2.5 Simplifying with Complex Numbers

Imaginary Numbers

For centuries, mathematicians kept running into problems that required them to take the square roots of negative numbers in the process of finding a solution. None of the numbers that mathematicians were used to dealing with (the "real" numbers) could be multiplied by themselves to give a negative. These square roots of negative numbers were a new type of number. The French mathematician René Descartes named these numbers "imaginary" numbers in 1637. Unfortunately, the name "imaginary" makes it sound like imaginary numbers don't exist. They do exist, but they seem strange to us because most of us don't use them in day-to-day life, so we have a hard time visualizing what they mean. However, imaginary numbers are extremely useful (especially in electrical engineering) and make many of the technologies we use today (radio, electrical circuits) possible.

The number i: i is the number whose square is -1. That is $\boxed{i=\sqrt{-1}}$ and $\boxed{i^2=-1}$.

We define the square root of a negative number as follows:

$$\sqrt{-x}=\sqrt{-1\cdot x}=\sqrt{-1}\cdot\sqrt{x}=i\sqrt{x} \text{ or } \sqrt{x}\cdot i.$$

Examples: Express in terms of i.

a) $\sqrt{-64}$

$\sqrt{+64\cdot -1}$

$\sqrt{64}\cdot\sqrt{-1}$

$i\sqrt{64}$

$i\cdot 8$ or $\boxed{8i}$

b) $\sqrt{-12}$

$\sqrt{-1\cdot 12}$

$i\sqrt{12}$

$2\cdot 2\cdot 3$

$\boxed{2i\sqrt{3}}$

c) $-\sqrt{+49}$

$-1\cdot i\cdot\sqrt{49}$

$-1\cdot i\cdot 7$

$\boxed{-7i}$

d) $-\sqrt{+18}$

$-1\cdot i\sqrt{18}$

$3\cdot 3\cdot 2$

$-1\cdot i\cdot 3\sqrt{2}$

$\boxed{-3i\sqrt{2}}$

Imaginary Number: A number that can be written in the form $a+bi$, where a and b are real numbers and $b\neq 0$.
Any number with an i in it is imaginary.

Complex Number: A number that can be written in the form $\boxed{a+bi}$ where a and b are real numbers. (a or b or both can be 0.) **The set of complex numbers is the set containing all of the real numbers and all of the imaginary numbers.**

Adding or Subtracting Complex Numbers

★ i acts like any other variable in addition and subtraction problems. Distribute any negative signs and combine like terms (add or subtract the real parts and add or subtract the imaginary parts). Write your answer with the real part first, then the imaginary part.

Subtract means add the opposite

Examples: Add or subtract and simplify.

a) $(2+5i)+(1-3i)$ $\boxed{3+2i}$

b) $(4-3i)+(+2+5i)$ $\boxed{6-8i}$

c) $(-3-7i)+(+6)$ $\boxed{3-7i}$

d) $5i+(-1+i)$ $\boxed{-1+6i}$

Multiplying Complex Numbers

$i=\sqrt{-1}$

$i^2=(\sqrt{-1})^2=-1$

Multiplying Complex Numbers:

- To multiply imaginary numbers, first write any square roots of negative numbers in terms of i.
- Multiply as usual by distributing, FOILing, and using exponent rules. Treat i like any other variable.
- Use the fact that $\boxed{i^2=-1}$. Anywhere you see an i^2, change it to a -1.
  - $8i^2=8(-1)=-8$
  - $-3i^2=(-3)(-1)=3$

* simplify first and then multiply!

Examples: Multiply and simplify. If the answer is imaginary, write it in the form $a+bi$.

* can't multiply if radicands are negative.

a) $\sqrt{-9}\cdot\sqrt{-4}$

$3i\cdot 2i$

$6i^2$

$6(-1)=\boxed{-6}$

b) $\sqrt{-3}\cdot\sqrt{-5}$

$i\sqrt{3}\cdot i\sqrt{5}$

$i^2\sqrt{15}$

$-1\cdot\sqrt{15}=\boxed{-\sqrt{15}}$

c) $-2i\cdot 7i$

$-14i^2$

$-14(-1)$

$\boxed{14}$

d) $-3i\cdot i\sqrt{5}$

$-3i^2\sqrt{5}$

$-3(-1)\sqrt{5}$

$\boxed{3\sqrt{5}}$

e) $3i(2-i)$

$6i-3i^2$

$-3(-1)$

$6i+3 \Rightarrow \boxed{3+6i}$

f) $(7+3i)(9-8i)$

$63-56i+27i-24i^2$

$+24$

$63-29i-24(-1)=\boxed{87-29i}$

g) $(2-i)^2$ rewrite it

$(2-i)(2-i)$

$4-2i-2i+i^2$

$4-4i-1$

$\boxed{3-4i}$

h) $(3-4i)(3+4i)$

$9+12i-12i-16i^2$

$9-16i^2=9+16$

(-1)

$\boxed{25}$

Simplify a Power of i: Express the given power of i in terms of powers of i^2, and use the fact that $\boxed{i^2=-1}$

Examples: Simplify each expression.

Exp. before multiply

a) i^{22}

$(i^2)^{11}$

$(-1)^{11}=\boxed{-1}$

b) i^{33}

$i\cdot i^{32}$

$i\cdot(i^2)^{16}$

$i(-1)^{16}=i\cdot 1=\boxed{i}$

c) i^{72}

$(i^2)^{36}$

$(-1)^{36}=\boxed{1}$

d) i^{47}

$i\cdot i^{46}$

$i(i^2)^{23}$

$i\cdot(-1)^{23}$

$i\cdot -1=\boxed{-i}$