1, 4, 9, 81, $\frac{1}{36}$ , $\frac{16}{25}$ , etc. 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169  Examples: Simplify each of the following:
a) $\sqrt[4]{196} = 14$ b) $\sqrt{625}$ 25 c) $\sqrt[49]{81} = 49$ $\sqrt[4]{9}$
<ul> <li>nth Root: A number that you raise to the nth power (multiply by itself n times) to end up with a is called an nth root of a. In symbols, k = <sup>n</sup>√a if k" = a.</li> <li>Index: In the expression <sup>n</sup>√a, n is called the index. It tells you what root to take.</li> </ul>
Examples: Simplify each expression, if possible.  a) $\sqrt[3]{125}$ 5 b) $\sqrt[481]{81}$ 2 c) $\sqrt[5]{32}$ 2 d) $\sqrt[3]{8x^6y^3}$ 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
$\frac{3 - 125}{-5 \cdot -5 \cdot -5}$ Simplified Radical Expressions:

Square Root: A number that you square (multiply by itself) to end up with a is called a square root

TZSR ZSis radicand

Example **Radical Sign:** The symbol  $\sqrt{\ }$ . The radical sign is used to indicate the *principal* (positive) square

Perfect squares: Numbers that are the squares of rational numbers. Examples:

## To Simplify a Radical Expression with Index n Using a Factor Tree:

Rewrite the expression as the product of separate nth roots.

1. Make a factor tree. Split the radicand into its prime factors.

No perfect nth power factors in the radicand No exponents in the radicand bigger than the index

To Simplify a Radical Expression with Index n by Factoring:

2.3. Simplifying Radical Expressions

of a. In symbols,  $k = \sqrt{a}$  if  $k^2 = a$ .

root of the number over which it appears.

Radicand: The number under the radical sign.

2. Circle groups of n identical factors.

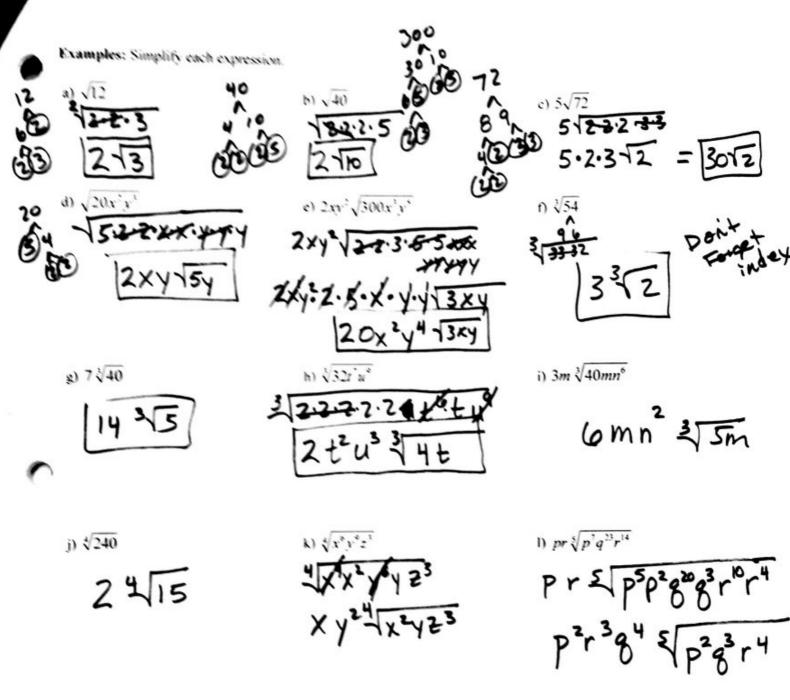
No fractions in the radicand The index is as small as possible

3. List the number or variable from each group only once outside the radical.

3. Simplify each expression containing the nth root of a perfect nth power.

- 4. Leave factors that are not part of a group under the radical.
- 5. Multiply the factors outside of the radical together. Do the same for the factors under the radical.

1. Write the radicand as the product of perfect nth powers and factors that are not perfect nth powers.



## Operations with Radicals

## Adding and Subtracting Radicals:

- Simplify each radical completely.
- Combine like radicals. When you add or subtract radicals, you can only combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.
   2x+3x = 5x

Like Radicals: Radicals with the same index and the same radicand

Examples: Determine whether the following are like radicals. If they are not, explain why not.

a) \$3 and \$2 NO

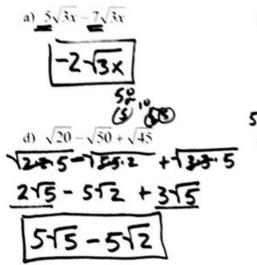
Different radioands

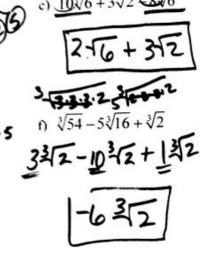
b) 4\5 and -3\5 Yes

same radioands same trulex c) A and Q V No

Different indexes

Examples: Add or subtract.





Don't make the following mistakes:

• 
$$\sqrt{2} + \sqrt{5} \neq \sqrt{7}$$

• 
$$\sqrt{9+16} \neq 3+4$$

• 
$$\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$$

\* must have some index Multiplying Radicals

The Product Rule for Radicals:

For any real numbers  $\sqrt[a]{a}$  and  $\sqrt[a]{b}$ ,  $\sqrt[a]{a} \cdot \sqrt[a]{b} = \sqrt[a]{a \cdot b}$ .

Caution: The product rule doesn't work if you are trying to multiply the even roots of negative numbers, because those roots are not real numbers. For example,  $\sqrt{-2} \cdot \sqrt{-8} \neq \sqrt{16}$ .

Re-write the radical in terms of i first, and then multiply.

For example, 
$$\sqrt{-2} \cdot \sqrt{-8} = i\sqrt{2} \cdot i\sqrt{8} = i^2 \sqrt{16} = (-1) \cdot \sqrt{16} = -4$$

Caution: The product only applies when the radicals have the same index:  $\sqrt[3]{5} \cdot \sqrt[4]{6} \neq \sqrt[12]{30}$ .

a)  $\sqrt{7} \cdot \sqrt{5}$ 

Examples: Multiply.

b) 
$$5\sqrt{2}\cdot\sqrt{8}$$

$$\frac{1}{3} \sqrt{3} \cdot \sqrt{3} = 3$$

h) 
$$2\sqrt[3]{10} \cdot 6\sqrt[3]{25}$$