2.3 Simplifying Radical Expressions

$\sqrt{\ }$ or $\sqrt[2]{\ }$

- ***Square Root:*** A number that you square (multiply by itself) to end up with a is called a square root of a. In symbols, $k = \sqrt{a}$ if $k^2 = a$. Example $\sqrt{25}$ $5 \cdot 5$ $= \boxed{5}$
- ***Radical Sign:*** The symbol $\sqrt{\ }$. The radical sign is used to indicate the *principal* (positive) square root of the number over which it appears.
- ***Radicand:*** The ~~number under the radi~~cal sign. $\sqrt{25}$ R 25 is radicand
- ***Perfect squares:*** Numbers that are the squares of rational numbers. Examples: 1, 4, 9, 81, $\frac{1}{36}$, $\frac{16}{25}$, etc. 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Examples: Simplify each of the following:

a) $\sqrt[2]{196} = \boxed{14}$ 14·14

b) $\sqrt{625}$ $\boxed{25}$ 25·25

c) $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \boxed{\frac{7}{9}}$

d) $\sqrt{y^4}$ $\boxed{y^2}$ $y^2 y^2$

e) $\sqrt{z^{14}}$ $\boxed{z^7}$ $z^7 \cdot z^7$

- ***nth Root:*** A number that you raise to the nth power (multiply by itself n times) to end up with a is called an nth root of a. In symbols, $k = \sqrt[n]{a}$ if $k^n = a$. $\sqrt[n]{\ }$ Example $\sqrt[4]{16}$ 4th root of 16
- ***Index:*** In the expression $\sqrt[n]{a}$, n is called the ***index***. It tells you what root to take.

$\sqrt[4]{16}$ 4 is the index

Examples: Simplify each expression, if possible.

a) $\sqrt[3]{125}$ $\boxed{5}$ $\underline{5} \cdot \underline{5} \cdot \underline{5}$

$\sqrt[3]{-125}$ $\boxed{-5}$ $\underline{-5} \cdot \underline{-5} \cdot \underline{-5}$

b) $\sqrt[4]{81}$ $\boxed{3}$ $\underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3}$

c) $\sqrt[5]{32}$ $\boxed{2}$ $\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2}$

d) $\sqrt[3]{8x^6y^3}$ $\underline{2 \cdot 2 \cdot 2}$ $\underline{x^2 x^2 x^2}$ $\underline{y\, y\, y}$ $\boxed{2x^2y}$

Simplified Radical Expressions:

- No perfect nth power factors in the radicand
- No exponents in the radicand bigger than the index
- No fractions in the radicand
- The index is as small as possible

To Simplify a Radical Expression with Index *n* by Factoring:

1. Write the radicand as the product of perfect nth powers and factors that are not perfect nth powers.
2. Rewrite the expression as the product of separate nth roots.
3. Simplify each expression containing the nth root of a perfect nth power.

To Simplify a Radical Expression with Index *n* Using a Factor Tree:

1. Make a factor tree. Split the radicand into its prime factors.
2. Circle groups of n identical factors.
3. List the number or variable from each group only *once* outside the radical.
4. Leave factors that are not part of a group under the radical.
5. Multiply the factors outside of the radical together. Do the same for the factors under the radical.

Examples: Simplify each expression.

a) $\sqrt{12}$

$\sqrt[2]{\cancel{2}\cdot\cancel{2}\cdot 3}$

$\boxed{2\sqrt{3}}$

b) $\sqrt{40}$

$\sqrt{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 5}$

$\boxed{2\sqrt{10}}$

c) $5\sqrt{72}$

$5\sqrt{\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{3}}$

$5\cdot 2\cdot 3\sqrt{2} = \boxed{30\sqrt{2}}$

d) $\sqrt{20x^2y^3}$

$\sqrt{5\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{y}\cdot\cancel{y}\cdot y}$

$\boxed{2xy\sqrt{5y}}$

e) $2xy^2\sqrt{300x^3y^5}$

$2xy^2\sqrt{\cancel{2}\cdot\cancel{2}\cdot 3\cdot\cancel{5}\cdot\cancel{5}\,\cancel{xx}x\,\cancel{yyyy}y}$

$2xy^2\cdot 2\cdot 5\cdot x\cdot y\cdot y\sqrt{3xy}$

$\boxed{20x^2y^4\sqrt{3xy}}$

f) $\sqrt[3]{54}$

$\sqrt[3]{\cancel{3}\cancel{3}\cancel{3}\,2}$

$\boxed{3\sqrt[3]{2}}$

Don't Forget index

g) $7\sqrt[3]{40}$

$\boxed{14\sqrt[3]{5}}$

h) $\sqrt[3]{32t^7u^9}$

$\sqrt[3]{\cancel{2\cdot 2\cdot 2}\cdot 2\cdot 2\,\cancel{t^6}\,t\,\cancel{u^9}}$

$\boxed{2t^2u^3\sqrt[3]{4t}}$

i) $3m\sqrt[3]{40mn^6}$

$6mn^2\sqrt[3]{5m}$

j) $\sqrt[4]{240}$

$2\sqrt[4]{15}$

k) $\sqrt[4]{x^6y^9z^3}$

$\sqrt[4]{\cancel{x^4}x^2\cancel{y^8}y\,z^3}$

$xy^2\sqrt[4]{x^2yz^3}$

l) $pr\sqrt[5]{p^7q^{23}r^{14}}$

$pr\sqrt[5]{p^5p^2q^{20}q^3r^{10}r^4}$

$p^2r^3q^4\sqrt[5]{p^2q^3r^4}$

Operations with Radicals

Adding and Subtracting Radicals:

1. Simplify each radical completely.
2. Combine like radicals. When you add or subtract radicals, you can *only* combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.

$2x + 3x = 5x$

Like Radicals: Radicals with the same index *and* the same radicand.

Examples: Determine whether the following are like radicals. If they are not, explain why not.

a) $\sqrt[2]{3}$ and $\sqrt[2]{2}$ NO

Different radicands

b) $4\sqrt{5}$ and $-3\sqrt{5}$ Yes

same radicands
same index

c) $2\sqrt[4]{x}$ and $\sqrt[6]{x}$ NO

Different indexes

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Examples: Add or subtract.

a) $5\sqrt{3x} - 7\sqrt{3x}$

$\boxed{-2\sqrt{3x}}$

b) $4\sqrt{11} + 8\sqrt{11}$

$\boxed{12\sqrt{11}}$

c) $10\sqrt{6} + 3\sqrt{2} - 8\sqrt{6}$

$\boxed{2\sqrt{6} + 3\sqrt{2}}$

d) $\sqrt{20} - \sqrt{50} + \sqrt{45}$

$\sqrt{2\cdot2\cdot5} - \sqrt{5\cdot5\cdot2} + \sqrt{3\cdot3\cdot5}$

$2\sqrt{5} - 5\sqrt{2} + 3\sqrt{5}$

$\boxed{5\sqrt{5} - 5\sqrt{2}}$

e) $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$

$10\sqrt{2} + 4\sqrt{5\cdot5\cdot5\cdot2\cdot2}$

$10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$

$\boxed{10\sqrt{2} + 10\sqrt{5}}$

f) $\sqrt[3]{54} - 5\sqrt[3]{16} + \sqrt[3]{2}$

$3\sqrt[3]{2} - 10\sqrt[3]{2} + 1\sqrt[3]{2}$

$\boxed{-6\sqrt[3]{2}}$

Don't make the following mistakes:

- $\sqrt{2} + \sqrt{5} \neq \sqrt{7}$
- $\sqrt{9+16} \neq 3+4$
- $\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$

Multiplying Radicals * must have same index

The Product Rule for Radicals:

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{a\cdot b}$.

Caution: The product rule doesn't work if you are trying to multiply the even roots of <u>negative</u> numbers, because those roots are not real numbers. For example, $\sqrt{-2}\cdot\sqrt{-8} \neq \sqrt{16}$.

Re-write the radical in terms of i first, and then multiply.
For example, $\sqrt{-2}\cdot\sqrt{-8} = i\sqrt{2}\cdot i\sqrt{8} = i^2\sqrt{16} = (-1)\cdot\sqrt{16} = -4$

Caution: The product only applies when the radicals have the same index: $\sqrt[3]{5}\cdot\sqrt[4]{6} \neq \sqrt[12]{30}$.

Examples: Multiply.

a) $\sqrt{7}\cdot\sqrt{5}$

$\sqrt{7\cdot5}$

$\boxed{\sqrt{35}}$

b) $5\sqrt{2}\cdot\sqrt{8}$

$5\sqrt{2\cdot8}$

$5\sqrt{16} = 5\sqrt{2\cdot2\cdot2\cdot2}$

$5\cdot4 = \boxed{20}$

c) $2\sqrt{5}\cdot7\sqrt{15}$

$14\sqrt{5\cdot5\cdot3}$

$70\sqrt{3}$

d) $\sqrt{3}\cdot\sqrt{3}$ $= 3$

$\sqrt{3\cdot3}$

$\sqrt{9} = \boxed{3}$

e) $(\sqrt{8})^2$

$\sqrt{8}\cdot\sqrt{8} = \sqrt{64}$

$\sqrt{8\cdot8} = \boxed{8}$

f) $(3\sqrt{11})^2$

$3\sqrt{11}\cdot3\sqrt{11}$

$9\sqrt{11\cdot11}$

$9\cdot11 = \boxed{99}$

g) $\sqrt[3]{3}\cdot\sqrt[3]{9}$

$\sqrt[3]{3\cdot3\cdot3}$

$\boxed{3}$

h) $2\sqrt[3]{10}\cdot6\sqrt[3]{25}$

$5\cdot12\sqrt[3]{2\cdot5\cdot5\cdot5}$

$\boxed{60\sqrt[3]{2}}$