b = 15$.

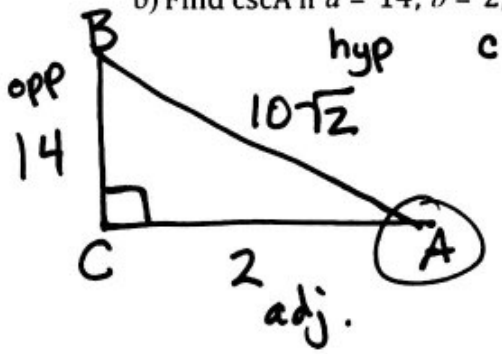



CAH

$\cos A = \frac{15}{17}$

b) Find csc A if $a = 14$, $b = 2$, and $c = 10\sqrt{2}$.

SOH




$\csc = \frac{H}{O}$ Inverse sine

$\csc A = \frac{5}{\frac{10\sqrt{2}}{14}}$

$\csc A = \frac{5\sqrt{2}}{7}$

**Examples:** Use a calculator to approximate each value to four decimal places. Make sure your calculator is in degree mode.

a) sin 120° .8660

b) cos 350° .9848

c) tan −30° −.5774

d) cot 280° $\frac{1}{\tan 280} = -.1763$

e) sec 360° $\frac{1}{\cos 360} = 1$

e) csc 360° $\frac{1}{\sin 360} =$ undefined