2.4 Radical Expressions, Multiply and Divide (Rationalizing the Denominator)

$(4\sqrt{3})^2$

Question: Can you ~~add and~~ subtract radicals the same way you multiply and divide them?

e.g.) Since $\boxed{\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}}$, does $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$? **NO!!!!!!!!!!**

Don't make the following mistakes:

- $\sqrt{x^2-4} \ne x-2$
- $(\sqrt{x}+\sqrt{y})^2 \ne x+y$

Multiplying Radical Expressions: Use the Product Property. Use the Distributive Property and FOIL to multiply radical expressions with more than one term.

Examples: Multiply and Simplify.

a) $\sqrt{3}(5+\sqrt{30})$

30 · 3

$5\sqrt{3}+\sqrt{90}$

$\sqrt{3\cdot3\cdot5\cdot2}$

$\boxed{5\sqrt{3}+3\sqrt{10}}$

b) $\sqrt{2}(\sqrt{6}-3\sqrt{2})$

$\sqrt{12}-3\sqrt{4}$

$\sqrt{2\cdot2\cdot3}$ -6

$\boxed{2\sqrt{3}-6}$

c) $(\sqrt{5}-\sqrt{6})(\sqrt{7}+1)$

$\sqrt{35}+1\sqrt{5}-\sqrt{42}-1\sqrt{6}$

7·5   7·2·3

$\boxed{\sqrt{35}+\sqrt{5}-\sqrt{42}-\sqrt{6}}$

d) $(5\sqrt{3}-4\sqrt{2})(\sqrt{3}+\sqrt{2})$

$5\sqrt{9}+5\sqrt{6}-4\sqrt{6}-4\sqrt{4}$

15   -8

$\boxed{7+\sqrt{6}}$

e) $(4\sqrt{3}-1)^2$ rewrite it and FOIL

$\boxed{(4\sqrt{3}-1)(4\sqrt{3}-1)}$

$16\sqrt{9}-4\sqrt{3}-4\sqrt{3}+1$

48   +1

$\boxed{49-8\sqrt{3}}$

f) $(\sqrt{2}+5)(\sqrt{2}-5)$

$2\ \sqrt{4}-5\sqrt{2}+5\sqrt{2}-25$

$\boxed{-23}$

Dividing Radicals ÷

The Quotient Rule for Radicals

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, where $b\ne 0$, $\boxed{\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$.

Examples: Simplify.

a) $\sqrt{\frac{9}{25}}=\frac{\sqrt{9}}{\sqrt{25}}=\boxed{\frac{3}{5}}$

b) $\sqrt[3]{\frac{x^3}{27}}=\frac{\sqrt[3]{x^3}}{\sqrt[3]{27}}=\boxed{\frac{x}{3}}$

3·3·3

c) $\sqrt{\frac{m^3}{16n^2}}=\frac{\sqrt{m^3}}{\sqrt{16n^2}}=\boxed{\frac{m\sqrt{m}}{4n}}$

mmm

d) $\sqrt[3]{\frac{250y^{11}}{8x^6}}=\frac{\sqrt[3]{2\cdot5\cdot5\cdot5\, y^{11}}}{\sqrt[3]{2\cdot2\cdot2\, x^6}}$

$=\boxed{\frac{5y^3\sqrt[3]{2y^2}}{2x^2}}$

250 → 125 · 2 → 5·5·5

Examples: Divide and, if possible, simplify.

a) $\dfrac{\sqrt{\cancel{72}\,36}}{\sqrt{\cancel{2}\,1}} = \dfrac{\sqrt{36}}{\sqrt{1}} = \dfrac{6}{1} = \boxed{6}$ b) $\dfrac{\sqrt{\cancel{50}\,25\,x}}{2\sqrt{\cancel{2}\,1}} = \dfrac{\sqrt{25x}}{2\sqrt{1}} = \boxed{\dfrac{5\sqrt{x}}{2}}$ c) $\dfrac{7\sqrt[3]{\cancel{48}\,8\,x^4y^{8-2}}}{\sqrt[3]{\cancel{6}\,1\,y^2}}$

$\dfrac{7\sqrt[3]{8x^4y^6}}{\sqrt[3]{1}}$

$y^2\,x \cdot 2 \cdot 7\sqrt[3]{\cancel{2\cdot2\cdot2}\,\cancel{xxx}\,x\,\cancel{y^6}}$

$\boxed{14xy^2\sqrt[3]{x}}$

Rationalizing Denominators with One Term:

Rationalizing the denominator means to write the expression as an equivalent expression but without a radical in the denominator. To do this, multiply by 1 under the radical or multiply by 1 outside the radical to make the denominator a perfect power.

$\dfrac{\sqrt{2}}{\sqrt{3}} \cdot 1 = \dfrac{\sqrt{2}}{\sqrt{3}}$

Examples: Rationalize each denominator.

a) $\sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{6}}{\cancel{\sqrt{9}}\,3}$ $\boxed{\dfrac{\sqrt{6}}{3}}$

b) $\dfrac{5}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{5\sqrt{3}}{\cancel{\sqrt{9}}\,3}$ $\boxed{\dfrac{5\sqrt{3}}{3}}$

c) $\dfrac{5}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{5\sqrt{5}}{2\cancel{\sqrt{25}}\,5}$ $\dfrac{\cancel{5}\,1\sqrt{5}}{2\,\cancel{10}}$ $\boxed{\dfrac{\sqrt{5}}{2}}$

d) $\dfrac{3-\sqrt{5}}{\sqrt{11}} \cdot \dfrac{\sqrt{11}}{\sqrt{11}}$ $\boxed{\dfrac{3\sqrt{11}-\sqrt{55}}{11}}$

Rationalizing Denominators with Two Terms:

To do this, multiply by 1 under the radical or multiply by 1 outside the radical to make the denominator a perfect power. However, since the denominator now has two terms, we will have to multiply by the **conjugate** of the denominator.

Conjugate of a binomial Radical Expression: Conjugates have the same first term, with the second terms being opposites. For example, these two expressions are conjugates: $(3-\sqrt{2})$ and $(3+\sqrt{2})$.

What happens when you multiply these conjugates together?

$(3-\sqrt{2})(3+\sqrt{2}) =$

$9 \cancel{+3\sqrt{2}} \cancel{-3\sqrt{2}} - \sqrt{4}$

$9 - 2 = \boxed{7}$

Examples: Find the conjugate of each number.

a) $4+\sqrt{5}$

$4-\sqrt{5}$

b) $-3-\sqrt{7}$

$-3+\sqrt{7}$

c) $\sqrt{15}$

Examples: Rationalize each denominator by multiplying by the conjugate.

a) $\dfrac{4}{(2+\sqrt{2})}\cdot\dfrac{(2-\sqrt{2})}{(2-\sqrt{2})}$ FOIL

$$\frac{8-4\sqrt{2}}{4\cancel{-2\sqrt{2}+2\sqrt{2}}-\cancel{\sqrt{4}}^{2}}$$

2

$$\frac{8^{\div 2}-4^{\div 2}\sqrt{2}}{2\div 2}$$ ← Simplify

$$\frac{4-2\sqrt{2}}{1}$$

$$\boxed{4-2\sqrt{2}}$$

b) $\dfrac{5}{(8-\sqrt{3})}\cdot\dfrac{(8+\sqrt{3})}{(8+\sqrt{3})}$

$$\frac{40+5\sqrt{3}}{64\cancel{+8\sqrt{3}-8\sqrt{3}}-\cancel{\sqrt{9}}_{3}}$$

$$\boxed{\frac{40+5\sqrt{3}}{61}}$$

c) $\dfrac{(5-\sqrt{3})}{(2+\sqrt{5})}\cdot\dfrac{(2-\sqrt{5})}{(2-\sqrt{5})}$ FOIL FOIL