2.2

Rules of Exponents

The following properties are true for all real numbers a and b and all integers m and n, provided that no denominators are 0 and that 0^0 is not considered.

1 as an exponent: $a^1 = a$ e.g.) $7^1 = 7,\ \pi^1 = \pi,\ (-10)^1 = -10$

0 as an exponent: $a^0 = 1$ e.g.) $2^0 = 1,\ 27^0 = 1,\ \left(-\frac{5}{8}\right)^0 = 1$

The Product Rule: $a^m \cdot a^n = a^{m+n}$ e.g.) $x^2 \cdot x^5 = x^{2+5} = x^7$

XX XXXXX

The Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n}$ e.g.) $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

multiply

The Power Rule: $(a^m)^n = a^{mn}$ e.g.) $(x^2)^5 = x^{(2)(5)} = x^{10}$

Raising a product to a power: $(ab)^n = a^n b^n$ e.g.) $(2k)^4 = 2^4 \cdot k^4 = 16k^4$

Raising a quotient to a power: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ e.g.) $\left(\dfrac{p}{q^2}\right)^3 = \dfrac{p^3}{(q^2)^3} = \dfrac{p^3}{q^6}$

Negative exponents: $a^{-n} = \dfrac{1}{a^n}$ e.g.) $2^{-3} = \dfrac{1}{2^3},\ 7x^3y^{-4} = \dfrac{7x^3}{y^4}$

$\dfrac{1}{a^{-n}} = a^n$ e.g.) $\dfrac{1}{x^{-9}} = x^9,\ \dfrac{b}{c^{-3}d} = \dfrac{bc^3}{d}$

$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$ e.g.) $\left(\dfrac{2}{v}\right)^{-3} = \left(\dfrac{v}{2}\right)^3 = \dfrac{v^3}{2^3} = \dfrac{v^3}{8}$

Rational exponents: $a^{n/d} = \sqrt[d]{a^n}$ e.g.) $2^{\frac{3}{5}} = \sqrt[5]{2^3}$

exponents negative exponents

To simplify an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Examples: Simplify the following expressions.

a) $m^{?} \cdot m^{?}$ m^{12}

b) [illegible]

$15a^{6}b^{8}$

c) [illegible] r^{6}

d) [illegible] $= \boxed{\frac{1}{p^{4}}}$

e) [illegible]

$\frac{2y}{3x^{4}}$

f) $(-2)^{4}$

$-2 \cdot -2 \cdot -2 \cdot -2$

$\boxed{16}$

g) -2^{4}

$-1(2 \cdot 2 \cdot 2 \cdot 2)$

$\boxed{-16}$

h) [illegible]

$5x^{-2}y^{2} = \boxed{\frac{5y^{2}}{x^{2}}}$

i) $\frac{1}{6^{-2}} = 6^{2} = \boxed{36}$

j) $9^{-3} \cdot 9^{8}$

$x^{-3} \cdot x^{8} = x^{5}$

$\boxed{9^{5} = 59049}$

k) [illegible]

$\frac{x^{5}y^{4}}{5}$

l) [illegible]

$\frac{y^{4}}{y^{5}} = \frac{1}{y}$

$\boxed{\frac{1}{y}}$

m) $(y^{-5})^{7} = y^{-35}$

$\boxed{\frac{1}{y^{35}}}$

n) $(-2x)^{3}$

$(-2)^{3}x^{3}$

$-2 \cdot -2 \cdot -2$

$\boxed{-8x^{3}}$

o) [illegible]

$3^{-2}x^{-10}y^{2}$

$\frac{y^{2}}{3^{2}x^{10}}$

$\boxed{\frac{y^{2}}{9x^{10}}}$

p) [illegible]

$\frac{y^{-6}z^{-9}}{5^{-3}} = \frac{5^{3}}{y^{6}z^{9}}$

$5 \cdot 5 \cdot 5$

$= \boxed{\frac{125}{y^{6}z^{9}}}$

✓ Fractional Exp.

2.2 Rational Exponents

If n is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number then $\boxed{a^{1/n} = \sqrt[n]{a}.}$

★ **The denominator of the exponent tells you what type of root to take.**

Examples: Write an equivalent expression using radical notation and, if possible, simplify.

a) $25^{1/2}$

$\sqrt[2]{25^1} = \sqrt{25}$

$5 \cdot 5$

$\boxed{5}$

b) $64^{1/3}$

$\sqrt[3]{64^1}$

$4 \cdot 4 \cdot 4$

$= \boxed{4}$

c) $(xy^2z)^{1/6}$

$\boxed{\sqrt[6]{xy^2z}}$

d) $(36x^{10})^{1/2}$

$\sqrt[2]{36x^{10}}$

$6 \cdot 6 \; x^5 x^5$

$\boxed{6x^5}$

e) $2x^{1/4}$

$2\sqrt[4]{x}$

f) $(2x)^{1/4}$

$\sqrt[4]{2x}$

Backwards

Fractional exponent

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[7]{2xy}$

$(2xy)^{1/7}$

b) $\sqrt[4]{\frac{ab^3}{7}}$

$\left(\frac{ab^3}{7}\right)^{1/4}$

c) $\sqrt[2]{3z}$

$(3z)^{1/2}$

d) $3\sqrt{z}$

$3z^{1/2}$

e) $\sqrt[5]{xy^2z}$

$(xy^2z)^{1/5}$

Positive Rational Exponents

If m and n are positive integers (where $n \neq 1$) and $\sqrt[n]{a}$ exists, then $\boxed{a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$

e.g.) $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ or $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

$2 \cdot 2 \cdot 2$ $4 \cdot 4 \cdot 4$

Examples: Write an equivalent expression using radical notation and simplify.

a) $t^{5/6}$

$\sqrt[6]{t^5}$

5 on outside or underneath

or $\left(\sqrt[6]{t}\right)^5$

b) $9^{3/2}$

$\left(\sqrt[2]{9}\right)^3$

3^3

$\boxed{27}$

c) $64^{2/3}$

$\left(\sqrt[3]{64}\right)^2$

4^2

$\boxed{16}$

d) $(2x)^{3/4}$

$\left(\sqrt[4]{2x}\right)^3$

or $\sqrt[4]{(2x)^3}$

e) $2x^{3/4}$

$2\sqrt[4]{x^3}$

or $2\left(\sqrt[4]{x}\right)^3$

Backwords

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[3]{x^5}$

$x^{5/3}$

b) $\sqrt[7]{9^2}$

$9^{2/7}$

c) $\left(\sqrt[5]{6n}\right)^3$

$(6n)^{3/5}$

d) $6\sqrt[5]{n^3}$

$6(n)^{3/5}$

same $6n^{3/5}$

e) $\left(\sqrt[4]{2m}\right)^2$

$(2m)^{2/4}$

$(2m)^{1/2}$

Negative Rational Exponents

For any rational number m/n, and any nonzero real number $a^{m/n}$, $\boxed{a^{-m/n} = \frac{1}{a^{m/n}}}$.

★ **The sign of the base is not affected by the sign of the exponent.**

Examples: Write an equivalent expression using positive exponents and, if possible, simplify.

a) $49^{-1/2}$ $= \frac{1}{49^{1/2}} = \frac{1}{\sqrt[2]{49}} = \boxed{\frac{1}{7}}$

b) $(3mn)^{-2/5}$ $\frac{1}{(3mn)^{2/5}}$ ↑ leave in parentheses

c) $7x^{-2/3}$ $= \frac{7}{x^{2/3}}$

Laws of Exponents: The laws of exponents apply to rational exponents as well as integer exponents.

Examples: Use the laws of exponents to simplify.

a) $2^{2/5} \cdot 2^{1/5}$ $2^{\frac{2}{5}+\frac{1}{5}} = \boxed{2^{\frac{3}{5}}}$

b) $\frac{x^{7/3}}{\cancel{x^{4/3}}}$ $-4/3$ $= x^{3/3} = x^1 = \boxed{x}$

c) $(19^{2/5})^{5/3}$ multiply $19^{\frac{2}{\cancel{5}}\cdot\frac{\cancel{5}}{3}}$ $\boxed{19^{\frac{2}{3}}}$

d) $x^{1/2} \cdot x^{2/3}$ ← common den. or calc. $x^{\frac{1}{2}+\frac{2}{3}}$ $x^{\frac{3}{6}+\frac{4}{6}} = \boxed{x^{\frac{7}{6}}}$

e) $\cancel{y^{-4/7}} \cdot y^{6/7}$

f) $\frac{z^{3/4}}{z^{2/5}}$ $z^{\frac{3}{4}-\frac{2}{5}}$ $z^{\frac{15}{20}-\frac{8}{20}}$ $\boxed{z^{\frac{7}{20}}}$

g) $\frac{x^{3/4} \cdot x^{1/6} \cdot y^{1-\frac{1}{2}}}{y^{1/2}}$ $\frac{3}{4}+\frac{1}{6} = \frac{11}{12}$ $\boxed{x^{11/12}y^{1/2}}$

h) $\frac{(2x^{2/5}y^{-1/3})^5}{x^2 y}$ $2^5 x^{\frac{2}{\cancel{5}}\cdot\frac{\cancel{5}}{1}} y^{-\frac{1}{3}\cdot\frac{5}{1}}$ $-\frac{5}{3}$ $\frac{32\cancel{x^2}}{\cancel{x^2}\, y^{\frac{3}{3}+\frac{5}{3}}}$ $\boxed{\frac{32}{y^{\frac{8}{3}}}}$

To Simplify Radical Expressions using the Rules of Exponents:

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

Examples: Use rational exponents to simplify. Do not use exponents that are fractions in the final answer.

a) $\sqrt[8]{z^4}$ $z^{4/8} = z^{1/2} = \sqrt[2]{z}$

b) $\left(\sqrt[3]{a^2bc^4}\right)^9$ $(a^2bc^4)^{9/3}$ $\boxed{a^6b^3c^{12}}$

c) $\sqrt{x} \cdot \sqrt[4]{x}$ $x^{\frac{1}{2}} \cdot x^{\frac{1}{4}}$ $x^{\frac{3}{4}} = \boxed{\sqrt[4]{x^3}}$

d) $\sqrt[6]{y^2} \cdot \sqrt[9]{y}$ $y^{2/6} \cdot y^{1/9}$ $y^{1/3} \cdot y^{1/9}$ $y^{1/3+1/9} = y^{4/9} = \boxed{\sqrt[9]{y^4}}$

e) $\frac{\sqrt[3]{k}}{\sqrt[3]{k^2}}$

f) $\frac{\sqrt[8]{m^4}}{\sqrt[6]{m}}$ $= \frac{m^{4/8}}{m^{1/6}}$ $1/2 - 1/6$ $m^{1/3}$ $\sqrt[3]{m}$

g) $\sqrt[4]{\sqrt[5]{x}}$ multiply $(x^{1/5})^{1/4}$ $x^{1/20}$ $\boxed{\sqrt[20]{x}}$

h) $\boxed{\sqrt[3]{2} \cdot \sqrt[5]{3}}$ $2^{1/3} \cdot 3^{1/5}$